Fixpoint Logics and Automata: A Coalgebraic Approach

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A long and fertile tradition in theoretical computer science, going back to the work of Büchi and Rabin, links the field of (fixpoint) logic to that of automata theory. In particular, automata operating on potentially infinite structures such as streams, trees, graphs or transition systems, provide an invaluable tool for the specification and verification of the ongoing behavior of systems. An interesting phenomenon in this branch of automata theory is that some of key results (such as determinization) hold for stream automata only, while many others hold of stream, tree and graph automata alike, and can even be proved for automata operating on yet more complex structures. This naturally begs the question whether the theory of automata operating on infinite objects can be lifted a higher level of generality.

The aim of the talk is to advocate Coalgebra as a nice framework for the development of such a theory. The basic observation is that streams, trees and transition systems are all examples of coalgebras of a certain type. In general, a coalgebra consists of a pair consisting of a set S of states together with a transition map from S to the set FS—here F is the type of the coalgebra, given as a functor F on the category Set (with sets as objects and functions as arrows). Universal Coalgebra is the emerging mathematical theory of such state-based evolving systems, in which concepts such as behavior, indistinguishability, invariants, etc can be modelled in a natural way.

In the talk we give a quick introduction to coalgebra, and we introduce various kinds of automata that are supposed to operate on coalgebras, generalizing the well-known automata that operate on streams, trees, etc. The criterion under which such an automaton accepts or rejects a pointed coalgebra is formulated in terms of a two-player parity game, and with each kind of coalgebra automaton we may naturally associate a language of coalgebraic fixpoint logic. Concretely, we show that some of the central results in automata theory can be generalized to the abstraction level of coalgebras. As examples of such results, we will see that the class of recognizable languages of coalgebras is closed under taking unions, intersections, projections, and complementation. We also prove that if a coalgebra automaton accepts some coalgebra it accepts a finite one of bounded size. Many of these results are based on an explicit construction which transforms a given alternating F-automaton into an equivalent nondeterministic one, of bounded size. Finally, we compare various notions of coalgebra automata, and discuss the foundations of a universal theory of automata.

The point behind the introduction of automata at this level of abstraction is that, in the spirit of Universal Coalgebra, we may gain a deeper understanding of automata theory by studying properties of automata in a uniform manner, parametric in the type of the recognized structures.