AUSTRALASIAN JOURNAL OF COMBINATORICS
Volume **37** (2007), Pages 225–232

On the upper bound of the minimum length of 5-dimensional linear codes

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Abstract

We consider an upper bound of minimum length $n_q(5,d)$ of linear codes with dimension 5 using projective geometry, and we find a new upper bound: $n_q(5,d) \le g_q(5,d)+1$ for some values of d.

1 Introduction and Preliminaries

Let $\mathbb{F}_q$ be a finite field with q elements. An $[n,k,d]_q$ code is a linear subspace in $\mathbb{F}_q^n$ with dimension k and the minimum Hamming distance d over $\mathbb{F}_q$. Optimal linear code problem is to find $n_q(k,d)$, the smallest value n for which there exists an $[n,k,d]_q$ code for given k and d. The following bound is called the Griesmer bound $g_q(k,d)$ as a lower bound on $n_q(k,d)$;

$$n_q(k,d) \ge g_q(k,d) := \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil,$$

where $\lceil x \rceil$ denotes the smallest integer $\ge x$. A code C is called an optimal linear code if the above equality holds.

For given q and k, the following theorem provides a starting point for finding the value of $n_q(k,d)$ for each d.

Theorem 1 ([5]) *Let $d = sq^{k-1} - \sum_{i=1}^{p} q^{u_i-1}$ such that $k > u_1 \ge u_2 \ge \cdots \ge u_p$ with $u_i > u_{i+q-1}$ for $1 \le i \le p-q+1$, where $s = \lceil \frac{d}{q^{k-1}} \rceil$. If*

$$\sum_{i=1}^{\min\{s+1,p\}} u_i \le sk,$$

* This work was supported by the Korea Research Foundation Grant. (KRF-2004-037-C00004)

then $n_q(k,d) = g_q(k,d)$.

When $k = 1, 2$, obviously, $n_q(k,d) = g_q(k,d)$ for any d by Theorem 1. When $k = 3, 4$, the value $n_q(k,d)$ is completely determined for any d for $q \leq 4$ in [6]. Moreover, we can find many more results about the values $n_q(k,d)$ in [11].

In this paper, we are interested in linear codes of dimension 5. When $k = 5$, by Theorem 1, we have $n_q(5,d) = g_q(5,d)$ for the following values of d and q:

$$\begin{cases} q^4 - q^3 - q + 1 \leq d \leq q^4 - q^3 \text{ or } q^4 - 2q^2 + 1 \leq d \leq q^4 \text{ for } q \geq 3, \\ 2q^4 - 2q^3 - q^2 + 1 \leq d \leq 2q^4 \text{ for } q \geq 3, \\ d \geq 3q^4 - 4q^3 + 1 \text{ for } q \geq 5. \end{cases}$$

Therefore we consider the value $n_q(5,d)$ for which d is different from the above range, and we prove $n_q(5,d) \leq g_q(5,d) + 1$ for the following values of d and q.

(1) $q^4 - 3q^2 + 1 \leq d \leq q^4 - 2q^2$ for $q \geq 4$,

(2) $2q^4 - 2q^3 - 2q^2 + 1 \leq d \leq 2q^4 - 2q^3 - q^2$ for $q \geq 3$,

(3) $3q^4 - 4q^3 - q^2 + 1 \leq d \leq 3q^4 - 4q^3$ for $q \geq 5$.

We remark that $n_2(5,d)$ is completely determined for any d ([8]). If d is in the range in (1), then $n_3(5,d)$ is given in [11].

As a notational convention, P, P_i, Q etc. stand for points in $\mathbb{P}^{k-1}$. Similarly, l, l_i (respectively δ, δ_i, Δ, Δ_i) etc. stand for lines (resp. planes, solids) in $\mathbb{P}^{k-1}$. We denote by θ_j the number of points in a j-dimensional subspace in $\mathbb{P}^{k-1}$, i.e., $\theta_j = \frac{q^{j+1}-1}{q-1} = q^j + \cdots + q + 1$ for $j \geq 0$. Here $\theta_0 = 1$. For a subset $S \subset \mathbb{P}^4$, $\langle S \rangle$ denotes the linear span of S.

Let C be an $[n,k,d]_q$ code with a generator matrix G. Now C is said to be non-degenerate if any column of G is nonzero. Thus if C is a non-degenerate code, each column of G can be regarded as a point in $\mathbb{P}^{k-1}$. The formal sum of all columns of G as points in $\mathbb{P}^{k-1}$ is called a 0-cycle of the code C, which we denote by $\mathcal{X}_C$. If one chooses another generator matrix G' of the same code C, then two 0-cycles of C corresponding to G and G', respectively, are projectively equivalent. Conversely, two codes are equivalent ones if their 0-cycles are projectively equivalent. Letting $m(P) \geq 0$ denote the number of times the point P occurrs as a column of G, we have $\mathcal{X}_C = \sum_{P \in \mathbb{P}^{k-1}} m(P)P$.

For any subset $S \subset \mathbb{P}^{k-1}$, we denote the restriction $\mathcal{X}_C$ to S by $\mathcal{X}_C(S) := \sum_{P \in S} m(P)P$. The symbol $[S]$ denotes the 0-cycle

$$[S] := \sum_{P \in S} P,$$

which can be identified with the set S. We use the notation $\mathrm{Supp}\mathcal{X}_C = \{P \in \mathbb{P}^{k-1} \mid m(P) \geq 1\}$.

For a 0-cycle $\mathcal{X}_C = \sum_{P \in \mathbb{P}^{k-1}} m(P)P$ corresponding to a given code C, let $\gamma_0 := \max\{m(P) \mid P \in \mathbb{P}^{k-1}\}$ and $c(S) := \deg \mathcal{X}_C(S)$. Then we express the parameters n and d of C in terms of the coefficients in the 0-cycle $\mathcal{X}_C$ as follows:

$$n = \deg \mathcal{X}_C := \sum_{P \in \mathbb{P}^{k-1}} m(P),$$
$$d = n - \max_{H \in \mathbb{P}^{k-1^*}} c(H),$$

where $\mathbb{P}^{k-1^*}$ means the set of all hyperplanes in $\mathbb{P}^{k-1}$.

The concept of minihyper (F, w) with weight function was defined in [4], [10] and [12]. In this paper, we use the terminology 0-cycle instead of (F, w). Then their definition can be expressed as follows. For $r \geq 2$, a 0-cycle $\mathcal{X} = \sum_{P \in \mathbb{P}^r} m(P)P$ defines $\{f, m; r, q\}$-minihyper if

$$f = \deg \mathcal{X} = \sum_{P \in \mathbb{P}^r} m(P),$$
$$m = \min_{H \in \mathbb{P}^{r*}} c(H).$$

When $k = 5$, we consider only the case $\gamma_0 \leq 3$, since $n_q(5, d) = g_q(5, d)$ for $\gamma_0 \geq 4$ by Theorem 1.

2 Construction of codes of length $g_q(5, d) + 1$

Now we construct a class of codes with length $g_q(5, d) + 1$ by generalizing the idea in the proof of Theorem C in [2].

Lemma 2 *There exists a collection of $q^2 + 1$ planes in $\mathbb{P}^4$ passing through a point P such that any two planes in the collection intersect only at P, and $q^2 + 1$ is maximal possible.*

Proof. Let $\mathcal{T} = \{\delta_1, \delta_2, \ldots, \delta_r\}$ be a collection of planes through a point P such that $\delta_i \cap \delta_j = \{P\}$ if $i \neq j$. Since $\mathbb{P}^4 \supseteq \cup\mathcal{T}$, we have $\theta_4 \geq |\cup\mathcal{T}| = 1 + \sum_{i=1}^{r}(|\delta_i| - 1) = 1 + r(q^2 + q)$, whence $r \leq q^2 + 1$.

On the other hand, let H_0 be a hyperplane in $\mathbb{P}^4$ such that $P \notin H_0$. Then by Theorem 4.1 in [7], we note that there exists a spread $\mathcal{S} = \{l_1, l_2, \ldots, l_{q^2+1}\}$ of exactly $q^2 + 1$ mutually disjoint lines in H_0. Let $\mathcal{S}_P = \{\langle l_i, P\rangle \mid 1 \leq i \leq q^2 + 1\}$. Then, clearly $\langle l_i, P\rangle \cap \langle l_i, P\rangle = \{P\}$ if $i \neq j$. Thus the lemma is proved. ■

Now, using Lemma 2, we construct three classes of minihypers in Lemma 3, 5 and 7. Then we state three theorems, Theorem 4, 6 and 8 in which we prove the existence of $[g_q(5, d) + 1, 5, d]_q$ code for given d and q in (1), (2) and (3), respectively.

Lemma 3 *There exists a* $\{2\theta_2+\alpha\theta_1+\beta-1,\ 2q+1+\alpha;\ 4,q\}$*-minihyper with* $w(P)\le 1$ *for any* P *in* $\mathbb{P}^4$*, where* α,β *with* $0\le\alpha,\beta\le q-1$.

Proof. Fix integers α and β such that $0\le\alpha,\beta\le q-1$. For a point P_0 in $\mathbb{P}^4$, by Lemma 2, we can choose $\alpha+3$ planes $\delta_0,\delta_1,\ldots,\delta_{\alpha+2}$ such that $\delta_i\cap\delta_j=\{P_0\}$ for $0\le i<j\le\alpha+2$. Now, choose a line l_i in each plane δ_i for $3\le i\le\alpha+2$ which does not contain the point P_0. Consider a 0-cycle

$$\mathcal{X}_1=[\delta_1]+[\delta_2]-[P_0]+\sum_{i=3}^{\alpha+2}[l_i].$$

Then obviously $\deg(\mathcal{X}_1)=2\theta_2+\alpha\theta_1-1$ and $c(H)\ge 2q+1+\alpha$ for any hyperplane H in $\mathbb{P}^4$. Let H_0 be a hyperplane containing δ_0. Since $\langle\delta_0,\delta_i\rangle=\mathbb{P}^4$, $i=1,2$ and $\langle\delta_0,l_j\rangle=\mathbb{P}^4$, $3\le j\le\alpha+2$, we note that $\delta_i\not\subset H_0$ for $i=1,2$ and $l_j\not\subset H_0$ for $3\le j\le\alpha+2$. Thus $c(H_0)=2q+1+\alpha$. Therefore, the 0-cycle $\mathcal{X}_1$ is a $\{2\theta_2+\alpha\theta_1-1,\ 2q+1+\alpha;\ 4,q\}$-minihyper.

Now, choose β points Q_j, $(1\le j\le\beta)$ in $\mathbb{P}^4-\mathrm{Supp}(\mathcal{X}_1)-H_0$. Let

$$\mathcal{X}_1'=\mathcal{X}_1+\sum_{j=1}^{\beta}[Q_j].$$

Then obviously the 0-cycle $\mathcal{X}_1'$ is a $\{2\theta_2+\alpha\theta_1+\beta-1,\ 2q+1+\alpha;\ 4,q\}$-minihyper, which completes the proof. ∎

Theorem 4 *If* $q\ge 4$ *and* $q^4-3q^2+1\le d\le q^4-2q^2$*, then*

$$n_q(5,d)\le g_q(5,d)+1.$$

Proof. For any d with $q^4-3q^2+1\le d\le q^4-2q^2$, there exist α and β such that $0\le\alpha,\beta\le q-1$ and $d=q^4-2q^2-(\alpha q+\beta)$.

Let C_1 be a code corresponding to the 0-cycle $\mathcal{Y}_1=[\mathbb{P}^4]-\mathcal{X}_1'$, where $\mathcal{X}_1'$ is the 0-cycle appeared in Lemma 3, that is,

$$\mathcal{Y}_1=[\mathbb{P}^4]-[\delta_1]-[\delta_2]-\sum_{i=3}^{\alpha+2}[l_i]-\sum_{j=1}^{\beta}[Q_j]+[P_0].$$

Then the length n of C_1 is $q^4+q^3-\theta_2-\alpha\theta_1-\beta+1$. Now we consider the minimum distance d of C_1. Since

$$\max_{H\in\mathbb{P}^{4*}}c(H)=\max_{H\in\mathbb{P}^{4*}}(\theta_3-\deg\mathcal{X}_1'(H))=\theta_3-\min_{H\in\mathbb{P}^{4*}}(\deg\mathcal{X}_1'(H)),$$

we have $d=n-(\theta_3-\min_{H\in\mathbb{P}^{4*}}\deg\mathcal{X}_1'(H))$. Since $\mathcal{X}_1'$ is a $\{2\theta_2+\alpha\theta_1+\beta-1,\ 2q+1+\alpha;\ 4,q\}$-minihyper by Lemma 3, $d=q^4-2q^2-\alpha q-\beta$. Since $q\ge 4$, we have $g_q(5,d)=q^4+q^3-\theta_2-\alpha\theta_1-\beta$ for $d=q^4-2q^2-\alpha q-\beta$, whence $n=g_q(5,d)+1$. Thus C_1 is a $[g_q(5,d)+1,5,d]_q$ code, which completes the proof. ∎

To prove Theorem 6 we need a minihyper with $w(P)\le 2$ for any point P in $\mathbb{P}^4$.

Lemma 5 *There exists a* $\{2\theta_3+\theta_2+\alpha\theta_1+\beta-1,\ 2q^2+3q+2+\alpha;\ 4,q\}$*-minihyper with* $w(P)\leq 2$ *for any* P *in* $\mathbb{P}^4$*, where* $0\leq\alpha,\beta\leq q-1$*.*

Proof. Fix integers α and β such that $0\leq\alpha,\beta\leq q-1$. For a point P_0 in $\mathbb{P}^4$, by Lemma 2, we can take $\alpha+2$ planes, say $\delta_1,\delta_2,\ldots,\delta_{\alpha+2}$ such that $\delta_i\cap\delta_j=\{P_0\}$ for $1\leq i<j\leq\alpha+2$. Choose a line l_j in each plane δ_j such that $P_0\notin l_j$ for $3\leq j\leq\alpha+2$. Let H_1 and H_2 be distinct hyperplanes containing the plane δ_2.

Consider a 0-cycle

$$\mathcal{X}_2=[H_1]+[H_2]+[\delta_1]+\sum_{i=3}^{\alpha+2}[l_i]-[P_0].$$

Then $\deg(\mathcal{X}_2)=2\theta_3+\theta_2+\alpha\theta_1-1$ and $c(H)\geq 2\theta_2+\theta_1+\alpha-1$ for any hyperplane H of $\mathbb{P}^4$. Let H_0 be a hyperplane containing δ_2 such that $H_0\neq H_1,H_2$. Since $\langle\delta_2,\delta_1\rangle=\mathbb{P}^4$ and $\langle\delta_2,l_j\rangle=\mathbb{P}^4$, $3\leq j\leq\alpha+2$, we note that $\delta_1\not\subset H_0$ and $l_j\not\subset H_0$ for $3\leq j\leq\alpha+2$. Thus we have $c(H_0)=2\theta_2+\theta_1+\alpha-1$. Choose β points Q_j $(1\leq j\leq\beta)$ in $\mathbb{P}^4-\mathrm{Supp}(\mathcal{X}_2)-H_0$. Let

$$\mathcal{X}_2'=\mathcal{X}_2+\sum_{j=1}^{\beta}[Q_j].$$

Then obviously the 0-cycle $\mathcal{X}_2'$ is a $\{2\theta_3+\theta_2+\alpha\theta_1+\beta-1,\ 2q^2+3q+2+\alpha;\ 4,q\}$-minihyper, which completes the proof. ■

Theorem 6 *If* $q\geq 3$ *and* $2q^4-2q^3-2q^2+1\leq d\leq 2q^4-2q^3-q^2$*, then*

$$n_q(5,d)\leq g_q(5,d)+1.$$

Proof. For any d with $2q^4-2q^3-2q^2+1\leq d\leq 2q^4-2q^3-q^2$, there exist α and β such that $0\leq\alpha,\beta\leq q-1$ and $d=2q^4-2q^3-q^2-(\alpha q+\beta)$.

Let C_2 be a code corresponding to the 0-cycle $\mathcal{Y}_2=2[\mathbb{P}^4]-\mathcal{X}_2'$, where $\mathcal{X}_2'$ is the 0-cycle appeared in Lemma 5, that is,

$$\mathcal{Y}_2=2[\mathbb{P}^4]-[H_1]-[H_2]-[\delta_0]-\sum_{i=3}^{\alpha+2}[l_i]-\sum_{j=1}^{\beta}[Q_j]+[P_0].$$

Then the length n of C_2 is $2q^4-\theta_2-\alpha\theta_1-\beta+1$. Now we consider the minimum distance d of C_2. Since

$$\max_{H\in\mathbb{P}^{4*}}c(H)=\max_{H\in\mathbb{P}^{4*}}(2\theta_3-\deg\mathcal{X}_2'(H))=2\theta_3-\min_{H\in\mathbb{P}^{4*}}(\deg\mathcal{X}_2'(H)),$$

we have $d=n-(2\theta_3-\min_{H\in\mathbb{P}^{4*}}\deg\mathcal{X}_2'(H))$. Since $\mathcal{X}_2'$ is a $\{2\theta_3+\theta_2+\alpha\theta_1+\beta-1,\ 2q^2+3q+2+\alpha;\ 4,q\}$-minihyper by Lemma 5, we have $d=2q^4-2q^3-q^2-\alpha q-\beta$. Since $q\geq 3$, we have $g_q(5,d)=2q^4-\theta_2-\alpha\theta_1-\beta$ for $d=2q^4-2q^3-q^2-\alpha q-\beta$, whence $n = g_q(5,d)+1$. Thus C_2 is a $[g_q(5,d)+1, 5, d]_q$ code, which completes the proof. ■

Next we construct a minihyper with $w(P) \le 3$ for any point P in $\mathbb{P}^4$ to prove Theorem 8.

Lemma 7 *There exists a* $\{4\theta_3 + \alpha\theta_1 + \beta - 1,\ 4\theta_2 + \alpha - 1;\ 4, q\}$*-minihyper with* $w(P) \le 3$ *for any* P *in* $\mathbb{P}^4$*, where* $0 \le \alpha, \beta \le q-1$.

Proof. Fix integers α and β such that $0 \le \alpha, \beta \le q-1$. For a point P_0 in $\mathbb{P}^4$, by Lemma 2, there exists a collection $\mathcal{D}$ of q^2+1 planes through P_0 such that any two planes in $\mathcal{D}$ intersect only at P_0. We take any 4 planes in $\mathcal{D}$, say δ_i $(i = 1,2,3,4)$. Then we take hyperplanes H_i satisfying the following conditions;

(i) H_i contains δ_i for $i = 1,2,3,4$, respectively.

(ii) H_2 does not contain the line $H_1 \cap \delta_4$.

(iii) H_3 does not contain the line $H_1 \cap \delta_2$.

(iv) H_4 does not contain the line $H_1 \cap H_2 \cap H_3$.

Indeed, it is easy to prove that such hyperplanes exist, $H_1 \cap H_2$ is a plane, $H_1 \cap H_2 \cap H_3$ is a line which is not contained in δ_4, and $H_1 \cap H_2 \cap H_3 \cap H_4 = \{P_0\}$.

Next, we take $\alpha+1$ planes in $\mathcal{D} - \{\delta_1, \delta_2, \delta_3, \delta_4\}$, say δ_0, δ_j $(j = 5, \ldots, \alpha+4)$ which does not contain the lines constructed by three of H_i for $i = 1,2,3,4$. For $5 \le j \le \alpha+4$, choose a line l_j in each plane δ_j such that $P_0 \notin l_j$.

Let

$$\mathcal{X}_3 = [H_1] + [H_2] + [H_3] + [H_4] + \sum_{i=5}^{\alpha+4} [l_i] - [P_0].$$

Then $\deg(\mathcal{X}_3) = 4\theta_3 + \alpha\theta_1 - 1$ and $c(H) \ge 4\theta_2 + \alpha - 1$ for any hyperplane H of $\mathbb{P}^4$. Moreover, $w(P) \le 3$ for any $P \in \mathbb{P}^4$, by the choice of H_i and l_j.

Let H_0 be a hyperplane containing δ_0. Since $\langle \delta_0, \delta_i \rangle = \mathbb{P}^4$, $1 \le i \le 4$ and $\langle \delta_0, l_j \rangle = \mathbb{P}^4$, $5 \le j \le \alpha+4$, we have $c(H_0) = 4\theta_2 + \alpha - 1$. Next, we choose β points Q_j $(1 \le j \le \beta)$ in $\mathbb{P}^4 - \mathrm{Supp}(\mathcal{X}_3) - H_0$. Let

$$\mathcal{X}_3' = \mathcal{X}_3 + \sum_{j=1}^{\beta} [Q_j].$$

Then obviously the 0-cycle $\mathcal{X}_3'$ is a $\{4\theta_3 + \alpha\theta_1 + \beta - 1,\ 4\theta_2 + \alpha - 1;\ 4, q\}$-minihyper, which completes the proof. ■

Theorem 8 *If* $q \ge 5$ *and* $3q^4 - 4q^3 - q^2 + 1 \le d \le 3q^4 - 4q^3$*, then*

$$n_q(5,d) \le g_q(5,d) + 1.$$

Proof. For any d with $3q^4-4q^3-q^2+1 \le d \le 3q^4-4q^3$, there exist α and β such that $0 \le \alpha, \beta \le q-1$ and $d = 3q^4-4q^3-(\alpha q+\beta)$.

Let C_3 be a code corresponding to the 0-cycle $\mathcal{Y}_3 = 3[\mathbb{P}^4] - \mathcal{X}'_3$, where $\mathcal{X}'_3$ is the 0-cycle appeared in Lemma 7, that is,

$$\mathcal{Y}_3 = 3[\mathbb{P}^4] - \sum_{i=1}^{4}[H_i] - \sum_{i=5}^{\alpha+4}[l_i] - \sum_{j=1}^{\beta}[Q_j] + [P_0].$$

Then the length n of C_3 is $3\theta_4 - 4\theta_3 - \alpha\theta_1 - \beta + 1$. Now we consider the minimum distance d of C_3. Since

$$\max_{H\in\mathbb{P}^{4*}} c(H) = \max_{H\in\mathbb{P}^{4*}} (3\theta_3 - \deg \mathcal{X}'_3(H)) = 3\theta_3 - \min_{H\in\mathbb{P}^{4*}} (\deg \mathcal{X}'_3(H)),$$

we have $d = n - (3\theta_3 - \min_{H\in\mathbb{P}^{4*}}(\deg \mathcal{X}'_3(H))$. Since $\mathcal{X}'_3$ is a $\{4\theta_3 + \alpha\theta_1 + \beta - 1,\ 4\theta_2 + \alpha - 1;\ 4, q\}$-minihyper by Lemma 7, we have $d = 3q^4 - 4q^3 - \alpha q - \beta$. Since $q \ge 5$, we have $g_q(5,d) = 3\theta_4 - 4\theta_3 - \alpha\theta_1 - \beta$ for $d = 3q^4 - 4q^3 - \alpha q - \beta$, whence $n = g_q(5,d)+1$. Thus C_3 is a $[g_q(5,d)+1, 5, d]_q$ code, which completes the proof. ■

Remark Maruta [9] proved that $n_q(5,d) = g_q(5,d)+1$ when

$$q^4 - 2q^2 - q + 1 \le d \le q^4 - 2q^2 \text{ for } q \ge 3,$$
$$2q^4 - 2q^3 - q^2 - q + 1 \le d \le 2q^4 - 2q^3 - q^2 \text{ for } q \ge 3,$$
$$3q^4 - 4q^3 - q + 1 \le d \le 3q^4 - 4q^3 \text{ for } q \ge 5,$$

which is corresponding to the case $\alpha = 0$ and $0 \le \beta \le q-1$ in our theorems Theorem 4, 6 and 8, respectively.

Also, in [1], [2] and [3], they proved that $n_q(5,d) = g_q(5,d)+1$ when

$$q^4 - 2q^2 - 2q + 1 \le d \le q^4 - 2q^2 - q \text{ for } q \ge 5,$$
$$2q^4 - 2q^3 - q^2 - 2q + 1 \le d \le 2q^4 - 2q^3 - q^2 - q \text{ for } q \ge 5,$$
$$3q^4 - 4q^3 - 2q + 1 \le d \le 3q^4 - 4q^3 - q \text{ for } q \ge 11,$$

which is corresponding to the case $\alpha = 1$ and $0 \le \beta \le q-1$ in our theorems Theorem 4, 6 and 8, respectively.

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(Received 13 Feb 2006)