

An Image Steganography Scheme Using 3D-Sudoku

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ABSTRACT. *Steganography, an information-hiding technique, is that embedding secret information into a cover-media to generate a meaningful stego-media. This paper proposes a novel image steganography scheme using 3D-Sudoku. In this paper, a cyclically moving algorithm is used to construct a 3D-Sudoku. The data-embedding phase is that the pixels of the cover-image as the coordinate of 3D-Sudoku are modified in the rule of a minimal distortion to indicate the position of a given secret data. And in the data-extraction phase, the modified pixels of the stego-image as the coordinate of 3D-Sudoku are applied to directly extract the embedded secret data. The experimental results show that the visual quality of stego-image in the proposed scheme is slightly less than that in the steganography scheme using 2D-Sudoku, but the embedding capacity in the proposed scheme is higher than that in the steganography scheme using 2D-Sudoku.*

Keywords: Steganography; Cover-media; Stego-media; 3D-Sudoku

1. **Introduction.** With the development of networks, the security of some important digital information attracts considerable attention in the transmission process. Therefore, more and more data hiding schemes have been proposed to solve the problem of information security. Steganography is an application of data hiding, in which a digital image [1], file [2], audio [3] or video [4] can be concealed within another digital media which is imperceptible and undetectable. Its advantage is that the cover media carrying the secret information does not attract attention. In other words, the stego-image is hard to be distinguished by human eyes.

For digital images, much redundancy in the spatial domain is available to embed secret data. Thus, many data hiding techniques [5, 6, 7, 8, 9, 10, 11, 12] based on digital images have been studied extensively in recent decades. In 1989, a classical information hiding method named least significant bit (LSB) replacement was proposed [9]. The main idea of LSB is that using secret bits to replace the least significant bits of the cover pixels. Because the impact of flipping the least significant bits is very small, a good visual quality can be achieved in a sufficient embedding capacity. In 2006, Mielikainen [10] improved the LSB method by introducing a binary function to choose a desired value to add or subtract one from the cover pixels and four embedding rules. It achieves the same payload as

LSB method with fewer modifications to the cover image. However, the advantages of modification-directions haven't been fully treated. Therefore, Zhang and Wang (2006) [11] proposed an Exploiting Modification Direction (EMD) method which further extended Mielikainens method. In their scheme, n cover pixels can carry a secret digit in a base- $(2n+1)$ numeral system by using EMD method. Due to its full use of modification-directions, their scheme provided a relatively high embedding efficiency. In 2008, inspired from Zhang and Wang's method and the solutions of Sudoku, Chang et al. proposed an information hiding scheme using Sudoku [12]. Sudoku [13, 14] is a logic-based 2D-matrix with the size of 9×9 and contains nine 3×3 sub-matrices. The scheme achieved a high embedding capacity and increased the security at the same time.

In this paper, a novel image steganography scheme is proposed by using 3D-Sudoku to further improve the embedding capacity. A cyclically moving algorithm used to construct $8 \times 8 \times 8$ 3D-Sudoku has been proposed in the scheme [15]. In [15], the 3D-Sudoku matrix was applied to a low-bit rate speech codec. It inspired us to use the 3D-Sudoku to conceal confidential information more effectively. In the data-embedding phase, the constructed $8 \times 8 \times 8$ 3D-Sudoku is expanded to $256 \times 256 \times 256$ 3D-Sudoku in our scheme. The pixels of the cover-image are used as the coordinate of 3D-Sudoku to find three 8×8 planes (i.e., X -plane, Y -plane, and Z -plane) and one $4 \times 4 \times 4$ sub-cube. Then the given secret data can be found in three 8×8 planes (i.e., X -plane, Y -plane, and Z -plane) and one $4 \times 4 \times 4$ sub-cube. These pixels are modified as the coordinate of the given secret data in the 3D-Sudoku in the rule of a minimal distortion. In the extraction phase, the modified pixels can directly indicate the position of the secret data in the 3D-Sudoku and it is convenient to extract the secret data. From the experimental results, our proposed scheme can achieve the goals of high embedding capacity and acceptable visual quality. In addition, because the 3D-Sudoku has a larger number of possible solutions than 2D-Sudoku, the proposed scheme is more secure.

The remainder of the paper is organized as follows. Section 2 introduces the related work, including traditional Sudoku, and Chang et al.'s information hiding method [12]. Section 3 presents the details of our proposed method. The experimental results obtained from tests of our proposed method are presented in Section 4. Finally, our conclusions of this paper are presented in Section 5.

2. Related work.

2.1. Traditional Sudoku. Sudoku is a logic-based combinatorial number-placement puzzle. It is initially popular in Japan in 1986 and was widespread in 2005. Traditional Sudoku is a 2-dimension matrix with the size of 9×9 and contains nine 3×3 sub-matrices. Each element in the traditional Sudoku matrix is filled with one of the numbers 1 to 9. FIGURE 1 shows an example of a traditional Sudoku solution. The rules of constructing a Sudoku matrix are described as follows.

- (1) Each row of Sudoku matrix contains nine different digits from 1 to 9.
- (2) Each column of Sudoku matrix contains nine different digits from 1 to 9.
- (3) Each 3×3 sub-matrix contains nine different digits from 1 to 9.

2.2. Chang et al.'s method. In 2008, Chang et al. proposed an information hiding scheme to improve embedding capacity by using the traditional Sudoku [12]. In the scheme, firstly, all digits of the traditional Sudoku matrix subtract one, i.e., each row, each column, and each 3×3 sub-matrix contain nine different digits from 0 to 8. Then the new Sudoku matrix is expanded to obtain a 256×256 Sudoku matrix M as shown in FIGURE 2. Secondly, the secret bit stream is segmented and every segment contains six secret bits. And then each 6-bit segment is converted into two digits with base-9 numeral

2	1	5	6	4	7	3	9	8
3	6	8	9	5	2	1	7	4
7	9	4	3	8	1	6	5	2
5	7	6	2	7	4	9	3	1
1	4	2	5	9	3	8	6	7
9	7	3	8	1	6	4	2	5
8	2	1	7	3	9	5	4	6
6	5	9	4	2	8	7	1	3
4	3	7	1	6	5	2	8	9

FIGURE 1. An example of a traditional Sudoku solution

system. For example, a 6-bit segment 011001_2 is converted into two digits 27_9 , i.e., $s_1=2$, $s_2=7$. Therefore, for all secret bits, the converted secret digits can be denoted by the sequence $S = \{s_1, s_2, \dots, s_n\}$, where n is the number of all secret digits.

In the embedding phase, a pixel pair (p_1, p_2) of the cover-image as the coordinate of matrix M is used to construct three sequences from matrix M , i.e., the row sequence SR , the column sequence SC , and the sub-matrix sequence SM . For a pixel pair (p_1, p_2) , three sets of sequences SR , SC , and SM are constructed by following rules.

Rule 1: Constructing the row sequence SR .

(1) If $3 < p_2 < 252$, then $SR = \{M(p_1, p_2 - 4), M(p_1, p_2 - 3), M(p_1, p_2 - 2), M(p_1, p_2 - 1), M(p_1, p_2), M(p_1, p_2 + 1), M(p_1, p_2 + 2), M(p_1, p_2 + 3), M(p_1, p_2 + 4)\}$.

(2) If $0 \leq p_2 \leq 3$, then $SR = \{M(p_1, 0), M(p_1, 1), M(p_1, 2), M(p_1, 3), M(p_1, 4), M(p_1, 5), M(p_1, 6), M(p_1, 7), M(p_1, 8)\}$.

(3) If $252 \leq p_2 \leq 255$, then $SR = \{M(p_1, 247), M(p_1, 248), M(p_1, 249), M(p_1, 250), M(p_1, 251), M(p_1, 252), M(p_1, 253), M(p_1, 254), M(p_1, 255)\}$.

Rule 2: Constructing the column sequence SC .

(1) If $3 < p_1 < 252$, then $SC = \{M(p_1 - 4, p_2), M(p_1 - 3, p_2), M(p_1 - 2, p_2), M(p_1 - 1, p_2), M(p_1, p_2), M(p_1 + 1, p_2), M(p_1 + 2, p_2), M(p_1 + 3, p_2), M(p_1 + 4, p_2)\}$.

(2) If $0 \leq p_1 \leq 3$, then $SC = \{M(0, p_2), M(1, p_2), M(2, p_2), M(3, p_2), M(4, p_2), M(5, p_2), M(6, p_2), M(7, p_2), M(8, p_2)\}$.

(3) If $252 \leq p_1 \leq 255$, then $SC = \{M(247, p_2), M(248, p_2), M(249, p_2), M(250, p_2), M(251, p_2), M(252, p_2), M(253, p_2), M(254, p_2), M(255, p_2)\}$.

Rule 3: constructing the sub-matrix sequence SM .

(1) If $0 < p_1 < 255$ and $0 < p_2 < 255$, then $SM = \{M(x, y), M(x, y + 1), M(x, y + 2), M(x + 1, y), M(x + 1, y + 1), M(x + 1, y + 2), M(x + 2, y), M(x + 2, y + 1), M(x + 2, y + 2)\}$, where $x = \lfloor \frac{p_1}{3} \rfloor \times 3, y = \lfloor \frac{p_2}{3} \rfloor \times 3$.

(2) If $p_1 = 255$ or $p_2 = 255$, then $SM = \{Empty\}$.

For a given secret digit s_i ($i=1,2,\dots,n$), three coordinates (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) can be found from three sequences SR , SC , and SM , respectively. The pixel pair (p_1, p_2) is modified as (p'_1, p'_2) in the rule of a minimal distortion to indicate the position (x_{min}, y_{min}) of the given secret digit s_i in the matrix M .

In the extraction phase, the pixel pair (p'_1, p'_2) of the stego-image as the coordinate is to extract the embedded secret digit with the same Sudoku matrix M used in the embedding phase.

		P_y																								
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	...	252	253	254	255		
P_x	0	1	0	4	5	3	6	2	8	7	1	0	4	5	3	6	2	8	7	...	1	0	4	5		
	1	2	5	7	8	4	1	0	6	3	2	5	7	8	4	1	0	6	3	...	2	5	7	8		
	2	6	8	3	2	7	0	5	4	1	6	8	3	2	7	0	5	4	1	...	6	8	3	2		
	3	4	6	5	1	6	3	8	2	0	4	6	5	1	6	3	8	2	0	...	4	6	5	1		
	4	0	3	1	4	8	2	7	5	6	0	3	1	4	8	2	7	5	6	...	0	3	1	4		
	5	8	6	2	7	0	5	3	1	4	8	6	2	7	0	5	3	1	4	...	8	6	2	7		
	6	7	1	0	6	2	8	4	3	5	7	1	0	6	2	8	4	3	5	...	7	1	0	6		
	7	5	4	8	3	1	7	6	0	2	5	4	8	3	1	7	6	0	2	...	5	4	8	3		
	8	3	2	6	0	5	4	1	7	8	3	2	6	0	5	4	1	7	8	...	3	2	6	0		
	9	1	0	4	5	3	6	2	8	7	1	0	4	5	3	6	2	8	7	...	1	0	4	5		
	10	2	5	7	8	4	1	0	6	3	2	5	7	8	4	1	0	6	3	...	2	5	7	8		
	11	6	8	3	2	7	0	5	4	1	6	8	3	2	7	0	5	4	1	...	6	8	3	2		
	12	4	6	5	1	6	3	8	2	0	4	6	5	1	6	3	8	2	0	...	4	6	5	1		
	13	0	3	1	4	8	2	7	5	6	0	3	1	4	8	2	7	5	6	...	0	3	1	4		
	14	8	6	2	7	0	5	3	1	4	8	6	2	7	0	5	3	1	4	...	8	6	2	7		
	15	7	1	0	6	2	8	4	3	5	7	1	0	6	2	8	4	3	5	...	7	1	0	6		
	16	5	4	8	3	1	7	6	0	2	5	4	8	3	1	7	6	0	2	...	5	4	8	3		
	17	3	2	6	0	5	4	1	7	8	3	2	6	0	5	4	1	7	8	...	3	2	6	0		
...		
252	1	0	4	5	3	6	2	8	7	1	0	4	5	3	6	2	8	7	...	1	0	4	5			
253	2	5	7	8	4	1	0	6	3	2	5	7	8	4	1	0	6	3	...	2	5	7	8			
254	6	8	3	2	7	0	5	4	1	6	8	3	2	7	0	5	4	1	...	6	8	3	2			
255	4	6	5	1	6	3	8	2	0	4	6	5	1	6	3	8	2	0	...	4	6	5	1			

FIGURE 2. An example of the expanded matrix M solution

3. The proposed scheme. In this section, the details of the proposed scheme will be introduced. A cyclically moving algorithm is proposed by [15] to construct a 3D-Sudoku. Our goal is to improve the embedding capacity using 3D-Sudoku and achieve acceptable visual quality.

3.1. Construct a 3D-Sudoku matrix. Before embedding secret data, an $8 \times 8 \times 8$ 3D-Sudoku matrix which contains eight $4 \times 4 \times 4$ sub-cubes will be constructed. According to the rules of constructing traditional Sudoku, some rules should be referred to design 3D-Sudoku as follows:

- (1) Each plane of X -plane contains 64 different digits from 0 to 63.
- (2) Each plane of Y -plane contains 64 different digits from 0 to 63.
- (3) Each plane of Z -plane contains 64 different digits from 0 to 63.
- (4) Each sub-cube contains 64 different digits from 0 to 63.

An example of 3D-Sudoku is shown in FIGURE 3. For the convenience to describe the process of constructing 3D-Sudoku, the 3D-Sudoku is regarded as a 3-dimensional coordinate system and some symbols are applied. They are

- $P(X, Y, Z)$: the planes of 3D-Sudoku,
- $C(X, Y, Z)$: the sub-cubes of 3D-Sudoku.

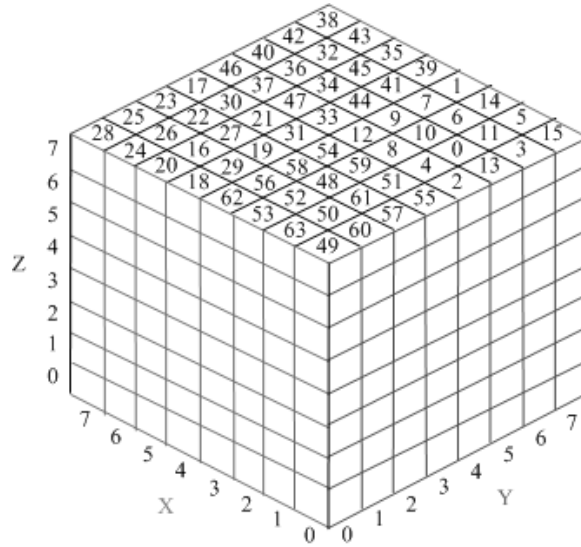


FIGURE 3. An example of 3D-Sudoku in the 3-dimensional coordinate system

According to the above rules, a cyclically moving algorithm will be introduced to construct 3D-Sudoku in the following.

Step 1. Construct the plane $P(0 : 7, 0 : 7, 0)(Z = 0)$. Firstly, the plane $P(0 : 7, 0 : 7, 0)$ is divided into four sub-planes (i.e., $P(0 : 3, 0 : 3, 0)$, $P(0 : 3, 4 : 7, 0)$, $P(4 : 7, 0 : 3, 0)$, and $P(4 : 7, 4 : 7, 0)$). For the sub-plane $P(0 : 3, 0 : 3, 0)$, it is randomly filled with different numbers from 0 to 15. Then, $P(0 : 3, 0 : 3, 0)$ adds 16 to generate $P(4 : 7, 0 : 3, 0)$, adds 32 to generate $P(0 : 3, 4 : 7, 0)$, and adds 48 to generate $P(4 : 7, 4 : 7, 0)$. An example of the constructed plane $P(0 : 7, 0 : 7, 0)$ is shown in FIGURE 4.

Y	Z=0							
7	45	35	47	34	61	51	63	50
6	41	39	33	44	57	55	49	60
5	42	38	46	40	58	54	62	56
4	32	43	37	36	48	59	53	52
3	13	3	15	2	29	19	31	18
2	9	7	1	12	25	23	17	28
1	10	6	14	8	26	22	30	24
0	0	11	5	4	16	27	21	20
	0	1	2	3	4	5	6	7

FIGURE 4. An example of the constructed plane $P(0:7,0:7,0)$

Step 2. Construct the remaining seven planes $P(0 : 7, 0 : 7, Z)(Z = 1, 2, \dots, 7)$. The operation of constructing the remaining seven planes is denoted by Eq. (1).

$$P(0 : 7, 0 : 7, k) = (P(0 : 7, 0 : 7, 0) + 16 \times k) \bmod 64, k \in [1, 7] \tag{1}$$

Step 3. For all sub-cubes, the cyclically moving algorithm is applied to make them satisfy Rule (4) described in this section. The sub-cubes $C(0 : 3, 0 : 3, 0 : 3)$, $C(4 : 7, 0 :$

$3, 0 : 3$), $C(0 : 3, 4 : 7, 0 : 3)$, $C(4 : 7, 4 : 7, 0 : 3)$, $C(0 : 3, 0 : 3, 4 : 7)$, $C(4 : 7, 0 : 3, 4 : 7)$, $C(0 : 3, 4 : 7, 4 : 7)$, and $C(4 : 7, 4 : 7, 4 : 7)$ are marked as No.1 to No.8, respectively. According to ascending Z coordinate, each sub-cube is layered into four 4×4 planes. The four planes are marked as Layer1 to Layer4 in order. For a 4×4 plane, there is one cyclically moving operation including both one cyclically upward moving and one cyclically left moving. FIGURE 5 shows an example of one cyclically moving operation. The number of cyclically moving operation is different for different layers of different sub-cubes. TABLE 1 lists the number of cyclically moving operations for each layer of sub-cubes.

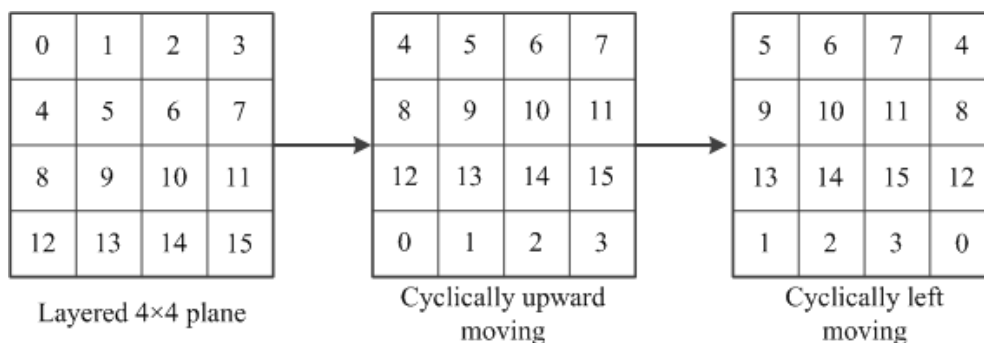


FIGURE 5. An example of one cyclically moving operation

TABLE 1. The number of cyclically moving operations for each layer of sub-cubes

Layer	Sub-cubes							
	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8
1	0	0	0	0	2	3	3	2
2	1	0	1	0	2	3	3	2
3	1	1	1	1	2	3	3	2
4	0	1	0	1	2	3	3	2

3.2. The data-embedding phase. The main idea of the proposed scheme is that three pixels of cover-image are modified to imply the position of a given secret digit in the 3D-Sudoku. Because the range of all pixels in digital image is from 0 to 255, the $8 \times 8 \times 8$ 3D-Sudoku is expanded to $256 \times 256 \times 256$ 3D-Sudoku matrix M' . The embedding phase is described in the following.

Step 1. The secret bit stream is segmented and every segment contains 6 secret binary bits. Then each 6-bit segment is converted into a decimal number as the secret digit sd_i . All secret digits are denoted as the sequence $SD = \{sd_1, sd_2, \dots, sd_i, \dots, sd_N\}$, where N is the number of all secret digits.

Step 2. Each three pixels of cover-image are grouped to compose the sequence $SI = \{(p_1, p_2, p_3)_1, (p_4, p_5, p_6)_2, \dots, (p_a, p_b, p_c)_i, \dots, (p_{3m-2}, p_{3m-1}, p_{3m})_m\}$, where $m = \lfloor \frac{H \times W}{3} \rfloor$, and H and W are the height and width of the cover-image, respectively.

Step 3. For a given secret digit sd_i , the pixel group $(p_a, p_b, p_c)_i$ will be selected to embed this secret digit. The coordinate value of the pixel group $(p_a, p_b, p_c)_i$ in the 3D-Sudoku matrix M' is denoted by $M'(p_a, p_b, p_c)$.

Step 3.1. If $3 \leq p_a, p_b, p_c \leq 251$, the pixel group $(p_a, p_b, p_c)_i$ as the coordinate of $256 \times 256 \times 256$ 3D-Sudoku matrix M' is to find three 8×8 planes P ($P(X, 0 : 7, 0 : 7)$, $P(0 : 7, Y, 0 : 7)$,

and $P(0 : 7, 0 : 7, Z)$) and one $4 \times 4 \times 4$ sub-cube $C(X, Y, Z)$ according to the Rule 1 to Rule 4 as shown in Table 3. Otherwise, the pixel group $(p_a, p_b, p_c)_i$ as the coordinate of matrix M' is only to construct one $4 \times 4 \times 4$ sub-cube C according to the Rule 4 as shown in TABLE 2.

TABLE 2. Number of blocks that used different path selection modes when thresholds were different

Rules	The constructed three planes and one sub-cube
1	X-plane: $P(X, 0:7, 0:7) = M'(p_a, p_b-3:p_b+4, p_c-3:p_c+4)$
2	Y-plane: $P(0:7, Y, 0:7) = M'(p_a-3:p_a+4, p_b, p_c-3:p_c+4)$
3	Z-plane: $P(0:7, 0:7, Z) = M'(p_a-3:p_a+4, p_b-3:p_b+4, p_c)$
4	Sub-cube: $C(X, Y, Z) = M'(\lfloor \frac{p_a}{4} \rfloor : \lfloor \frac{p_a}{4} \rfloor + 3, \lfloor \frac{p_b}{4} \rfloor : \lfloor \frac{p_b}{4} \rfloor + 3, \lfloor \frac{p_c}{4} \rfloor : \lfloor \frac{p_c}{4} \rfloor + 3)$

Step 3.2. In the three planes and one sub-cube, the corresponding coordinates of $M'(p_a, p_b, p_c)$ are found, i.e., (x_{11}, y_{11}, z_{11}) , (x_{12}, y_{12}, z_{12}) , (x_{13}, y_{13}, z_{13}) , (x_{14}, y_{14}, z_{14}) , and the corresponding coordinates of the given secret digit sd_i are found, i.e., (x_{21}, y_{21}, z_{21}) , (x_{22}, y_{22}, z_{22}) , (x_{23}, y_{23}, z_{23}) , (x_{24}, y_{24}, z_{24}) . The minimal distortion between (x_{1t}, y_{1t}, z_{1t}) and (x_{2t}, y_{2t}, z_{2t}) ($t = 1, 2, 3, 4$) is computed by Eq. (2).

$$D_{min} = \operatorname{argmin}(|x_{1t} - x_{2t}| + |y_{1t} - y_{2t}| + |z_{1t} - z_{2t}|), t = 1, 2, 3, 4. \quad (2)$$

Assuming the distortion between (x_{1i}, y_{1i}, z_{1i}) and (x_{2i}, y_{2i}, z_{2i}) ($i = 1, 2, 3, 4$) is minimal. Their differences are computed by Eq. (3).

$$\begin{cases} d_1 = x_{2i} - x_{1i} \\ d_2 = y_{2i} - y_{1i} \\ d_3 = z_{2i} - z_{1i} \end{cases} \quad (3)$$

Step 3.2. According to the minimal distortion D_{min} , the pixel group $(p_a, p_b, p_c)_i$ is modified as $(p'_a, p'_b, p'_c)_i$ to embed the given secret digit sd_i . The process of modifying the pixel group of cover-image should be described by Eq. (4).

$$\begin{cases} p'_a = p_a + d_1 \\ p'_b = p_b + d_2 \\ p'_c = p_c + d_3 \end{cases} \quad (4)$$

Step 4. All the pixel group $(p_a, p_b, p_c)_i$ are modified as $(p'_a, p'_b, p'_c)_i$ to generate the stego-image.

In order to clearly express the process of embedding secret digit, an example is given in the following. Assuming a 6-bit secret segment is 001110_2 . It is converted into the decimal number 14_{10} . Assuming a pixel group of cover-image is $(156, 101, 79)$. It is used as the coordinate of 3D-Sudoku to find the three planes and one sub-cube as shown in FIGURE 6. The value of $M'(156, 101, 79)$ is 44. It is obviously that the distortion of pixels modification is minimal in plane $P(0 : 7, 0 : 7, Z)$. The set of differences (d_1, d_2, d_3) is $(-2, 2, 0)$. Therefore, the pixel group $(156, 101, 79)$ of cover-image is modified as the pixel group $(154, 103, 79)$ of stego-image to embed the secret segment 001110_2 .

3.3. The data-extraction phase. In the extraction phase, the position of the secret digit in the $256 \times 256 \times 256$ 3D-Sudoku is indicated by the three pixels of stego-image, so it is convenient to extract the secret digit. All pixels of stego-image are used to compose a sequence in the raster scanning order. The sequence is grouped and each group contains

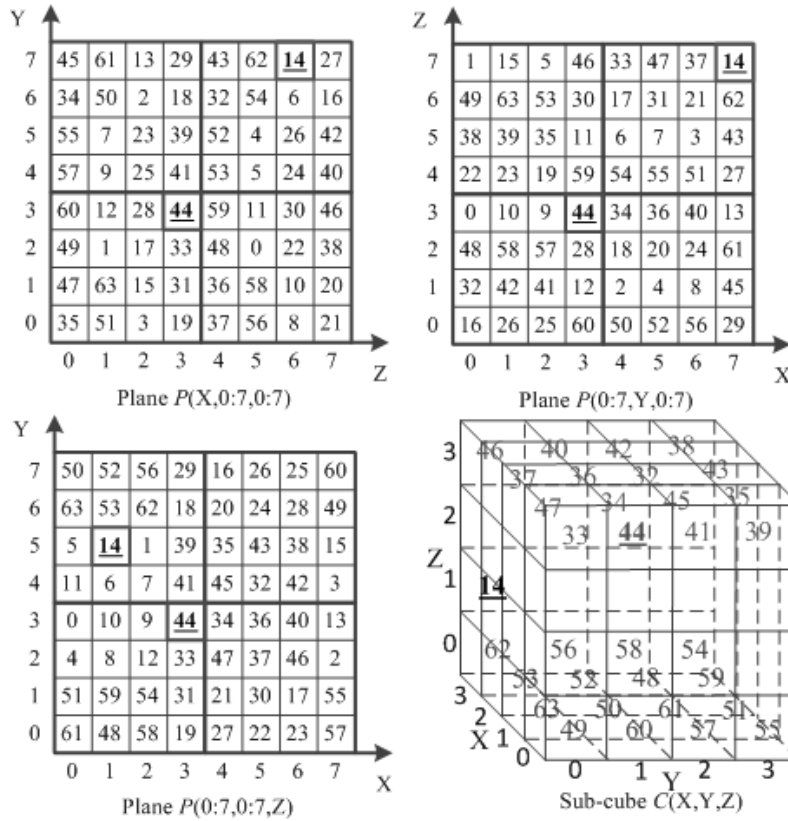


FIGURE 6. Three planes and one sub-cube

three pixels. Then the 3-pixel group as the coordinate of 3D-Sudoku is to extract the embedded secret digit.

4. Experimental Results. In this section, the experimental results are presented to demonstrate the performances of our proposed scheme. Six 512×512 grayscale images as shown in FIGURE 7 are used as the test images, i.e., Lena, Peppers, F16, Boat, Goldhill, and Baboon. The simulation environment is equipped with the MATLAB platform. In the experiment, two factors (i.e., the quality of stego-image and embedding capacity) are used to evaluate the performances of a data hiding scheme. For the quality of stego-image, it is measured by using the peak-signal-to-noise ratio (PSNR). Generally speaking, a higher PSNR reflects that the stego-image has a better visual quality. The PSNR is defined as follows.

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right) (dB), \tag{5}$$

where MSE is the mean squared error between the original image and the recovered image with the size of $H \times W$, and it is defined as:

$$MSE = \frac{1}{H \times W} \sum_{i=1}^H \sum_{j=1}^W [p_{(i,j)} - q_{(i,j)}]^2. \tag{6}$$

where $p_{i,j}$ and $q_{i,j}$ denote the pixel values of the original image and the recovered image, respectively.

For embedding capacity, it is measured by using the bit-per-pixel (bpp). The bit-per-pixel means that how many bits can be embedded into a cover pixel. A larger embedding

capacity EC indicates that a larger number of secret bits can be embedded into cover-image. The bit-per-pixel is defined in the following to denote the embedding capacity EC .

$$EC = \frac{|S|}{H \times W} (bpp), \quad (7)$$

where $|S|$ is the number of all embedded secret bits, and H and W are the height and width of the cover-image, respectively.

TABLE 3 lists the PSNR and the embedding capacity EC of our proposed scheme and Chang et al.'s scheme [12]. From the experimental results, our proposed scheme can achieve a higher embedding capacity than Chang et al.'s scheme. But the visual quality of Chang et al.'s scheme is better than that of our proposed scheme. This is because visual quality and embedding capacity are always inversely proportional. In our scheme, the three pixels of cover-image as the coordinate of 3D-Sudoku are modified to embed a secret digit, that is $\log_2 64$ -bit secret digit is embedded into each three pixels. The embedding capacity is 524286 bits and denoted by 2 bpp. However, in Chang et al.'s scheme, the two pixels as the coordinate of 2D-Sudoku are modified to embed secret digit, that is $\lfloor \log_2 9 \rfloor$ -bit secret digit is embedded into two pixels. The embedding capacity is 393216 bits and denoted by 1.5 bpp. Therefore, our proposed scheme increases 0.5 bpp (131070 bits). Although the PSNR of the proposed scheme is lower than that of Chang et al.'s scheme, it still reaches to 41 dB on average which indicates that the visual quality is relatively high. As a result, our proposed scheme achieves a higher embedding capacity in the case of a good visual quality. In addition, a flexible mechanism can be chosen by users to determine the embedding capacity and PSNR. That is, the PSNR can be improved by reducing the embedding capacity.



FIGURE 7. Six tested 512×512 standard gray images

5. Conclusions. In this paper, a novel image steganography scheme is proposed to improve the embedding capacity by using 3D-Sudoku. The 3D-Sudoku is multifarious (i.e., the 3D-Sudoku has a large number of possible solutions). Therefore, our proposed scheme has a very high security. In addition, because the pixels of cover-image are modified in the 8×8 plane or $4 \times 4 \times 4$ sub-cube, our proposed scheme can embed a 6-bit secret digit into

TABLE 3. The embedding capacity and PSNR of the proposed scheme and Chang et al.'s scheme [12]

Method	Performances	Image					
		Lena	Peppers	F16	Boat	Goldhill	Baboon
Proposed	Capacity(bpp)	2	2	2	2	2	2
	PSNR(dB)	41.31	41.30	41.28	41.23	41.29	41.25
Chang et al.'s	Capacity(bpp)	1.5	1.5	1.5	1.5	1.5	1.5
	PSNR(dB)	44.96	44.67	44.99	44.90	44.85	44.68

three pixels and achieves a higher embedding capacity than the scheme using 2D-Sudoku. And the visual quality of the proposed scheme can still reach 41dB on average which satisfies that the stego-image is hard to be identified by our human eyes. Compared with the data hiding scheme using 2D-Sudoku, our proposed scheme using 3D-Sudoku is more suitable for the high-payload-needed image which achieves acceptable visual quality.

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