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A Simulation Method of Atmospheric Turbulence Based on Von Karman Model

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Abstract. *In recent years virtual test technology has been widely used to evaluate the performance of aircraft. In these virtual flight experiments, virtual wind field environment is a necessary and important issue. In all kinds of wind field, atmospheric turbulence is most primary and essential. Nowadays, in virtual team-flight test the new requirement for three-dimensional (3D for short) atmospheric turbulence has become more and more intense. Therefore, building a reasonable and verisimilar 3D atmospheric turbulence field is a very important step in the virtual flight test of aircraft. Most classical methods which have been used to build one-dimensional atmospheric turbulence field are not available in 3D atmospheric turbulence simulation, while several existing 3D atmospheric turbulence simulation methods are not applicable in generating large-scale 3D atmospheric turbulence rapidly. In this paper we proposed a new simulation method for 3D atmospheric turbulence, which is based on the correlation function method and Von Karman model. The correlation function method can make recursive calculation by use of frequency spectrum directly, so it has more accurate calculation precision and does not need large memory to store the whole wind field data. The recursive formulate are deduced by use of the self-correlation function, and the generating steps of 3D atmospheric turbulence based on the proposed method are detailed in this paper. Simulation results indicate that the relativity of 3D atmospheric turbulence field based on the proposed method is near to the theoretic value.*

Keywords: Three-dimensional atmospheric turbulence; Von Karman model; Correlation function.

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1. **Introduction.** Now more and more virtual test technology is used in the process of aircraft design. Virtual flight test can reduce costs and shorten the development cycle. In virtual test, virtual environment is indispensable. The atmospheric turbulence filed (ATF) is an important issue in an aeronautical experimentation [1][2]. Therefore, it is very necessary to research ATF simulation method. In virtual test, the final requirement of the virtual environment is to construct a three-dimensional space like with a real environment. According to the statistical results in reference[3] about atmospheric turbulence of multiple districts, atmospheric turbulence field presents discontinuous block distribution, which maximum level goes to 300~400km and the maximum thickness can be up to 1400~2200m. So the atmospheric turbulence models in virtual environment not only need to meet the requirement of the spectral characteristics and the three dimensional space characteristics of the atmospheric turbulence, but also need to have capacity to generate a wide range atmospheric turbulence field. At the same time, atmospheric turbulence models in virtual environment also need to meet the real-time requirement, in order to meet with some virtual tests which contain physical/semi-physical volunteers.

The most widely used ATF simulation method is filtering the Gaussian random noise signal [4][5][6]. By using this method, the turbulence winds can be evaluated in a region along the aircraft trajectory. But this method is a 1D-ATF simulation method which only guarantees the relativity of ATF along the aircraft trajectory. When there are more than one aircraft in the virtual test, the 1D-ATF cannot guarantee the relativity between different flight trajectories [7]. In this case, we need a 3D-ATF. At present, some researchers have studied the 3D-ATF simulation technology. Hong made the three-dimensional space grid discretization [8], and generated 3D-ATF by using a method based on correlation coefficient matrix decomposition; but when the space is larger, the grid number increasing, this method takes up computer memory seriously, and can't generate larger 3D-ATF. Gao proposed a method which generated three dimensional atmospheric turbulence directly in frequency domain by using a method based on time domain transformation, then get the 3D-ATF data of time domain after three dimensional inverse Fourier transform [9]. But using this method once calculation merely generates an atmospheric turbulence data in all frequency, it can't generate atmospheric turbulence according to simulation step in real time, so it can't meet the instantaneity requirement of the virtual test. Therefore, the reference [10] comes up with that we can use that method to generate atmospheric turbulence data in advance, and store the data in database in order to meet the need of simulation, but when we do as this way we need to store much more data, and it will influence the flexibility of the virtual test.

Lu advanced a turbulence simulation method based on spatial correlation function [11], which has been successfully applied in 1D-ATF and 2D-ATF simulations and can generate the turbulence data in a large spatial scale [12]. Wu proposed a new method to build the 3D-ATF based on the correlation function [13], it can generate large scale 3D-ATF based on iterative computation. In paper [13], the 3D-ATF is built on the base of Dryden model. Gao proposed a numerical simulation method for 3D atmospheric turbulence which is based on Monte Carlo method [15]. In this paper, we adopt another famous turbulence model-Von Karman to build the 3D-ATF, and proposed the simulating method based on Von Karman model.

2. The fundamental assumption and the mathematic description of atmospheric turbulence.

The causes of generating atmospheric turbulence are very complicated, so its difficult to give a mathematic description by the way of describing random process. It usually be regarded as a random process in flight simulation, and be described by using a method which describes random process, namely, be described by statistical features. In order to facilitate the study of the problem, we also idealize atmospheric turbulence in simulation and aircraft virtual tests, making some basic assumption. In this section, at first we will introduce some basic assumptions about atmospheric turbulence in flight simulation. Secondly the statistical characterization and the definition of the description of random process will be discussed. Finally, we will give the common atmospheric turbulence correlation function and spectrum in flight simulation.

2.1. The basic assumptions about atmospheric turbulence in flight simulation.

There are several basic assumptions about atmospheric turbulence in aircraft simulation as follows [3]:

1. Stationary and uniformity assumption: the statistical characteristics of atmospheric turbulence don't change with time and position;
2. Isotropy assumption: the statistical characteristics of atmospheric turbulence don't change with the coordinate system, that is to say, it is irrespective to direction;

3. Gauss distribution assumption: atmospheric turbulence is Gaussian distribution, in other words, the magnitude of velocity obeys normal distribution;
4. Freeze field assumption: aircraft's flight velocity far more than turbulence velocity and it's variation, the time of flight for quite a long distance is very short, so change in the velocity of turbulence is very small, and can be negligible. That is to say, when we deal with the problem of turbulence effects on the aircraft, we can freeze the turbulence.

2.2. **Atmospheric turbulence correlation function and spectrum function.** In flight simulation, we usually use correlation function or spectrum function to describe atmospheric turbulence. Spectrum function is the Fourier transform of the correlation, so these two descriptive methods are equal.

H.L.Dryden and T.Von Karman respectively carried on statistics and deduction according to measured data by different theory system. H.L.Dryden provided the longitudinal and transversal correlation [3], as (1) and (2) follows; then deduce spectrum function, as (3) follows.

$$f(\xi) = \exp(-\xi/L) \tag{1}$$

$$g(\xi) = (1 - 0.5\xi/L)\exp(-\xi/L) \tag{2}$$

$$\left.\begin{aligned} \Phi_{uu}(\Omega) &= \sigma_u^2 \tfrac{L_u}{\pi} \tfrac{1}{1+(L_u\Omega)^2} \\ \Phi_{vv}(\Omega) &= \sigma_v^2 \tfrac{L_v}{\pi} \tfrac{1+12(L_v\Omega)^2}{[1+4(L_v\Omega)^2]^2} \\ \Phi_{ww}(\Omega) &= \sigma_w^2 \tfrac{L_w}{\pi} \tfrac{1+12(L_w\Omega)^2}{[1+4(L_w\Omega)^2]^2} \end{aligned}\right\} \tag{3}$$

where ξ -special distance(m); L-atmospheric turbulence scale(m); -space frequency(rad/m); σ -atmospheric turbulence intensity(m/s); *u, v, w*-respectively representative physical quantity's components along with airframe's horizontal axis, vertical axis, diagonal axis.

Von Karman gave atmospheric turbulence's energy spectrum, as (4) follows; then deduced the spectrum and correlation function of the three turbulence components, as(5)-(7)follows.

$$E(\Omega) = \sigma^2 \frac{55L}{9\pi} \frac{(aL\Omega)^4}{[1+(aL\Omega)^2]^{17/6}} \tag{4}$$

$$\left.\begin{aligned} \Phi_{uu}(\Omega) &= \sigma_u^2 \tfrac{L_u}{\pi} \tfrac{1}{[1+(aL_u\Omega)^2]^{5/6}} \\ \Phi_{vv}(\Omega) &= \sigma_v^2 \tfrac{L_v}{\pi} \tfrac{1+(8/3)(2aL_v\Omega)^2}{[1+4(aL_v\Omega)^2]^{11/6}} \\ \Phi_{ww}(\Omega) &= \sigma_w^2 \tfrac{L_w}{\pi} \tfrac{1+(8/3)(2aL_w\Omega)^2}{[1+4(aL_w\Omega)^2]^{11/6}} \end{aligned}\right\} \tag{5}$$

$$f(\xi) = \frac{2^{2/3}}{\Gamma(1/3)} \zeta^{1/3} K_{1/3}(\zeta) \tag{6}$$

$$g(\xi) = \frac{2^{2/3}}{\Gamma(1/3)} \zeta^{1/3} [K_{1/3}(\zeta) - \frac{1}{2}\zeta K_{2/3}(\zeta)] \tag{7}$$

where, $\zeta = \xi/(aL)$, a=1.339, Γ is Gamma function, K is Bessel function.

Fig.1 and Fig.2 are respectively Dryden spectrum model and Von Karman spectrum model, which give the comparisons of the vertical space spectrum function and the vertical correlation function. From Fig.1, it can be seen that there are little differences between these spectrum models overall. However, in high frequency section, there are

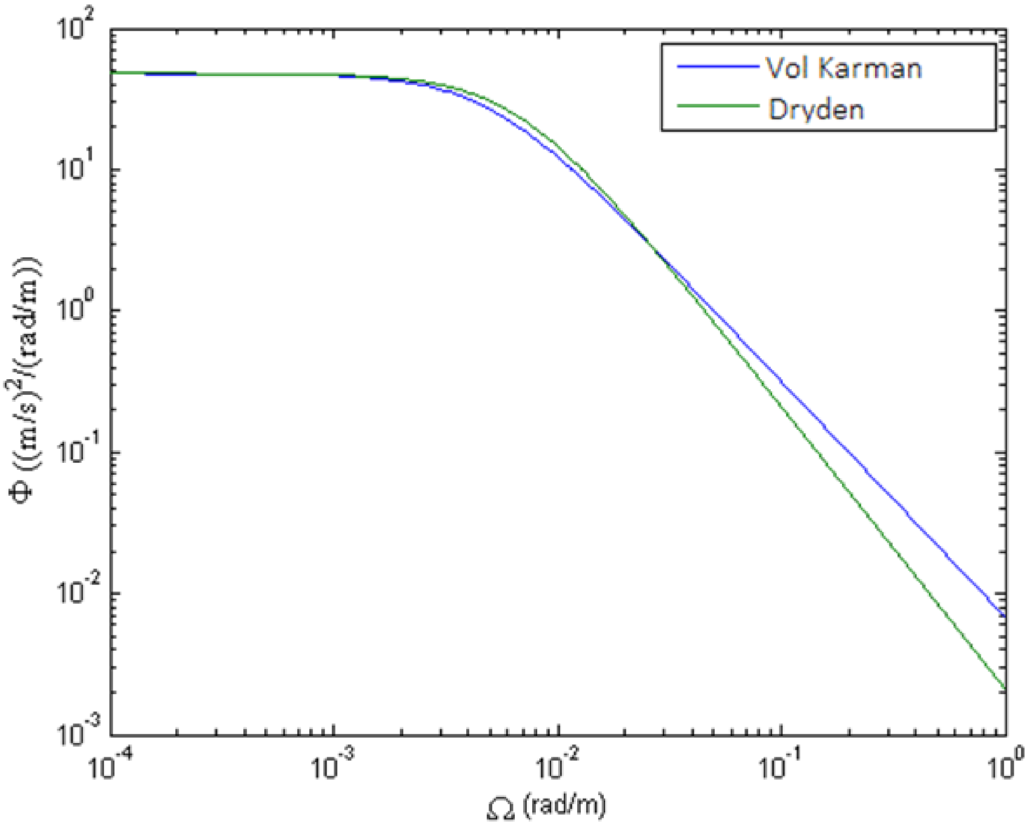


Figure 1. Comparison of the vertical space spectrum functions between Dryden and Von Karman

apparent differences between the two spectrums and Von Karman spectrum is more accurate. Therefore, it's better to use Von Karman model when the virtual tests comes to the flight quality of aircraft structure vibration as well as the structure fatigue of aircrafts.

Reference [11] proposed a three dimensional atmospheric turbulence modeling method based on Dryden model. From the above discussion, Von Karman model has more accurate characteristics in high frequency section, so we try to propose a three dimensional turbulence modeling method based on Von Karman model in this article.

3. **The ATF generating method based on correlation function.** The current simulation methods of 3D-ATF can't meet the requirements in virtual tests because there are many problems such as large calculating quantity, much occupancy of computer storage, poor real-time of simulation. The method of atmospheric turbulence simulation based on correlation function [11], producing atmospheric turbulence data by recursive model. With the advantages of less calculation, minimum occupancy of computer storage, this method can be used in real-time simulation, but it doesn't applicative to the simulation of 3D-ATF currently. In this section, we will introduce the theory of atmospheric turbulence simulation based on correlation function first. To generate 3D-ATF, we will establish the random process model which can produce 3D-ATF, based on that theory.

Early method of atmospheric turbulence simulation established ATF in accordance with atmospheric turbulence spectrum. The basic idea of the method like the Fig.3: letting white noise r through a forming filter G, then making the output w meet the requirement of the atmospheric turbulence spectrum by choosing appropriate filter parameters.

According to the theory of digital signal processing,

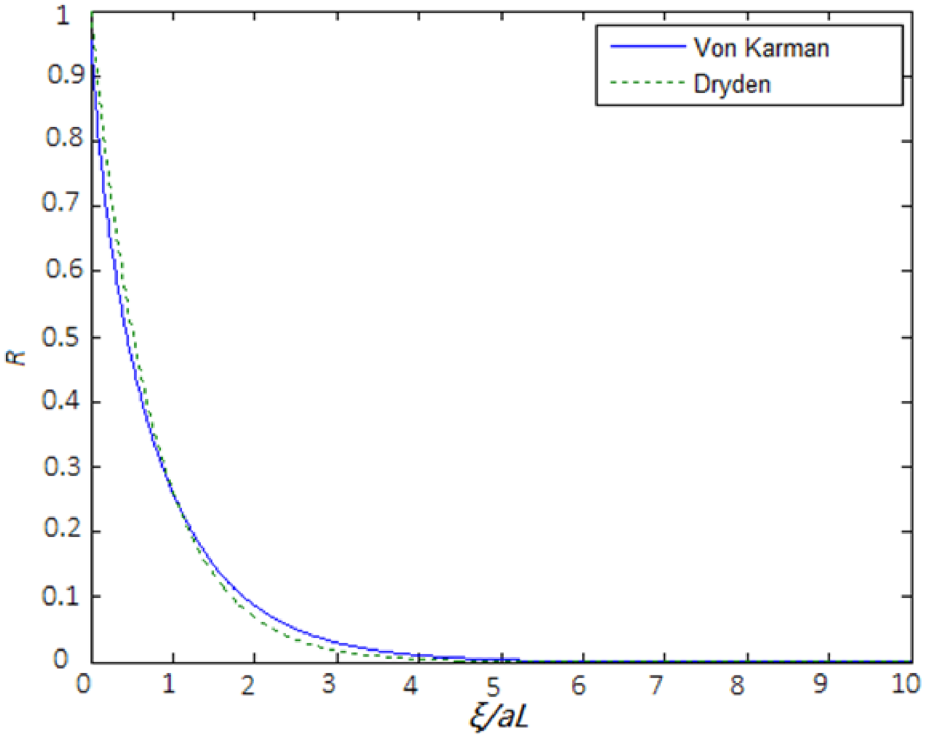


FIGURE 2. Comparison of the vertical correlation functions between Dryden and Von Karman

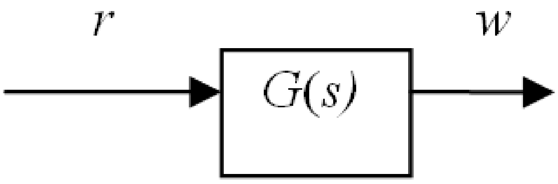


FIGURE 3. Illustration of generating turbulence by filtering white noise

$$\Phi_w(\omega) = |G(i\omega)|^2 \Phi_r(\omega) \tag{8}$$

While the spectrum of white noise is constant, so the spectrum of output is:

$$\Phi_w(\omega) = |G(i\omega)|^2 = G^*(i\omega)G(i\omega) \tag{9}$$

According to (9), making the atmospheric turbulence spectrum conjugated decomposition, thus getting the parameters of forming filter [14]. And then we will get a differential recursion formula which can output the value of atmospheric turbulence according to the forming filter, following formula:

$$w_i = aw_{i-1} + \sigma r \tag{10}$$

where, a, σ can be made determined by forming filter formula.

Therefore, if an initial atmospheric turbulence value w_0 is given, all atmospheric turbulence sequences can be get by the formula (10). This method needs to make the atmospheric turbulence spectrum to be factored, and the atmospheric turbulence spectrum is called to be rational expression. Therefore, this method can only produce the ATF of Dryden spectrum, while it is not suitable for Von Karman spectrum model.

Reference [11] proposes a method to generate ATF on the basis of correlation function. The relationship between Correlation function and spectrum function its Fourier transformation, so the ATF established by correlation function is equal to the ATF established by spectrum function. While the method to produce ATF based on correlation function avoid making the atmospheric turbulence spectrum to be factored, it not only can produce ATF of Dryden spectrum, but also can produce ATF of Von Karman, with better adaptability. This method constructs the producing ATF random processing model directly (self-recursion model), then solve the model coefficient by the correlation between each point, thus ensure to create the correlation of ATF.

3.1. **Random process model for 3D-ATF.** In flight simulation, atmospheric turbulence which can be seen as a random process is simulated by filtering the white noise [10]. By adjusting the parameters of filters, it can make the output signal meet the reference spectrum of atmospheric turbulence. To get the filter parameters, the method based on correlation function can solve the difference equation of the filter by the spatial correlation function directly. The main steps of this method are given as follows: 1) Giving the difference equation of filters. 2) Confirming the coefficients of the difference equation, according to the equations established by self-correlation and one step correlation function.

$$w(x) = aw(x-h) + \sigma_w r(x) \tag{11}$$

$$w(x,y) = a_1 w(x-h,y) + a_2 w(x,y-h) + a_3 w(x-h,y-h) + \sigma_w^{'} r(x,y) \tag{12}$$

Where, w(x) and w(x, y) are the turbulence value of the position x and (x, y), r(x) and r(x, y) are 1D and 2D white noise, h is grid size, a, a1, a2, a3, σ_w, $\sigma_w^{'}$ are parameters.

For 1D-ATF, the correlation function and one step correlation function can be expressed as

$$R_0 = E[w(x)w(x)] = E\{[aw(x-h) + \sigma_w r(x)]^2\} = a^2 R_0 + \sigma_w^2 \tag{13}$$

$$R_1 = E[w(x)w(x-h)] = E\{[aw(x-h) + \sigma_w r(x)]w(x-h)\} = aR_0 \tag{14}$$

When the direction of the turbulence is perpendicular to the coordinate axis, R_0 and R_1 can be calculated by the horizontal correlation function; when the direction of the turbulence is parallel with the coordinate axis, R_0 and R_1 can be calculated by the vertical correlation function.

For the 2D-ATF, the correlation function and one step correlation function (including horizontal, vertical, diagonal direction) can be expressed as

$$R_{00} = E[w(x,y)w(x,y)] = a_1 R_{10} + a_2 R_{01} + a_3 R_{11} + {\sigma_w^{'}}^2 \tag{15}$$

$$R_{01} = E[w(x,y)w(x,y+h)] = a_1 R_{11} + a_2 R_{00} + a_3 R_{10} \tag{16}$$

$$R_{10} = E[w(x,y)w(x+h,y)] = a_1 R_{00} + a_2 R_{11} + a_3 R_{01} \tag{17}$$

$$R_{11} = E[w(x,y)w(x+h,y+h)] = a_1 R_{01} + a_2 R_{10} + a_3 R_{00} \tag{18}$$

R_{00}, R_{01}, R_{10} and R_{11} can be calculated by the spatial correlation function.

The coefficients in the formula (11) and (12) can be solved from the formula (13) $\sim$ (18). As the formula (11) and (12) are recurrence formulas, we can generate 1D-ATF and 2D-ATF by formula (11) and (12) as long as the initial conditions are given.

The method of building 3D-ATF can be extended from 1D-ATF and 2D-ATF method. The filter of 3D-ATF can be expressed as

$$\begin{aligned} w(x,y,z) = a_1 w(x-h,y-h,z-h) + a_2 w(x-h,y-h,z) + a_3 w(x,y-h,z-h) + \\ a_4 w(x-h,y,z-h) + a_5 w(x,y,z-h) + a_6 w(x,y-h,z) + a_7 w(x-h,y,z) + \sigma_w r(x,y,z) \end{aligned} \tag{19}$$

The coefficients in formula (19) can be calculated by the correlation function and one step correlation function of 3D-ATF as following.

$$\begin{aligned}
R_{000} &= a_1R_{111} + a_2R_{110} + a_3R_{011} + a_4R_{101} + a_5R_{001} + a_6R_{010} + a_7R_{100} + \sigma_w^2 \\
R_{001} &= a_1R_{110} + a_2R_{111} + a_3R_{010} + a_4R_{100} + a_5R_{000} + a_6R_{011} + a_7R_{101} \\
R_{010} &= a_1R_{101} + a_2R_{100} + a_3R_{001} + a_4R_{111} + a_5R_{011} + a_6R_{000} + a_7R_{110} \\
R_{011} &= a_1R_{100} + a_2R_{101} + a_3R_{000} + a_4R_{110} + a_5R_{010} + a_6R_{001} + a_7R_{111} \\
R_{100} &= a_1R_{011} + a_2R_{010} + a_3R_{111} + a_4R_{001} + a_5R_{101} + a_6R_{110} + a_7R_{000} \\
R_{101} &= a_1R_{010} + a_2R_{011} + a_3R_{110} + a_4R_{000} + a_5R_{100} + a_6R_{111} + a_7R_{001} \\
R_{110} &= a_1R_{001} + a_2R_{000} + a_3R_{101} + a_4R_{011} + a_5R_{111} + a_6R_{100} + a_7R_{010} \\
R_{111} &= a_1R_{000} + a_2R_{001} + a_3R_{100} + a_4R_{010} + a_5R_{110} + a_6R_{101} + a_7R_{011}
\end{aligned} \tag{20}$$

Set

$$\mathbf{A} = \begin{bmatrix} R_{110} & R_{111} & R_{010} & R_{100} & R_{000} & R_{011} & R_{101} \\ R_{101} & R_{100} & R_{001} & R_{111} & R_{011} & R_{000} & R_{110} \\ R_{100} & R_{101} & R_{000} & R_{110} & R_{010} & R_{001} & R_{111} \\ R_{011} & R_{010} & R_{111} & R_{001} & R_{101} & R_{110} & R_{000} \\ R_{010} & R_{011} & R_{110} & R_{000} & R_{100} & R_{111} & R_{101} \\ R_{001} & R_{000} & R_{101} & R_{011} & R_{111} & R_{100} & R_{010} \\ R_{000} & R_{001} & R_{100} & R_{010} & R_{110} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} R_{001} & R_{111} & R_{010} & R_{100} & R_{000} & R_{011} & R_{101} \\ R_{010} & R_{100} & R_{001} & R_{111} & R_{011} & R_{000} & R_{110} \\ R_{011} & R_{101} & R_{000} & R_{110} & R_{010} & R_{001} & R_{111} \\ R_{100} & R_{010} & R_{111} & R_{001} & R_{101} & R_{110} & R_{000} \\ R_{101} & R_{011} & R_{110} & R_{000} & R_{100} & R_{111} & R_{101} \\ R_{110} & R_{000} & R_{101} & R_{011} & R_{111} & R_{100} & R_{010} \\ R_{111} & R_{001} & R_{100} & R_{010} & R_{110} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} R_{110} & R_{001} & R_{010} & R_{100} & R_{000} & R_{011} & R_{101} \\ R_{101} & R_{010} & R_{001} & R_{111} & R_{011} & R_{000} & R_{110} \\ R_{100} & R_{011} & R_{000} & R_{110} & R_{010} & R_{001} & R_{111} \\ R_{011} & R_{100} & R_{111} & R_{001} & R_{101} & R_{110} & R_{000} \\ R_{010} & R_{101} & R_{110} & R_{000} & R_{100} & R_{111} & R_{101} \\ R_{001} & R_{110} & R_{101} & R_{011} & R_{111} & R_{100} & R_{010} \\ R_{000} & R_{111} & R_{100} & R_{010} & R_{110} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} R_{110} & R_{111} & R_{001} & R_{100} & R_{000} & R_{011} & R_{101} \\ R_{101} & R_{100} & R_{010} & R_{111} & R_{011} & R_{000} & R_{110} \\ R_{100} & R_{101} & R_{011} & R_{110} & R_{010} & R_{001} & R_{111} \\ R_{011} & R_{010} & R_{100} & R_{001} & R_{101} & R_{110} & R_{000} \\ R_{010} & R_{011} & R_{101} & R_{000} & R_{100} & R_{111} & R_{101} \\ R_{001} & R_{000} & R_{110} & R_{011} & R_{111} & R_{100} & R_{010} \\ R_{000} & R_{001} & R_{111} & R_{010} & R_{110} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_4 = \begin{bmatrix} R_{110} & R_{111} & R_{010} & R_{001} & R_{000} & R_{011} & R_{101} \\ R_{101} & R_{100} & R_{001} & R_{010} & R_{011} & R_{000} & R_{110} \\ R_{100} & R_{101} & R_{000} & R_{011} & R_{010} & R_{001} & R_{111} \\ R_{011} & R_{010} & R_{111} & R_{100} & R_{101} & R_{110} & R_{000} \\ R_{010} & R_{011} & R_{110} & R_{101} & R_{100} & R_{111} & R_{101} \\ R_{001} & R_{000} & R_{101} & R_{110} & R_{111} & R_{100} & R_{010} \\ R_{000} & R_{001} & R_{100} & R_{111} & R_{110} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_5 = \begin{bmatrix} R_{110} & R_{111} & R_{010} & R_{100} & R_{001} & R_{011} & R_{101} \\ R_{101} & R_{100} & R_{001} & R_{111} & R_{010} & R_{000} & R_{110} \\ R_{100} & R_{101} & R_{000} & R_{110} & R_{011} & R_{001} & R_{111} \\ R_{011} & R_{010} & R_{111} & R_{001} & R_{100} & R_{110} & R_{000} \\ R_{010} & R_{011} & R_{110} & R_{000} & R_{101} & R_{111} & R_{101} \\ R_{001} & R_{000} & R_{101} & R_{011} & R_{110} & R_{100} & R_{010} \\ R_{000} & R_{001} & R_{100} & R_{010} & R_{111} & R_{101} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_6 = \begin{bmatrix} R_{110} & R_{111} & R_{010} & R_{100} & R_{000} & R_{001} & R_{101} \\ R_{101} & R_{100} & R_{001} & R_{111} & R_{011} & R_{010} & R_{110} \\ R_{100} & R_{101} & R_{000} & R_{110} & R_{010} & R_{011} & R_{111} \\ R_{011} & R_{010} & R_{111} & R_{001} & R_{101} & R_{100} & R_{000} \\ R_{010} & R_{011} & R_{110} & R_{000} & R_{100} & R_{101} & R_{101} \\ R_{001} & R_{000} & R_{101} & R_{011} & R_{111} & R_{110} & R_{010} \\ R_{000} & R_{001} & R_{100} & R_{010} & R_{110} & R_{111} & R_{011} \end{bmatrix}$$

$$\mathbf{A}_7 = \begin{bmatrix} R_{110} & R_{111} & R_{010} & R_{100} & R_{000} & R_{011} & R_{001} \\ R_{101} & R_{100} & R_{001} & R_{111} & R_{011} & R_{000} & R_{010} \\ R_{100} & R_{101} & R_{000} & R_{110} & R_{010} & R_{001} & R_{011} \\ R_{011} & R_{010} & R_{111} & R_{001} & R_{101} & R_{110} & R_{100} \\ R_{010} & R_{011} & R_{110} & R_{000} & R_{100} & R_{111} & R_{101} \\ R_{001} & R_{000} & R_{101} & R_{011} & R_{111} & R_{100} & R_{110} \\ R_{000} & R_{001} & R_{100} & R_{010} & R_{110} & R_{101} & R_{111} \end{bmatrix}$$

Then

$$a_i = \frac{|\mathbf{A}_i|}{|\mathbf{A}|} i = 1, 2, ..., 7 \tag{21}$$

$$\sigma_w = \sqrt{R_{000} - (a_1 R_{111} + a_2 R_{110} + a_3 R_{011} + a_4 R_{101} + a_5 R_{001} + a_6 R_{010} + a_7 R_{100})} \tag{22}$$

Now, we can get the random process model of 3D-ATF.

3.2. **Generation method of 3D-ATF.** There are three steps for generating a 3D-ATF.

Step1, According to the formula (11), the atmospheric turbulence values at axis are made by the simulation method of 1D-ATF. When the directions of the atmospheric turbulence and the axis are the same, formula parameters are calculated by using the vertical correlation function. And when the directions of the atmospheric turbulence and the axis are perpendicular, formula parameters are calculated by using the horizontal correlation function.

For Von Karman model, the vertical and horizontal correlation functions can be respectively expressed as

$$f(\xi) = \frac{2^{2/3}}{\Gamma(1/3)}\zeta^{1/3}K_{1/3}(\zeta) \tag{23}$$

$$g(\xi) = \frac{2^{2/3}}{\Gamma(1/3)}\zeta^{1/3}[K_{1/3}(\zeta) - \frac{1}{2}\zeta K_{2/3}(\zeta)] \tag{24}$$

Where, ξ is distance (m),.., a=1.339, L is the scale of ATF, Γ is Gamma function, K is Bessel function.

Step 2, With the atmospheric turbulence values at axis as the boundary conditions, the atmospheric turbulence values on three boundary surfaces, xoy, xoz and yoz are generated by using formula (22). Equation parameters are computed by the spatial correlation function. Spatial correlation function for Von Karman model can be obtained by Batchelor formula which is formed by the horizontal and vertical correlation functions. The formula model is given as

$$\begin{cases} R_{uu}(\xi_1,\xi_2,\xi_3) = \sigma_n^2 \frac{2^{2/3}}{\Gamma(1/3)}\zeta^{1/3}\left[K_{1/3}(\zeta) - \frac{1}{2}\zeta K_{2/3}(\zeta)\frac{\xi_2^2+\xi_3^2}{\xi^2}\right] \\ R_{vv}(\xi_1,\xi_2,\xi_3) = \sigma_n^2 \frac{2^{2/3}}{\Gamma(1/3)}\zeta^{1/3}\left[K_{1/3}(\zeta) - \frac{1}{2}\zeta K_{2/3}(\zeta)\frac{\xi_1^2+\xi_3^2}{\xi^2}\right] \\ R_{ww}(\xi_1,\xi_2,\xi_3) = \sigma_n^2 \frac{2^{2/3}}{\Gamma(1/3)}\zeta^{1/3}\left[K_{1/3}(\zeta) - \frac{1}{2}\zeta K_{2/3}(\zeta)\frac{\xi_1^2+\xi_2^2}{\xi^2}\right] \end{cases} \tag{25}$$

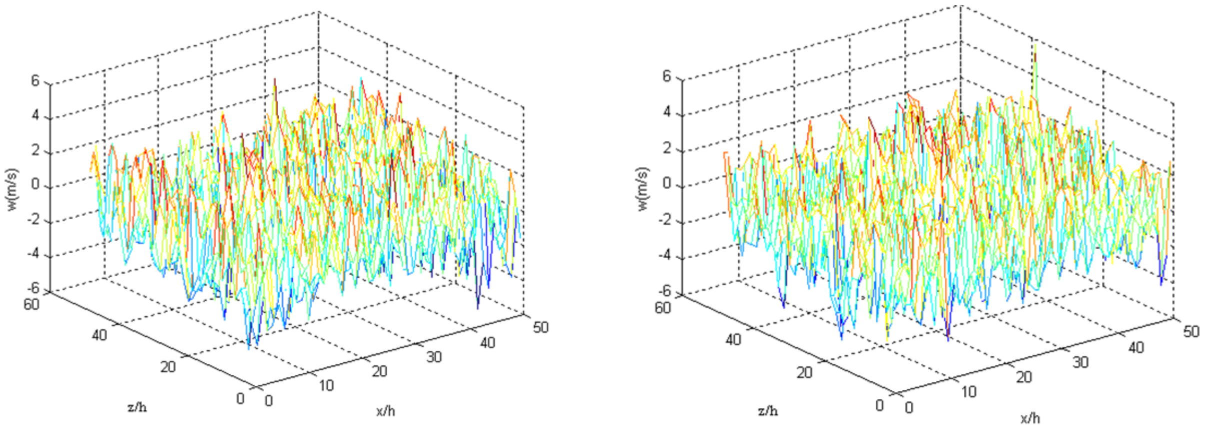
Where, σ is the intension of ATF(m/s), uv and w are separately denoting vertical, horizontal and erect heft.

Step 3, With the atmospheric turbulence values at axis and the values on boundary surfaces as the boundary conditions, the whole 3D-ATF are built using the formula (19). The correlation function is still account by using formula (25).

4. **Experiments and Analysis.**

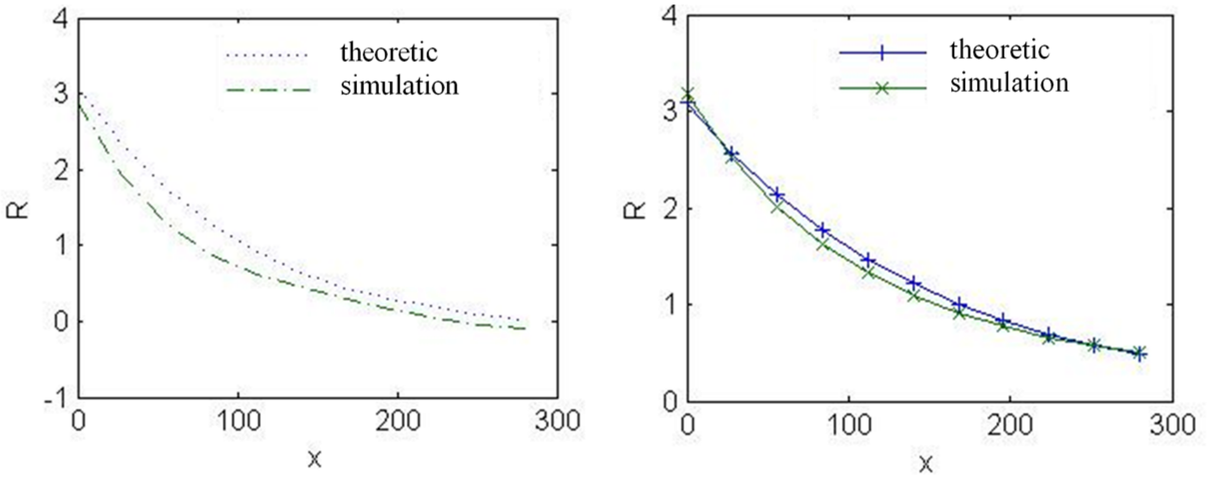
At this part, we use the proposed method to build a 3D-ATF, and the parameters is L=150m, σ=1.7585m/s, and h=70m. The grid number is range of 2000200015. Fig.4 a) and b) show the distribution of vertical component of turbulence field at 400 meters height and 800 meters height level. In these figures, the horizontal axes of x and z represent the grid number, and the vertical axis w is the turbulence strength in meters per second. From these figures, it is apparent that the changes of atmospheric turbulence field are random, which is corresponding with the characteristics of turbulence field.

The horizontal relativity and vertical relativity are respectively calculated and compared with the theoretical value, as presented in Fig.5 a) and Fig.5 b). The relativity difference in simulation value and theoretic value is employed as the evaluation parameter. It is clear that the trend of the relativity curves of both simulation algorithms is similar to the theoretic value. The relativity decreases with the increasing of the distance between points. It can be inferred that the ATF generated by the proposed simulation method have excellent statistical characteristic. For real-time simulation, the atmospheric turbulence data can be generated in advance and stored in the database. The simulation method for the u and v directions can refer to the w direction and the conclusion is the same.



a) H=400m b) *H*=800m

Figure 4. Profile of turbulence field with Von Karman spectrum at different height



a) horizontal relativity b) vertical relativity

Figure 5. Relativity Test

5. **Conclusion.** This paper proposed a simulation method of three-dimensional atmospheric turbulence, which is based on the correction function method and Von Karman model. The generation process of 3D-ATF is iterative computation, so this method can generate large scale turbulence field without calculating in advance and storing the huge amount of data. Finally, from the result of experiments, we can see that the 3D-ATF generated by our method have excellent statistical characteristic. Although the 3D-ATF is based on the Von Karman turbulence model, this method is not limited to be used in the flight simulation of Von Karman turbulence model. The method can also be applied in flight simulations with other turbulence models, as long as the spatial correlation function can be determined.

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