

RECOGNIZING VARIABLE SPATIAL ENVIRONMENTS
—THE THEORY OF COGNITIVE PRISM

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von

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To my parents

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- A.2 The fact that region \mathcal{X}_0 is of the same size as region \mathcal{X}_1 can be tested as follows: For any two regions \mathcal{A} and \mathcal{B} , if \mathcal{X}_1 can be moved to such a place that it is connected both with \mathcal{A} and \mathcal{B} , so can \mathcal{X}_0 , and there is no such a place for \mathcal{X}_1 that it is connected both with \mathcal{A} and \mathcal{B} , neither is \mathcal{X}_0 154

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Abstract

Objects are not statically located in environments. Therefore, a snapshot view of an environment at one moment may be quite different from its snapshot at other moments. However, people are capable of recognizing the environment. This thesis presents a theory of recognizing variable spatial environments — The Theory of Cognitive Prism. It collected theoretical and empirical evidences from several disciplines, such as cognitive psychology, neurology, psycho-linguistics, philosophy, and proposes a symbolic computational theory. This thesis has three parts: The first part is the description of the theory in natural language. It presents a commonsense knowledge of the snapshot view of a spatial environment—the “cognitive spectrum”, and of the process of recognizing a spatial environment by comparing two cognitive spectrums. The second part is the mereotopological formalism of the theory. It presents the formal structure of the cognitive spectrum and the process of comparing two formalized cognitive spectrums. The third part introduces a symbolic simulating system of the theory — the LIVE model.

The research questions are: (1) What are the spatial relations between extended objects through observation? (2) According to what principle is the reference ordering between extended objects formed? (3) What is the structure of a cognitive spectrum? (4) How shall two cognitive spectrums be compared? (5) How does the difference between two cognitive spectrums determine the degree of the compatibility between the perceived cognitive spectrum and the target one? (6) How shall the results of Question (1) to (5) be formalized?

Distance relations between extended objects are understood by the degree of the

extension from one extended object to the other; orientation relations between two extended objects are distance comparison between one extended object and sides of the other extended object. The reference ordering between two extended objects is based on the commonsense knowledge of relative stabilities of related objects. The cognitive spectrum is structured by extended objects and their spatial relations which obey the reference ordering. Two cognitive spectrums are compared based on the categories and locations of extended objects. The compatibility of two cognitive spectrums is determined by their differences and relative stabilities of related objects. Recognizing spatial environments is the comparison process between the cognitive spectrum of the perceived environment and the cognitive spectrum of target environment and the judgement of the compatibility between them. All above are mereotopologically formalized by the connectedness relationship **C**. The computational complexity of the formalized recognition process is *polynomial* **P**. A symbolic simulation system, the LIVE model, has been implemented in Lisp.

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Chapter 1

An introduction

Mr. Bertel has lately rented an unfurnished single-room apartment as his home, shown in Figure 1.1(a). The door of the apartment is in the middle of one wall. There is a big window opposite to the door.

On the next day Mr. Bertel buys a writing-desk, a bookshelf, and a big couch, Figure 1.1(b). He puts the writing-desk next to the window for good eyesight. He puts the bookshelf to the left side of the writing-desk. The couch that is used as bed at night is put left to the door.

On the third day Mr. Bertel buys a balloon as a chair, which is good for his back, a small tea-table, and a dining-table, Figure 1.1(c). The balloon is put in front of the writing-desk; the small tea-table is put in front of the couch; the dining-table is close to the wall opposite to the couch.

On the fourth day Mr. Bertel buys some flowers, a lamp, several tea-cups, some books, and a picture, Figure 1.1(d). He puts the lamp on the writing-desk, the flowers on the table. The picture is hung on the wall to decorate his new home. The books are put on the bookshelf. The tea-cups are placed on the tea-table.

On the fifth day Mr. Bertel's mother comes to see her son's new beautiful home.

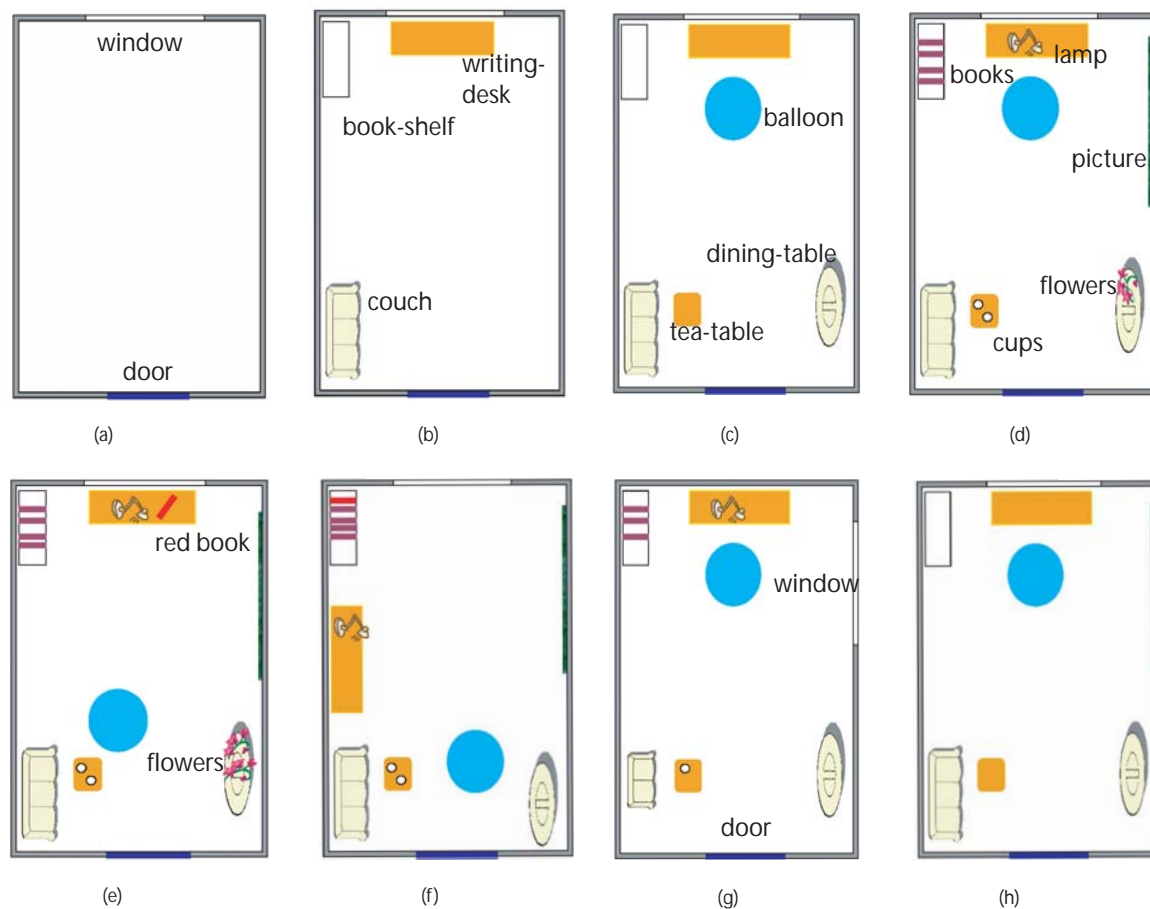


Figure 1.1: The layouts of Mr. Bertel's apartment (a), (b), (c), (d), (e), (f); the layout of Mr. Certel's apartment (g); the layout of Mr. Bertel's apartment as his mother remembers (h)

On the sixth day Mr. Bertel invites his neighbor, Mr. Certel, to his new home. Mr. Certel gives Mr. Bertel a small plastic-bound red book¹, which is put on the writing-desk, and a bunch of flowers that are put on the dining-table. Mr. Certel likes sitting on the balloon to chat with Mr. Bertel who is lying on the couch, Figure 1.1(e).

On the next day Mr. Bertel's mother comes and mumbles, why did her son put

¹Mao Tse Tung's Bible

the balloon to the tea-table? She moves the balloon back to the writing-desk without paying attention to the new book and the flowers from Mr. Certel.

On the eighth day the flowers are withered and thrown away. All the books are put on the bookshelf. In the evening Mr. Bertel moves his writing-desk to the right side of the bookshelf to make room for a party. The balloon is put near the table which is now at the right hand side of the door in the corner, Figure 1.1 (f).

On the ninth day Mr. Bertel's mother comes again. This time she goes into Mr. Certel's apartment by mistake, Figure 1.1(g), while Mr. Certel joined Mr. Bertel's party without closing his door, got tired, and fell asleep on Mr. Bertel's dining-table. Although Mr. Certel's apartment is very similar to Mr. Bertel's apartment, Mr. Bertel's mother notices that she is not in her son's home and leaves the apartment. When she goes to Mr. Bertel's apartment, she finds that the writing-desk is located between the bookshelf and the couch and that the table and the balloon are located differently as she expects, she wonders for a while and accepts it as her son's apartment. When she meets Mr. Bertel, she asks what happened last night.

How is Mr. Bertel's mother able to recognize that she is not in her son's apartment? The window in the perceived environment is located differently and the couch is a little shorter. How could her son have changed this? When Mr. Bertel's mother gets to Mr. Bertel's apartment, he is in the kitchen and Mr. Certel is in the bathroom. However, she is confident that she is in the right apartment, because if the writing-desk along with the balloon is put next to the window, and the table is moved closer to the picture, the apartment looks just like what she has seen before, Figure 1.1(h). She does not mind the absence of the flowers or the different number of tea-cups — one of the tea-cups was broken during the party.

1.1 The aim

The layout of Mr. Bertel's apartment changes to meet his needs, such as to chat with a friend or to hold a party. The configurations of our spatial environments often change, e.g., our offices, kitchens, dining-rooms, sitting-rooms. In our offices chairs are moved very often; books and stationeries are sometimes put on the desk and sometimes into drawers. At our homes plates and bowls are on the table during eating time, after that they are put in the sink and then in the closet. We can see all these changes; however, we do not suspect that they are not our offices or our homes just because of the changed layouts. The aim of this thesis is on the commonsense knowledge needed for recognizing variable spatial environments and to make computational modelling. It researches into the question of how to recognize a remembered spatial environment with regards to the nature of its variable layout.

1.2 How can we recognize variable spatial environments?

How can Mr. Bertel's mother distinguish Mr. Bertel's apartment from Mr. Certel's apartment? The answer appears simple: Because Mr. Bertel's apartment is different from Mr. Certel's. However, if we notice that Mr. Bertel's apartment is not a static environment, we understand the difficulty in the question: How does Mr. Bertel's mother distinguish her son's apartment from other rooms with regards to the changing layouts of all these rooms? Mr. Bertel's mother must have some knowledge of her son's apartment. What kind of knowledge does she have? How can she use the knowledge of Mr. Bertel's apartment to justify whether the perceived apartment is her son's apartment or not?

Some people say that they know which is their office since their office key can open the door; some people say that they know which is their office, because their puppy bear sits on the desk in the office; some say that they know which is their office, because their office door is the biggest on the floor; etc.

What these people say reflects the simplest situation to recognize a spatial environment. They recognize a representative figural object in the spatial environment, with which they can recognize the spatial environment. For example, when we see the Eiffel Tower, we know that we are in Paris; when we see Big Ben, we know that we are in London; when we see the Great Wall, we know that we are in Peking; when we see the Statue of Liberty, we know that we are in New York. However, what happens when the key can open many rooms? What happens when the puppy bear is in a drawer? etc.

This simplest method also fails for cognitive agents who cannot recognize such a representative figural object. For example, a robot has just visited your office and on your order it moves to the kitchen to fetch a cup of coffee. Consider the simplest situation: There is only a corridor connecting your office and the kitchen. The robot must go out of your office, pass the corridor, and enter the kitchen; and then take the same way back. During this time your chair is moved a bit; there are people or other robots walking on the corridor; and the coffee machine in the kitchen may have been moved. With all these variabilities in spatial environments current robots may take much more time, if they can achieve this, than your going to the kitchen and making the coffee yourself.

A better way to recognize spatial environments may be to consider the environment as a whole, and to compare the spatial layout at two different times. When

people get to a spatial environment, they have a snapshot view of the configuration of the environment. They compare the current perceived configuration with the one they remember, and make judgement of the transformation between them. Recognizing an environment can be interpreted as a kind of subjective judgment of the ease of transformation from the remembered configuration into the perceived configuration. If the transformation is very difficult, e.g., the perceived window is located at a different location, then a negative judgment will be made; if the transformation is easy, for example, differently located chairs and books, then a positive judgment will be made.

The following questions will, therefore, be addressed: How shall we represent a spatial configuration? How shall we represent a transformation from one configuration into another? And how shall we judge its ease?

1.3 Interdisciplinary perspectives

The representation of spatial configurations can be explored from different perspectives.

1.3.1 Neurology

Neurology provides cases of brain-impaired patients who lost some of their mental abilities. By examining what mental abilities they lost and what they can still achieve, we can find causal relations among mental abilities. For example, Wilson et al. (1999) reported a patient who could not retrieve images from her long-term memory. The patient lost the ability to retrieve images, thus she cannot recognize single objects as precisely as normal people. On the other hand, she can still recognize spatial

environments, like her home. This could be roughly understood by the findings in cognitive neuroscience that the visual system consists of at least two subsystems: the “what” cortical system and the “where” cortical system, e.g., Ungerleider and Mishkin (1982), Rueckl et al. (1989), Creem and Proffitt (2001). The patient’s “what” cortical system is partially damaged, but her “where” cortical system is normal. Recognizing spatial environments relies more on the “where” cortical system than on the “what” cortical system. This case shows that **recognizing spatial environments does not require much information on figural objects** and that **spatial location of objects are important for recognizing spatial environments**.

1.3.2 Cognitive psychology

Cognitive psychology explores human or animal behavior by performing empirical experiments. Tolman (1948) conducted a series of experiments on rats and showed that rats have “cognitive maps” about spatial environments.

Research of object recognition shows that humans identify objects² at certain level of categories, e.g., “basic level category” by Rosch et al. (1976), or “entry point level” by Jolicoeur et al. (1984). At the “basic level” humans are fastest to categorize instances, Rosch (1975), fastest to identify, Murphy and Smith (1982), and to spontaneously choose a name. The “basic level” object recognition shows that even normal people do not see everything of a single object, though they may see a bit more than the patient reported by Wilson et al. (1999). Thus, the results of neurology and the results of cognitive psychology have the convergent conclusion that **to recognize spatial environments requires to categorize objects, rather than to identify them**.

²Except the recognition of human faces

Psychological experiments on spatial relations found that humans use three kinds of spatial relations: topological relations, qualitative orientation relations, and qualitative distance relations, Piaget and Inhelder (1948).

Psychological research on mental spatial models showed that **spatial models are hierarchically structured and have distortions**, e.g., Tversky (1981), McNamara (1986), McNamara (1991), Tversky (1991), McNamara (1992), Tversky (1992).

1.3.3 Psycho-linguistics

Psycho-linguistics also provides a window to human cognition. The corresponding relationship between mental model of spatial environments and their language descriptions were discussed in Talmy (1983) and Tversky and Lee (1999). This provides theoretical support to explore a person's mental representation of spatial environments by examining her/his language descriptions of spatial layouts. The non-symmetric relation between the location object and the reference object in spatial linguistic descriptions supports the existence of relative stability. This thesis proposes that **people have commonsense knowledge of relative stabilities of objects in a spatial environment and that objects are referenced to more stable objects in an environment which leads to the hierarchical structure of the spatial configuration in the mind.**

1.3.4 Formal spatial ontologies

Grenon and Smith (2004) proposed that a dynamic spatial ontology should combine two distinct types of inventory of the entities and relationships in reality: On the one hand, a purely spatial ontology supporting snapshot views of the space at successive instants of time: SNAP; on the other hand, a purely spatiotemporal ontology of

change and process: SPAN. SPAN is a 4-dimensionalist ontology of processes — it has temporal parts and unfolds itself phase by phase. Following this perspective, recognizing a spatial environment relates two snapshots (one is remembered in mind, the other is currently perceived) and the spatiotemporal relation between them: Can they (or to what extent can they) participate into one SPAN?

Intuitively, we do not recognize a place by checking everything in it. The air in your office, a sheet of paper, apples on a table and contents of the dustbin do not help recognize your office. This thesis raises the ontological question for the task of recognizing spatial environments: **What exists in an environment that makes it to be that environment?**

Smith and Varzi (2000) distinguish two basic typologies of boundaries: *Bona fide* (or physical) boundary and *fiat* (in the sense of human decision or delineation) boundary. Two basic typologies of spatial objects follow from this distinction: *Bona fide* objects which have *bona fide* boundaries and *fiat* objects which only have *fiat* boundaries. This thesis distinguishes *fiat* ontologies from *bona fide* ontologies following Smith and Varzi (2000), Smith (2001). It proposes that **recognizing spatial environments relates to *fiat* ontologies through perception, cognition, and language expressions**. It has little to do with finger-print or foot-print checking, molecule analysis, or DNA testing.

1.3.5 Computational modelling

Computer science, especially artificial intelligence, provides methods of knowledge representation, symbolic system construction, and implementation to simulate cognitive phenomena. The philosophical assumption behind experimental computational modelling is that cognitive phenomena are computable, Newell and Simon (1972),

Newell (1980). The method of computational modelling has been successfully applied to cognitive structures and processes of environmental spatial knowledge, e.g., Kuipers (1977), Kuipers (1978), Yeap (1988), Yeap and Jefferies (1999), Klippel (2003a), Klippel (2003b), Klippel et al. (2004), Klippel and Richter (2004), Klippel and Montello (2004), Klippel et al. (2005a), Klippel et al. (2005b), Klippel et al. (2005c) and of geographic spatial knowledge, e.g., Barkowsky (2001), Barkowsky (2002), Barkowsky (2003), Engel et al. (2005), Barkowsky et al. (2005). This thesis applies this method to spatial knowledge focusing on a basic problem of spatial cognition: **How to recognize variable spatial environments?**

1.4 The assumption and the criteria

Creatures interact with their surrounding environments by their sensors. Pigeons navigate by their magnetic sensors; dogs search foods by their noses; blind bats fly in the sky by their ultrasonic-wave sensors; rattle snakes catch foods by their infrared-ray sensors; people mostly use their eyes. Creatures with different sensors live in different conceptual spaces. The conceptual space of rattle snakes is based on the partition of temperatures of different objects in the surrounding physical space; their conceptual space shall not be as splendid as the conceptual space of people in the daytime, but they still can find foods without illumination, while people can not see anything without light.

The stimuli received by sensors change with the changes of the surrounding environment, the locomotion of the creature, even the malfunctions of the sensors themselves. By looking at a clock, you will see different stimuli at any second, supposing

that its second hand moves per second and its other hands move accordingly. However, you will have the same concept for all these different stimuli: It is the same clock. The pattern of your home with the door open is different from that with the door closed, however, they are patterns of the same environment.

The starting point of this thesis is that a cognitive agent perceives a snapshot view of spatial environments, recognizes objects from this snapshot view, and remembers a target spatial environment in the mind. It researches into the question of the representation of a snapshot view and the question of the comparison between the representation of the current perceived snapshot view and the representation in the mind.

People observe spatial environments and remember them. The first criterion for the representation of snapshot views is, therefore, that **the representation of spatial environments shall be acquirable through observation**. People are able to describe spatial environments. The second criterion is therefore that **the representation shall provide a systematical way to give meanings to spatial linguistic descriptions**. People may have distortions of spatial relations. Third criterion is therefore that **the representation shall provide ways to explain formations of spatial distortions**. People recognize familiar spatial environments, e.g., their homes, offices, quickly and easily. The criteria for the comparison of two representations with regard to recognizing spatial environment is therefore that **the process shall be computable (the decision process shall end in an amount of time that is polynomial in the size of the input)**.

1.5 Results and contributions

Research results are as follows: Spatial relations between extended objects are understood by the connectedness relation with some extension objects — Distance relations between extended objects are understood by the connectedness relation between one extended object and the near extension region of the other extended object. Orientation relations between extended objects are understood by the connectedness relation between a near extension region of an extended object and a side of the other extended object. People have commonsense knowledge of the relative stability of spatial objects. A representation of spatial layouts is structured by objects and spatial relations following the principle of selecting reference objects that *the reference object is of higher relative stability than the location object and that the nearer objects are preferred to be reference objects*. The representation of a snapshot view is mereotopologically formalized. Recognizing a perceived snapshot view as target environment is interpreted as the compatibility between the representation of the perceived snapshot view and the representation of the target snapshot view. The degree of the compatibility is determined by the spatial differences between the two representations and the relative stabilities of related objects. Recognizing spatial environments is formalized as a particular relationship between two representations of snapshot views.

This research work contributes to qualitative spatial representation by the commonsense understanding and the mereotopological formalism of spatial relations between extended objects. It contributes to artificial intelligence by the commonsense understanding and the formalism on the representation of snapshot view of spatial environments and the reasoning on recognizing variable spatial environments. It opens

a new research direction in artificial intelligence/spatial cognition—vista spatial cognition.

1.6 The organization of this thesis

The thesis is structured as follows.

Chapter 2 presents the state of the art that pertains to recognizing variable spatial environments. I will review related work in cognitive psychology, object recognition, cognitive maps, formal spatial ontology, commonsense knowledge, and qualitative spatial representation.

Chapter 3 elaborates the research topics and research problems in detail based on the state of the art.

Chapter 4 presents the commonsense understanding of spatial relations, the relative stability of an extended object, relative spaces, the structure of snapshot views of environments, and the procedure of recognizing a spatial environment.

Chapter 5 formalizes the commonsense understanding in Chapter 4. It proposes a region-based representation and reasoning framework for recognizing spatial environments.

Chapter 6 introduces a symbolic computational system that pertains to the formalism in Chapter 5 — the LIVE model. The LIVE model is implemented in LispWorks 4.2 both on the Linux Susie 6.3 platform and on the Windows XP professional platform.

Chapter 7 summarizes and evaluates the results, and presents some future work.

Chapter 2

The state of the art

In this chapter I review research work in cognitive psychology, object recognition, cognitive maps, formal spatial ontology, commonsense knowledge, and qualitative spatial representation that pertain to recognizing spatial environments.

2.1 Psychological spaces

The structure of space can be described from the point of view of behavior.

(Piaget, 1954, p.212)

Montello (1993) classified four kinds of psychological spaces based on the way of observation: Figural space, vista space, environmental space, and geographical space. Vista space refers to a space that is projectively as large as the human body and that can be apprehended from one place without the need for locomotion. Typical vista spaces are single rooms, corridors, small valleys, etc. Tversky et al. (1999b) classified three kinds of psychological spaces: the space of the body, the space surrounding the body, and the space of navigation. The space of the body is *the space of our own actions and our sensations, experienced from the inside as well as the outside.*

It is schematized in terms of the natural parts of our body, our limbs;...the space surrounding the body is the space that can be seen from a single place, given rotation in place (Tversky et al., 1999b, p.517). The space surrounding the body is conceptualized in three dimensions; the space of navigation is the space that is too large to be seen from a single place and it is conceptualized in two dimensions.

For out-door spatial environments, there might be differences between Montello (1993)'s vista space and Tversky et al. (1999b)'s space surrounding the body. When you see the moon at night, it is located in a vista spatial environment at that moment, however, it might not be very suitable to say that it is located in the space surrounding you, as it is quite far away from your body. The notion of vista spatial environments only emphasis the visual perception; while the notion of the space surrounding the body also emphasis actions of bodies or limbs besides the visual perception. For recognizing in-door spatial environments, what people see is located in a reachable distance, therefore, Montello (1993)'s "vista spatial environment" is equivalent to Tversky et al. (1999b)'s "the space surrounding the body". "Spatial environments" in this thesis refer to Montello (1993)'s "vista spatial environment" and Tversky et al. (1999b)'s "the space surrounding the body".

2.2 Object recognition (Figural spatial cognition)

Humans have a visual system that demonstrates remarkable object recognition ability, Liter and Buelthoff (1996). They can recognize objects very accurately and very fast. Research in cognitive psychology on human's object perception shows: (1) People tend to categorize objects at a preferred category; (2) people recognize objects based on some grouping laws (Gestalt theory); (3) there are two correlated models for object

recognition: The view-dependent model and the view-independent model.

2.2.1 Object recognition at the preferred category

The physical and social environment of a young child is perceived as a continuum. It does not contain any intrinsically separate “things”. The child, in due course, is taught to impose upon this environment a kind of discriminating grid which serves to distinguish the world as being composed of a large number of separate things, each labelled with a name.

(Leach, 1964, p.34)

Humans’ visual system is capable of recognizing objects from stimuli. Three common visual abilities of the human visual system are *discrimination*, *categorization*, *identification*.

The normal meaning of recognizing an object is that the object is successfully *categorized* into a particular object class, Liter and Buelthoff (1996). A *category* is a number of objects that are considered equivalent. A *taxonomy* is a system by which categories are related to one another by class inclusions. A level of abstraction within a taxonomy refers to a particular level of inclusiveness, Rosch et al. (1976). For example, if something belongs to the category of kitchen chair, it must also belong to the category of chair; if something belongs to chair, it must belong to the category of piece of furniture. Thus, in this example, we have a simple taxonomy which has three levels: The top is the category of furniture, next is the category of chair, the lowest level is the category of kitchen chair.

Identification means that an object belongs to a category that only has one object. Normal cases for identification are the human-face recognition and the environment

recognition. When you recognize the face of your mother, your boss, or Bill Clinton, there is only one instance of your mother, your boss, or Bill Clinton. When you recognize the spatial environment of your current home, your current office, the main train station in Bremen, there is also one instance in the world to be your current home, your current office, or the main train station in Bremen. This thesis addresses the identification of a spatial environment, such as Mr. Bertel's home, your office, etc.

Discrimination means a yes-no judgment of whether two objects belong to the same category.

There is plenty of research showing that objects are recognized first at a particular level of abstraction, e.g., Rosch et al. (1976), Jolicoeur et al. (1984).

Rosch et al. (1976) argued that categories within taxonomies of concrete objects are structured such that there is generally one level of abstraction at which humans find it easiest to name objects and recognize them the fastest, namely "basic level category". Basic level of abstraction in a taxonomy is the level at which categories carry the most information, possess the highest cue validity, and are, thus, the most differentiated from others. Experiments in Rosch et al. (1976) showed that basic objects are the categories whose members: (a) Possess significant numbers of attributes in common; (b) have similar shapes; (c) are the first categorization made during perception of the environment, showing that basic objects have greater cognitive primacy than subordinate and superordinate categories.

Jolicoeur et al. (1984) proposed the notion of "entry point level", which means that every object has one particular level at which contact is made first with semantic memory. This level corresponds to the basic level in most cases. However, there are exceptions. For example, an expert airplane mechanic shifts the entry points of

airplanes toward subordinate levels:

One subject . . . a former airplane mechanic. His taxonomy was interesting. The lists of attributes common to airplanes produced by most subjects were paltry compared to the lengthy lists of additional attributes which he could produce. Furthermore, his motor programs as a mechanic were quite distinct for the attributes of the engines of different types of planes. Finally, his visual view of airplanes was not the canonical top and side images of the public; his canonical view was of the undersides and engines.

(Rosch et al., 1976, p.430)

2.2.2 Gestalt grouping laws

Gestalt psychology was started in Germany in 1912 by Max Wertheimer, Wolfgang Kohler and Kurt Koffka. The center tenet of Gestalt psychology is that the whole is more than the sum of its parts. The sound of a melody is much more than the sequence of its musical notes. If the melody is played one musical note after one musical note every one minute, people will be in difficulty in enjoying it. The same situation happens, when a film (a sequence of pictures) is played slower than 20 pictures per second. The Gestalt theorists maintained that it is the parts of the melody or the film that interact with one another and produce the nature of the whole, Rock and Palmer (1990).

To explain how human perceptions are formed, Gestalt theorists proposed the principle of “Praeganz” (regularity) that perception tends to use the simplest and most regular organization to group the perceived stimuli, e.g., Kohler (1929), Koffka (1935), Wertheimer (1958). A partly occluded object will appear to continue

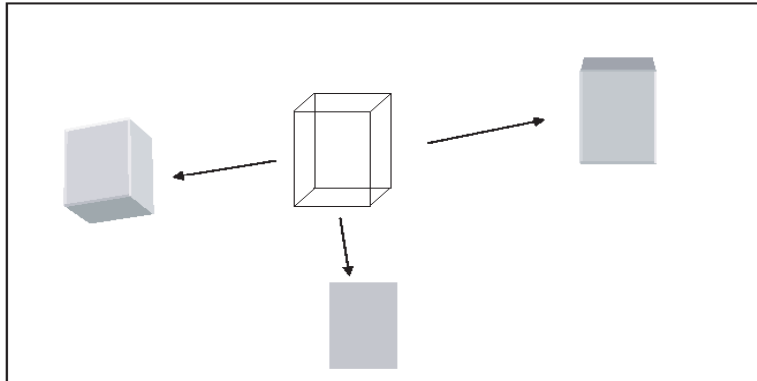


Figure 2.1: Three kinds of sides of a cube

behind an occluding object when such a continuation produces units that are more homogeneous in color or texture (grouping law of similarity), more smoothly contoured (grouping law of good continuation). Spelke (1990) summarized that young infants are sensitive to Gestalt grouping laws. For example, young infants can detect a misaligned contour in an array of elements with aligned contours. This shows that humans are sensitive to the Gestalt grouping law of good continuation at very young age.

2.2.3 View-dependent vs. view-independent models

We can distinguish these five types of behavior : (1) ..., (2) ..., (3)..., (4) the “reconstruction of an invisible whole from a visible fraction” (5)

(Piaget, 1954, p.14)

Intuitively, when we look around, we only perceive sides of spatial objects; when we perceive an object from different perspectives, different sides of the object are

perceived. In Figure 2.1, we can see at most three sides of the cube from a viewpoint; when we perceive the cube from different view points, we can see different sides. There is a debate in the community of object recognition: The view-independent school proposes that the representation of a 3-dimensional object should be regardless of the viewpoint of the observer, e.g., Marr and Nishihara (1978), Biederman (1987), Marr (1982), Biederman and Gerhardstein (1993), and Biederman and Gerhardstein (1995). In contrast, the view-dependent school proposes that the representation of the 3-dimensional object is to represent multiple images of an object from different view-points, e.g., Breuel (1992), Buelthoff and Edelman (1992), Humphreys and Khan (1992), Tarr (1995), and Tarr and Buelthoff (1995).

Marr and Nishihara (1978) and Marr (1982) proposed a view-independent theory of object recognition. According to this theory, object recognition is carried out in two stages. In the first stage a view-dependent $2\frac{1}{2}$ sketch is formed; in the second stage a view-independent 3-dimensional object representation is formed from this $2\frac{1}{2}$ sketch.

Biederman (1987) proposed the theory of “Recognizing an object By its Components”, namely RBC theory, which is built on Marr and Nishihara’s early work. The assumption is that a modest set of generalized-cone components, named *geons*, can be derived from contrasts of readily detectable properties of edges in a two-dimensional image. In RBC theory, an object is represented by its components and the qualitative spatial relations among them, and recognizing an object is carried out by recognizing component(s) of the object.

On the other hand, there is plenty of research providing evidence that humans’ object recognition performance is strongly viewpoint-dependent, e.g., Breuel (1992),

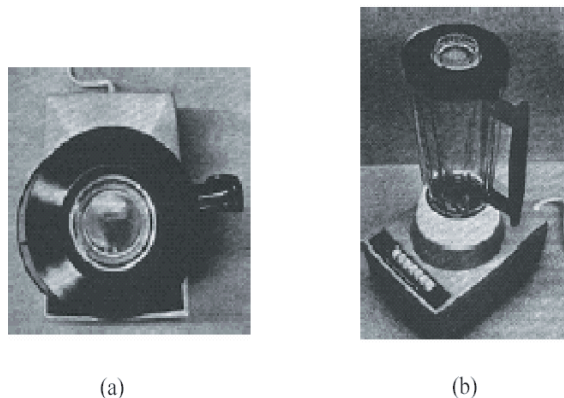


Figure 2.2: In (a) the mixer is viewed from top to down (a bird view); in (b) the same mixer is viewed normally (a field view). People are more easily to recognize the mixer with the field view (b) than the bird view (a). The picture is copied from (Biederman, 1987, p.144)

Humphreys and Khan (1992), Buelthoff and Edelman (1992), Edelman and Buelthoff (1992), and Tarr (1995).

According to view-dependent approaches, object recognition depends on what is observed during familiarization with this object. Objects can be more readily recognized from some familiar orientations compared with others, Palmer et al. (1981). For example, people have more difficulty in recognizing the mixer when they have a bird-view compared to a field-view of the machine, shown in Figure 2.2.

Humphreys and Khan (1992) conducted three psychological experiments to examine how novel 3-D objects are represented in long-term memory and how different views affect their representations. To address the issue of view dependency, subjects were trained to become familiar with objects in a specific orientation, then they were tested for recognizing these objects oriented in the familiar view and also novel views. The view-dependent approach and the view-independent approach make different

predictions about the accuracy and time taken to recognize objects as a function of orientation: No significant difference between recognition of familiar views and unfamiliar views supports the view-independent approach; significant difference supports the view-dependent approach. The three experiments show that at least under certain conditions, the visual system stores a viewpoint specific representation of objects.

Buelthoff and Edelman (1992) showed that human recognition is better described by two-dimensional view interpolation than by methods that rely on object-centered three-dimensional models and that humans recognize objects best when they see them from a familiar view and worse from other views.

After debates between view dependent and view independent approaches in human object recognition, i.e., Biederman and Gerhardstein (1995) and Tarr and Buelthoff (1995), Tarr and Buelthoff (1998) reviewed findings from psychophysics, neurophysiology, machine vision, and behavioral results on the view-dependent/independent object recognition and concluded that although the view-dependent approach holds great promise, it has potential pitfalls that may be best overcome by structural information. And they proposed a hybrid approach for object recognition, which incorporates the most appealing aspects of view dependent and view independent approaches.

The RBC theory is, to some extent, something between a view-independent approach and a view-dependent approach. On the one hand, it suggests that to recognize an object is to recognize some of its components, which has some flavor of a view-dependent approach, as from different viewpoints, different components should be perceived. On the other hand, it inherits the view-independent approach of Marr and Nishihara to reconstruct the components from 2-D images.

Getting into the debate of the view dependent/independent issue is beyond the

scope of this thesis. However, two general assertions are accepted by both of the two approaches: (1) An object can be recognized by parts of it; (2) there are structural relations among parts of an object.

2.3 Cognitive maps, frames, and other schemata

Physicians, engineers, mechanics and others use errors as signs of malfunctioning, that some system has broken down and is in need of repair. Not so for psychologists. Errors are viewed as natural products of the systems, and as such are clues to the way the system operates.

(Tversky, 1992, p.131)

In this section I review work on mental representations of spatial environments. These models have been given different names by different researchers. Trowbridge (1933) named them imaginary map; Shemyakin (1962) named them mental map; Apleyard (1969) named them environmental image; Boulding (1956) and Lynch (1960) named them spatial image; Lee (1968) named them spatial schema; Tolman (1948), Downs and Stea (1973), Kaplan (1973), and Kuipers (1975) named them cognitive map; Minsky (1975) named them frame; Tversky (1993) named them cognitive collage. In this section, I use *mental spatial representations* (MSR) to refer to all the above names in general. The following topics are reviewed: The existence of the represented world of MSR, the structure of MSR, the cognitive reference points in MSR, and the exploration of MSR from sentences.

2.3.1 The existence of the represented world of MSR

Tolman (1948) proved the existence of the represented world of mental spatial representations, namely cognitive maps, by serial experiments with rats. “Latent learning” experiments show that rats learn the maze structure which they run inside. “Vicarious trial and error” experiments show that rats can discriminate simple visual patterns and relate them with anticipations. “Searching for the stimulus” experiments show that rats often have to look actively for the significant stimuli in order to form its map and do not merely passively receive and react to all the stimuli which are physically present. “The hypothesis” experiments show that rats go through a maze by making systematic choices, which proved that rats have their “hypotheses” of spatial environments that results in their systematic choices (a rat may choose all right-hand doors, then all left-hand doors, then all dark doors). “Spatial orientation” experiments strikingly show that rats have higher level representations of spatial environments that allow them to re-orient themselves when the former path is blocked.

2.3.2 The partial hierarchical structure of the MSR

The structure of MSR is addressed directly or indirectly in cognitive modelling, e.g., Siegel and White (1975), Stevens and Coupe (1978), Byrne (1979), Davis (1981), Tversky (1981), McDermott (1981), McNamara et al. (1984), Hirtle and Jonides (1985), McNamara (1986), Tversky (1991), McNamara (1991), McNamara (1992), and in artificial intelligence, e.g., Minsky (1975), Kuipers (1977), Yeap (1988), Yeap and Jefferies (1999), Barkowsky (2002), Barkowsky (2003), and Barkowsky et al. (2005).

Three classes of MSR theories exist in the literature: (1) non-hierarchical theory,

e.g., Byrne (1979), which proposed that relations among spatial objects are mentally represented in a network; (2) strong hierarchical-theory, which proposed that spatial objects may group into “regions” leading to the hierarchical tree of regions and that spatial relations are not explicitly represented among regions at the same level in the hierarchical tree; (3) partial hierarchy-theory, e.g., Davis (1981), McDermott (1981), Kuipers (1978), and Stevens and Coupe (1978), which allowed spatial relations to be explicitly represented among regions at the same level in the hierarchical tree.

Byrne (1979) proposed that spatial relations among objects were mentally represented as a propositional network. For example, a mental representation of an urban environment could be a network of spatial objects with topological relations.

Stevens and Coupe (1978) and Tversky (1981) found that people make large systematic errors in judging spatial relations between two locations. For example, most people judge by mistake that San Diego (CA) is further east than Reno (NV) or that Madrid (Spain) is further south than Washington (DC) or that Seattle (USA) is further south than Montreal (Canada). To account for these errors, Stevens and Coupe (1978) proposed a partially hierarchical structure of mental spatial representation. That is, spatial relations between two locations are explicitly stored if the two locations are located in the same spatial region; spatial relations can be inferred by combining related spatial relations.

Another perspective of research that supports the existence of hierarchical structure is the influence of barriers to subjects’ cognitive performance, e.g., Kosslyn et al. (1974), Cohen et al. (1978), Thorndyke (1981), and Newcombe and Liben (1982).

Kosslyn et al. (1974) found that children exaggerate distances between two locations that have either a transparent or an opaque barrier between them and adults

make distance exaggeration when there is an opaque barrier. Newcombe and Liben (1982) found that subjects made exaggeration with rank order data. Cohen et al. (1978) found that cognitive maps of a familiar environment in 9-10 year old children and adults are strongly influenced by barriers, such as buildings, trees, and hills which tend to prohibit travelling. Distance estimation is based on the ease of travel. Distances are exaggerated when there are more barriers; distances are underestimated when there are less barriers. Barriers may influence the cognitive map by providing a method for chunking the space into subspaces which leads to the hierarchical structure of cognitive map.

Hirtle and Jonides (1985) used the Ordered Tree Algorithm, McKeithem et al. (1981), to show that people form a hierarchy of spatial objects on the basis of spatial and non-spatial attributes and that clusters in a hierarchy have consequences for performance in various tasks. McNamara (1986) systematically tested non-hierarchical and hierarchical schema with the results that partial-hierarchical schema theories are preferable.

In the AI community, Minsky (1975) proposed a partial theory of schema, namely the frame theory. A frame is a data structure consisting of nodes and relations which represents a situation. The top levels of a frame are fixed; the lower levels have many variables that can be assigned specific instances. Thus, a frame is also structured hierarchically. Kuipers (1975) represented the scenario of a cube world using the frame theory.

2.3.3 Cognitive reference points in the MSR

Wertheimer (1938) first suggested certain “ideal types” among perceptual stimuli

used as anchoring points for perception. An introspective judgment by Wertheimer was: A line of 85° was almost vertical, but a vertical line was not almost 85° .

Rosch (1975) examined the existence of such “ideal types” in natural categories, which were named “cognitive reference points”. She conducted psychological experiments for three domains: Colors, line orientations, and numbers. For the color system, “red”, “yellow”, “green”, and “blue” are preferred cognitive reference points: The desaturated red was judged “muddy” by subjects but still named “red”; and the off-hue red was judged “purplish”, but still “red” (Rosch, 1975, p.536). For decimal numbers, multiples of 10 are “ideal type” numbers. People say that “99.231 is around 100” rather than “100 is around 99.231”. For line orientations, vertical, horizontal and diagonal lines are reference orientations.

In spatial cognition some spatial objects are used as cognitive reference objects to locate other objects. Sadalla et al. (1980) and Couclelis et al. (1987) investigated reference points in large scale spatial environments. Sadalla et al. (1980) found that landmarks are used to define the location of adjacent spatial objects and that subjective distances between reference points and non-reference points are therefore asymmetrical. Couclelis et al. (1987) found that landmarks may be discriminable features of a route, or discriminable features of a region, or salient information in a memory task. Locations in large spatial environments are partitioned into sub-regions each having a reference point.

2.3.4 Exploring the structure of the MSR through spatial linguistic descriptions

Talmy (1983) discussed how language is effective for conveying spatial information.

He proposed that language schematizes space by selecting certain aspects of a reference scene to represent the whole, while discarding others. The schematization of indoor spatial environments in Ullmer-Ehrich (1982) discarded all small objects, like apples, cups, books, pens, etc. and only selected big ones. Foos (1980) investigated the construction of MSR of environmental spaces from language descriptions. The possibility of exploring the structure of MSR through linguistic descriptions is further strengthened by Schematization Similarity Conjecture that *to the extent that space is schematized similarly in language and cognition, language will be successful in conveying space* (Tversky and Lee, 1999, p.158).

2.4 Spatial ontologies

2.4.1 SNAP & SPAN ontologies

Grenon and Smith (2004) proposed two spatial ontologies, called SNAP and SPAN, to model different aspects of dynamic situations. SNAP and SPAN ontologies are partial in the sense that each is a window to the reality. SNAP is a snapshot ontology of endurants existing at a time. *SNAP entities have continuous existence in time, preserve their identity through change and exist in toto if they exist at all*¹. SPAN is a four-dimensionalist ontology of processes. *SPAN entities have temporal parts; unfold themselves phase by phase and exist only in their phases*². For example, the lobster, a nation, a population, an ocean are instances of SNAP ontologies; while the growth of a lobster, the history of a nation, the migration of a population, or the tide of an ocean are instances of SPAN ontologies.

¹from <http://www.spatial.maine.edu/actor2002/participants/smith-short.pdf>

²see the above note

2.4.2 *Fiat* boundaries & *fiat* objects

One reason for resisting scepticism in face of the fiat world turns on the fact that people kill each other over fiat borders, and they give their lives to defend them.

(Smith and Varzi, 2000, p.405)

Boundaries can be either physical or non-physical parts of spatial entities. For example, the surfaces of planets or footballs are parts of these spatial entities. Such boundaries have material constitution. However, not all the boundaries have material constitution. For example, national borders or province borders are not based on the material discontinuities between two nations or two provinces; the equator of the earth and the North Pole are based on some mathematical measurement. Accordingly, two basic typologies of boundaries are distinguished: *bona fide* (or physical) boundary and *fiat* (in the sense of human decision or delineation) boundary, Smith and Varzi (2000). Two basic typologies of spatial objects are followed: *bona fide* objects which have *bona fide* boundaries and *fiat* objects which only have *fiat* boundaries.

Some *fiat* boundaries are dependent on *bona fide* boundaries. For example, every *bona fide* spatial object has the *fiat* boundary as its closure. When two people shake hands or kiss each other, they do not share a common part of their skin (*bona fide* boundaries) rather parts of their *fiat* boundaries “coincide”. *Fiat* boundaries are boundaries which *exist only in virtue of the different sorts of demarcations effected cognitively by human beings* (Smith, 2001, p.135). They owe their existence both to acts of human decision (or *fiat*, to laws or political decrees, or to related human cognitive phenomena) and to real properties of the underlying factual material, Smith and Varzi (2000) and Smith (2001). The shores of the North Sea are *bona fide*



Figure 2.3: The *fiat* boundary around the stars of the Constellation “Great Bear”

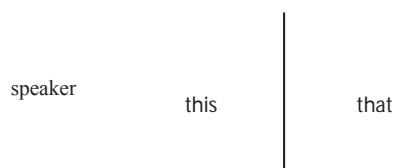


Figure 2.4: Ephemeral *fiat* boundary established by the use of indexical terms (Smith, 2001, p.142)

boundaries; but we conceive the North Sea as a *fiat* object nonetheless, because “(its objectivity) is not affected by the fact that it is a matter of our arbitrary choice which part of all the water on the earth’s surface we mark off and elect to call the ‘North Sea’ ”, Frege (1884). The constellation “Great Bear” is a *fiat* object which owes its existence both to some huge stars (factual material) and to human cognitive activity (creation of a *fiat* boundary around these stars which has the shape of a bear), shown in Figure 2.3. Natural language makes a good contribution to the generation of *fiat* boundaries, namely “Linguistic Fiats” (Smith, 2001, p.141). For example, natural language expressions like “this” and “that” create an ephemeral *fiat* boundary in space, shown in Figure 2.4. *To set out the constraints on drawing fiat boundaries is a task that is by no means trivial* (Smith, 2001, p.138), because drawing *fiat* boundaries (or creating *fiat* objects) is based on a commonsense understanding

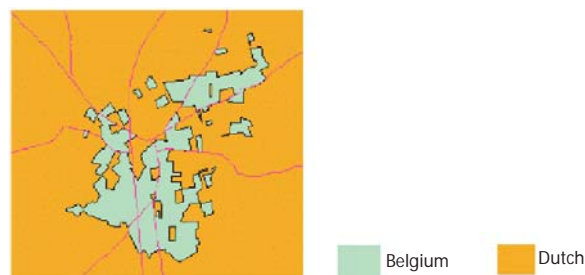


Figure 2.5: The Enclaves of Baarle-Hertog and Baarle-Nassau. The picture is copied from (Smith, 2001, p.156)

of reality. Some *fiat* boundaries may not have sharp boundaries in reality. Consider the example of the Belgian enclave of Baarle-Hertog and its neighbor, the Dutch community of Baarle-Nassau, shown in Figure 2.5. The brighter areas represent the community of Baarle-Hertog. The small darker areas depict the tiny Dutch enclaves of Baarle-Nassau. Each such enclave is surrounded by a portion of Belgian territory, which is surrounded once more by Dutch territory (Smith, 2001, p.156).

2.5 Modelling commonsense knowledge

Naive Physics is the body of knowledge that people have about the surrounding physical world. The main enterprises of Naive Physics are explaining, describing, and predicting changes to the physical world.

(Hardt, 1992, p.1147)

After recognizing that most artificial intelligence systems in the late 1970s were toy systems, Hayes (1978) coined the term *Naive Physics* and proposed that researchers should concentrate on modelling commonsense knowledge.

Kuipers (1977) and Kuipers (1978) constructed a model of commonsense knowledge of large-scale space. Kuipers (1979) defined commonsense knowledge as follows: “... (*commonsense*) knowledge about the structure of the external world that is acquired and applied without concentrated effort by any normal human that allows him or her to meet the everyday demands of the physical, spatial, temporal, and social environment with a reasonable degree of success.”

In the research field of Spatial Information Theory, Egenhofer and Mark (1995) coined the term *Naive Geography* that is concerned with formal models of the commonsense geographic world. It links the knowledge that people have about the surrounding geographic world and the formal representation and reasoning of that knowledge. Smith (1995) reviewed relations among commonsense knowledge, artificial intelligence, naive physics, physical science, etc. and set out the goal of developing a theory of the commonsense world.

2.6 Qualitative spatial representations

In this section I review some region-based qualitative spatial representation work that relates to the formalism presented in Chapter 5.

2.6.1 Classic topological relations

Smith (1994) proposed that topological relations are the most fundamental relations for cognitive science. Lynch (1960) used the metaphor of a rubber sheet for this fundamental relation in the sense that spatial entities can spread, shrink, twist, bend, etc. (like a rubber sheet) as long as the connection relations among them are preserved.

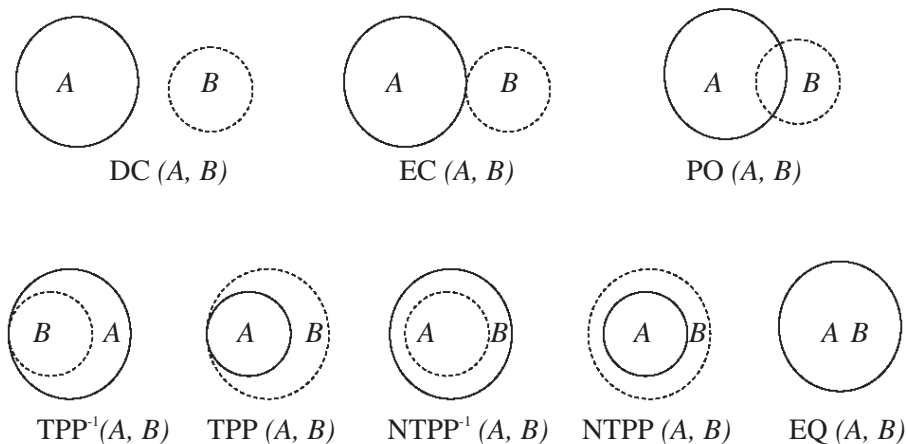


Figure 2.6: The RCC-8 relations between regions

Formalizing the classic topological relation was done simultaneously and independently at Cohn’s group, e.g., Randell et al. (1992), Cohn (1993), Gooday and Cohn (1994), Gotts (1994), Cohn (1995), Cohn and Gotts (1996), Bennett et al. (2000b), Bennett et al. (2000a), Bennett et al. (2002), Bennett (2003), Bennett (2004), etc. and at Egenhofer’s group, e.g., Egenhofer (1989), Egenhofer (1991), Egenhofer and Sharma (1993), Egenhofer (1993), Egenhofer (1994), Egenhofer and Franzosa (1995), Egenhofer and Franzosa (1991), Egenhofer (2005), etc.

Cohn’s school took the region rather than the point as the primitive unit in qualitative spatial representation and reasoning. They maintain that regions are more natural to represent indefiniteness that is *germane* to qualitative representation; spaces occupied by any real physical objects are always regions, not mathematical points; in common sense the word “point” means a small region rather than a real mathematical point, Cohn et al. (1997). The RCC theory was therefore developed for the topological relations between extended regions. The RCC-8 relations are shown in Figure 2.6.

Egenhofer’s school took the relational algebra perspective and represented a region \mathcal{X} by a pair of the boundary of \mathcal{X} (in the notion of “ $\delta\mathcal{X}$ ”) and interior of \mathcal{X} (in the notion of “ \mathcal{X}° ”). Topological relations between two regions are represented through the emptiness or non-emptiness of the intersection relations among their interiors, boundaries, and exteriors. The topological relation between two extended objects is represented by matrices shown in Figure 2.7.

2.6.2 Orientation representations between extended objects

Haar (1976) proposed a triangular model for the orientation relation between two extended objects. The space is partitioned into four mutually exclusive cones. The location of an object is represented by its centroid, which is the arithmetic mean of all the points of the object. The orientation relation is therefore determined by the centroid of the location object with regard to the four cones. However, the triangular model needs to be refined when the two extended objects are very close (or connected) or of irregular shapes, like the horseshoe-shape.

Guesgen (1989) extended Allen’s (1983) 1-dimensional model into 2-dimensional and 3-dimensional models. In the extension to the 2-dimensional model, extended objects are projected into x - and y - axis. On each axis there is a 1-dimensional relation between the projections of the two objects. Spatial relations between two extended objects are represented by the pair of 1-dimensional relations. On each axis, nine orientation relations can be distinguished; therefore, the model can distinguish $9 \times 9 = 81$ relations for 2-dimensional space. As the model approximates relations of extended objects by projecting them to two 1-dimensional models, there are certain cases where this model has problems. For example, the two rectangles in Figure 2.8

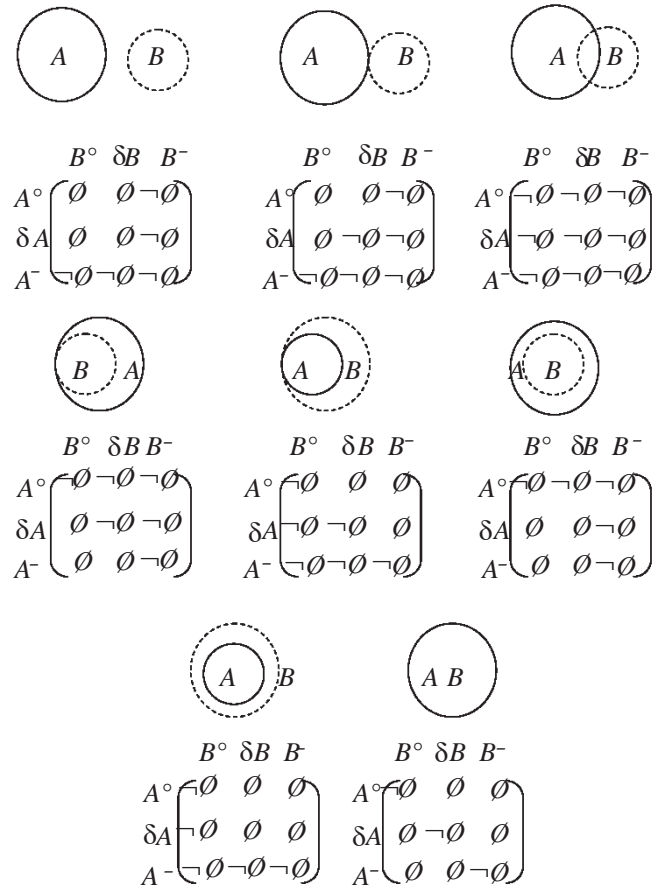


Figure 2.7: The geometric interpretation of 8 topological relations between regions with the connectedness relation, Egenhofer (1994)

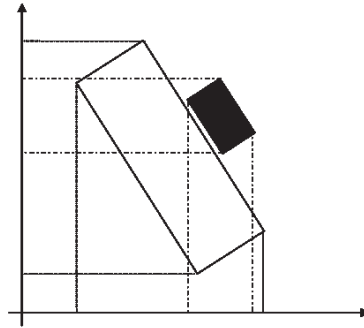


Figure 2.8: The ‘disconnected’-relation between the white rectangle and the black rectangle might not be suitably represented with the method in Guesgen (1989)

are disconnected, however, their projections on the two axes are connected.

Goyal (2000) proposed the *coarse direction-relation matrix* which partitions space around an extended object and records into which tiles an extended location object falls. By counting in the ratio of the location object in each tile, the model can capture more detailed orientation information, shown in Figure 2.9.

Schmidtke (2001) assumed that direction information is unsuitable to be combined with topological information and formalized directional localization relations between extended regions. Her work introduced a system of sectors based on an extended object and the directional location of another object is described based on the sector system. The work yields a geometry of directional locations for 2-dimensional objects.

2.6.3 Distance representations between extended objects

Geometrical concepts between two extended objects—“solids” was defined by the primitive relation—“can connect” in de Laguna (1922). For example, region \mathcal{A} is “longer than” region \mathcal{B} is interpreted as: There are regions \mathcal{W} and \mathcal{X} such that \mathcal{A} can connect \mathcal{W} and \mathcal{X} while \mathcal{B} cannot; the distance between \mathcal{A} and \mathcal{B} is *zero*, if there is no \mathcal{X} such

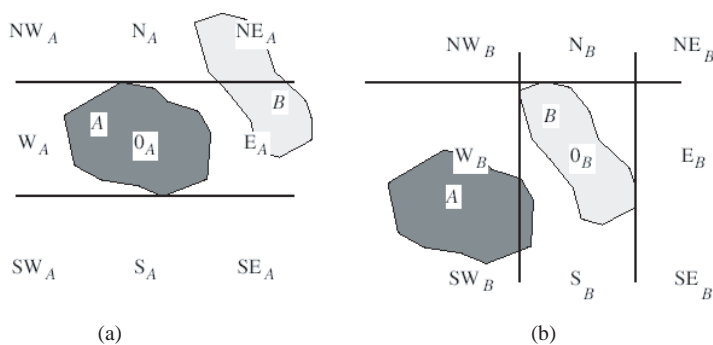


Figure 2.9: The cardinal direction relation between two extended objects, A and B, is interpreted by the ‘connected’-relation among the location object and the projection-based partitions of the reference object. The picture is copied from (Goyal, 2000, p.39)

that \mathcal{X} cannot connect \mathcal{A} and \mathcal{B} . This work suggests that distance relations can be included into the topological framework.

There are also recent attempts trying to integrate all of the three systems of spatial relations into one framework. However, this has not proven successful, for example, Brennan et al. (2004) tried to propose a spatial ontology that brings together three aspects of spatial knowledge, namely connectivity, proximity, and orientation. They attempted to model the “very close to each other” relation by splitting the “disconnected” (DC) relation of the RCC-8 theory into two relations: $DC_{=D}$ and $DC_{\neq D}$. However, the work ignores the “externally connected” (EC) relation in RCC-8. This results in the fact that $DC_{=D}$ is only a *façon de parler*³ of EC . This approach abstracts extended objects as points, called “site”, and introduces a pseudo-metric space with pseudo-distance D . Then for any site p , Brennan et al. (2004) proposed an influence area, denoted by $IA(p)$. To define the proximity relation, it introduced six kinds of “nearness” relations between a “site” and an “influence region” or between

³the art of speaking

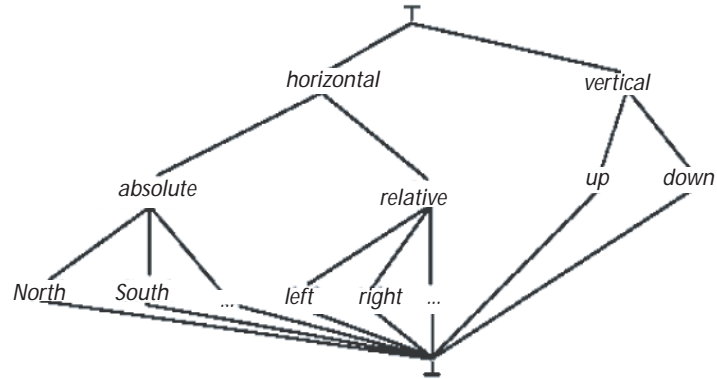


Figure 2.10: The lattice definition of spatial orientations as shown in (Brennan et al., 2004, p.173)

two “influence regions”. Therefore, this part is still “point” based and not consistent with its connectivity part. In the orientation part, their work is even more problematic. Without providing ontological definitions of spatial orientation relations, the work presented a “lattice definition of the relation hierarchy of orientation relations”, shown in Figure 2.10, and the composition of orientation relations of different reference systems equals to \perp , e.g., $left \wedge North = \perp$. This obviously violates our common sense and research results in cognitive psychology: Tversky et al. (1999a) reported that people are likely to switch perspectives in describing orientation relations, therefore, there can be more than one orientation relation between two extended objects, such as *the courtyard is left of the church*, or *the courtyard is south of the church*, Lee (2002).

Chapter 3

Research topics and research questions

3.1 The puzzle of recognizing environments

People receive features of stimuli from the surrounding environment, group them, and interpret them as objects. Recognizing objects means the perceived features can be grouped by a particular structure of features of an object (see 2.2.2). This follows that objects in the same category (grouped by the same structure of features) are equivalent and indistinguishable in isolation (see 2.2.1). Object categories are therefore multi-element categories. For example, people might have some difficulty in distinguishing twins. Figure 3.1 (left) shows the twin sisters from Taiwan: Sandy and Mandy. It would be quite difficult to identify which one is in the picture on the right.

Recognizing spatial environments, however, is different from recognizing single objects. Your home, your office, the entrance hall of your office building, etc. are of one-element category, although objects inside are of multi-element categories. That is to say that objects of multi-element categories come together and form an object

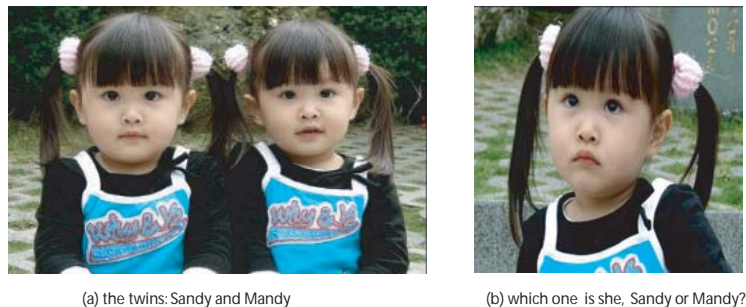


Figure 3.1: People might have difficulty in distinguishing the twin sisters — Sandy and Mandy

of one-element category. How come? The environment as a whole is more than a set of all the objects inside. A spatial configuration is like a melody — the recognized objects and their correlations structure the configuration, just like musical notes and their correlations structure a melody. Wilson et al. (1999) reported a memory impaired patient (LE), a sculptress suffered from autoimmune disorder¹. This impairs her visual short-term memory² with mental image generation. She could not retrieve images from her memory, thus, she could only remember contours of objects. Consequently, she failed to distinguish two windows with different images in the church and she even had difficulty in recognizing the face of her husband³. However, she can locate objects; and amazingly, she can recognize her home. This case provides evidence that it is the relations among objects that turn a set of objects of multi-element categories into a configuration of one-element category. Two research topics are the knowledge of the snapshot view of a spatial environment (representation) and the use of the knowledge to recognize spatial environments (reasoning).

¹systemic lupus erythematosus

²dissociation between spatial span and pattern span.

³Personal communication with Allan Baddeley.

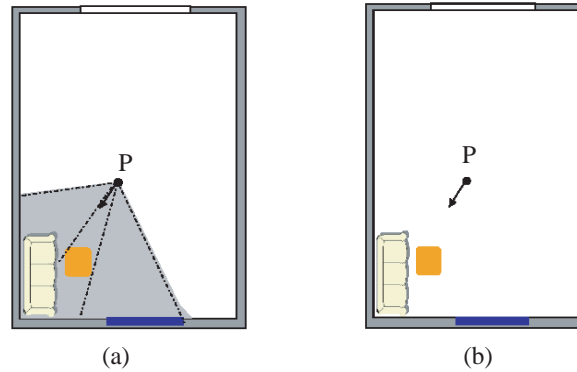


Figure 3.2: In (a) the observer stands at P and faces to the corner. He perceives only part of the stimuli of the objects. However, he can recognize the couch, the tea-table, the door, and the walls of the room, shown in (b)

3.2 The commonsense knowledge of spatial environments

The knowledge of a snapshot view of an environment is called “Mental Spatial Representation” (MSR) (see 2.3.1–2.3.3). It can be explored through its external source — the perceived environment, and its product — the spatial linguistic descriptions. People perceive light features, e.g., colors, brightness, from the surrounding environment. They group the features together and interpret them as objects (see 2.2.2 and 2.2.3). For example, in Figure 3.2, the observer stands at P and faces the corner, so he can only perceive parts of the couch, parts of the door, parts of the walls, and parts of the tea-table. The observer can group stimuli from partial images by remembered features and recognize a couch, a tea-table, a door, and walls of the room. People select part of the objects as the components of the knowledge of the environment, while neglecting others, like the dust (see 2.3.4). However, the knowledge about objects in the environment is not the knowledge of the whole environment. Spatial linguistic

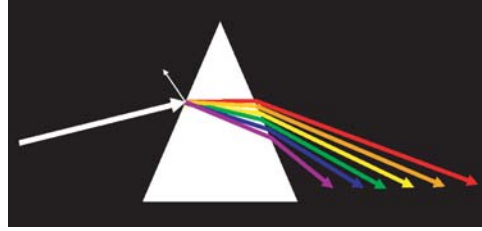


Figure 3.3: When white light passes through a triangular optical prism, a spectrum will be formed

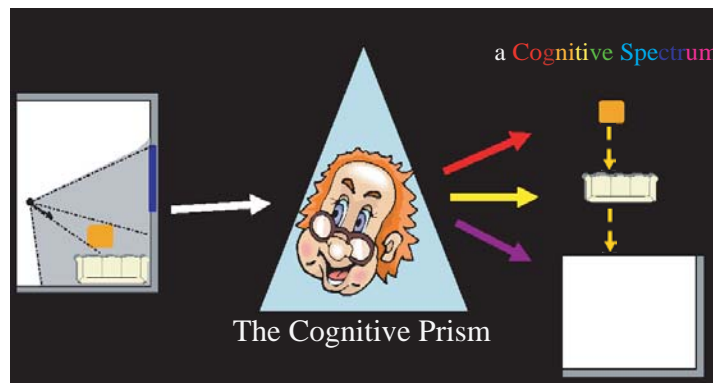


Figure 3.4: When a scene passes through a cognitive system, a cognitive spectrum will be formed. Dotted arrows represent the ordering

descriptions (see 2.3.4) suggest that it shall include spatial structures among the objects. For example, in Figure 3.2, the observer may say, “the tea-table is in front of the couch; the couch is in the corner”. The structure among objects includes spatial relations, such as “in front of” and “in”, and the reference ordering, e.g., the tea-table is referenced to the couch, and the couch is referenced to the corner (see 2.3.3). The knowledge of a snapshot view of an environment is, therefore, at least composed of categorized objects and structural relations (including spatial relations and reference ordering relations) that are acquired through observation.

When a beam of light reaches an optical prism, part of the light will be reflected,

others will pass through, and light that passes through will be arranged with a particular ordering (refraction) and a spectrum will be formed, shown in Figure 3.3. When a scene enters the human eyes, some objects will be neglected, and the selected objects will be arranged with a particular ordering in the mind. The human cognitive system works like an optical prism which neglects some objects and re-arranges the selected by some properties, shown in Figure 3.4. It is, therefore, called “a cognitive prism” and the knowledge (the product of the cognitive prism) is called “a cognitive spectrum”. It is a particular spatial configuration which is composed of objects, relations, and ordering.

The research questions are: (1) What are the spatial relations between extended objects through observation? (addressed in 4.2 and 4.3) (2) According to what property is the reference ordering formed? (addressed in 4.4 and 4.5) (3) What is the structure of a “cognitive spectrum”? (addressed in 4.6)

3.3 Recognizing spatial environments

When we talk about recognizing something, we talk about two worlds: One is the world before the eyes, namely, the perceived world; the other is the world in the memory, namely, the remembered world. Recognition means that the perceived world is believed to be the same as the remembered world.

If the remembered world is some remembered cognitive spectrums of a target environment, then these remembered cognitive spectrums can be transformed from one to another. Particularly, if they are temporally sequenced, then the first one and the last one span the target environment within a temporal duration, others in between demarcate traces of objects’ movements during this temporal duration. From

the perspective of spatial ontologies (see 2.4), the remembered cognitive spectrums are SNAP ontologies, and they participate into the SPAN of the target environment at different temporal points.

If the observer perceives an environment for some time, he will have a sequence of cognitive spectrums of the perceived world. Recognizing the perceived environment as the remembered one means that the perceived sequence of cognitive spectrums is believed to be those that follow the sequence of cognitive spectrums of the target environment that is remembered in the mind. That is, SNAPs of the perceived environment and SNAPs of the target environment participate into the same SPAN at two temporal durations — the remembered sequence is before the perceived one. This is equivalent to that the latest cognitive spectrum in the remembered sequence was before the first cognitive spectrum in the perceived sequence, and can be transformed to it. Because other cognitive spectrums in the remembered sequence transformed into the latest one and the first cognitive spectrum in the perceived sequence transforms into other ones in the perceived sequence, recognizing a spatial environment is equivalent to that a remembered cognitive spectrum is believed to transform into a perceived cognitive spectrum. One cognitive spectrum transforms into the other, if they are exactly the same, or the difference between them can be diminished by the replacements of the some objects in them. The ease of the replacements of related objects determines the degree of the confidence of the recognition result.

The research questions are: (4) How shall be two cognitive spectrums compared? (addressed in 4.7.1, 4.7.2, 4.7.3, and 4.7.4) (5) How does the difference between two cognitive spectrums effect the degree of the confidence of the belief that the perceived cognitive spectrum is transformed from the remembered one? (addressed in 4.7.5)

3.4 A computational approach to recognizing spatial environments

To model the commonsense understanding of recognizing spatial environment, we will develop a computational theory and formalize spatial relations between extended objects. In most of the current representation models of distances and orientations, objects are either represented by points, e.g., Hernández (1994), Schlieder (1995), Zimmermann (1993) or by a directed segment, Freksa (1992). This is reasonable for geographical spaces and large-scale spaces: For geographical spaces the distances among stars or planets are much greater than the sizes of these objects, therefore, the point-based approach is suitable; for large-scale spatial environments locations are abstracted into points, thus, objects in the location are certainly abstracted into points, and objects in the street can be abstracted into a directed segment. However, for *visa* spatial environments (or the space surrounding the body), these geographical-oriented and large-scale-oriented point-based approaches have some problems.

Firstly, there is an incompatibility between “vista” and “point”: To model a *vista* spatial environment is to model knowledge of something visible. The basic knowledge of visible extended objects is their shapes, colors, sizes, etc. If an extended object is represented by a point, then all the basic knowledge is missing, as the knowledge that a point represents is only an imagined extremely tiny mathematical location with a name, if such a location exists at all.

Secondly, there is an inconsistency of the treatment between spaces occupied by objects (occupied spaces) and spaces among them (unoccupied spaces). The two kinds of spaces are correlated and in the same granularity in *vista* spatial environments.

People recognize occupied spaces, and the unoccupied spaces are the things that attach to the occupied spaces. Therefore, if there is a zooming system that zooms out occupied spaces into points, then it will also zoom out unoccupied spaces into points. This turns the whole vista environment into one point — this is a typical representation style for large-scale spaces.

Therefore, the first research question is: (1)' How can we formalize the spatial relations between extended objects through observation? (addressed in 5.2 and 5.3) In corresponding to the questions (2) to (5), other research questions in the computational part are: (2)' How can we formalize the reference ordering relation? (addressed in 5.4) (3)' What is the formal structure of a “cognitive spectrum”? (addressed in 5.5 and 5.6) (4)' How shall two formal structures of cognitive spectrums be compared? (addressed in 5.7.1-5.7.5) (5)' How to formalize the degree of the confidence of the belief that the perceived cognitive spectrum is transformed from the remembered one? (addressed in 5.7.6)

3.5 Towards The Theory of Cognitive Prism

Answering questions (1)–(5) and (1)'–(5)' results in a computational theory on recognizing variable spatial environments – The Theory of Cognitive Prism: When a cognitive system observes a spatial environment, it will select part of the objects in the environment, while neglecting others, and subjectively re-arrange the selected objects, forming a cognitive spectrum. To recognize a spatial environment, it compares the currently perceived cognitive spectrum with the cognitive spectrum of the target environment in its memory. The recognition result is determined by the ease of the transformation from the remembered cognitive spectrum to the perceived one.

Chapter 4

Recognizing spatial environments: A commonsense approach

This chapter presents a commonsense approach to recognizing variable spatial environments. It is structured as follows: Section 4.1 presents the starting point; section 4.2 proposes spatial relations between extended objects; section 4.3 presents the notion of the relative space; section 4.4 proposes the notion of the relative stability; section 4.5 proposes the principle of reference between extended objects; section 4.6 presents the structure of a cognitive spectrum; section 4.7 presents the relations between two cognitive spectrums that pertains to recognize spatial environments; section 4.8 summaries The Theory of Cognitive Prism.

4.1 Knowledge about extended objects based on observation

When Mr. Bertel's mother stood at the entrance door of her son's apartment, she saw view-dependent images of the objects — some partially blocked by others. However, she can recognize objects based on parts of one or more sides of them.

4.1.1 Preferred categories of objects based on observation

Recognizing objects means categorization. To recognize objects is to put them into the preferred categories (see 2.2.1). This owes both to real properties of the underlying factual material and to acts of human decision. When you see a chair near your kitchen table, you have a recognized chair in your mind; when you see a dog in a picture, you have an imagined dog in your mind; when you see a film, you have imagined spaces based on the sequence of the pictures. Your recognizing the chair near your kitchen table owes to the light reflection from the chair and to your recognition activity. Your recognizing the dog in the picture owes to the color distribution in the picture and to your knowledge of a dog in the mind. Imagined or recognized objects are, therefore, *fiat* objects (see 2.4.2) and exist in the mind. They sleep somewhere in the memory, and are awoken either by some external stimuli or by certain mental desires.

4.1.2 Sides of recognized objects

People see sides of objects from different perspectives and recognize them (see 2.2.3). Sides are distinguished and named qualitatively, such as the *left* side, the *right* side, the *back* side, the *upper* side, the *bottom* side. Sides are parts of the boundary of an extended object. Your face is the *front* side of your head. Your two ears are located on the *left* side and the *right* side of your head. The boundaries of the *front* side, the *left* side, and the *right* side are *fiat*, as there are no physical discontinuities between the *front* side and the *left* side, and between the *front* side and the *left* side of your head. Neighborhood sides are partially overlapped as when you are seen from the front, parts of your *left* side and your *right* side will also be seen. When you are seen from the left, part of the your *front* side will be seen.

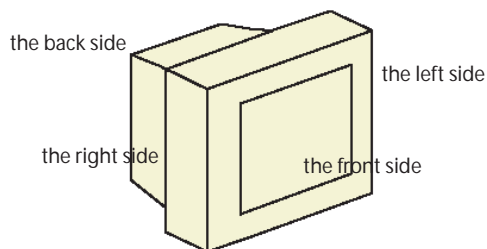


Figure 4.1: Different sides of a TV set

4.1.3 Spatial relations among sides

Sides of an object have some simple spatial relations. When you walk clockwise from the *front* side to the *back* side of the TV set, the surface that you have passed is the *right* side of the TV set. When you walk clockwise from the *back* side to the *front* side of the TV set, the surface that you have passed is the *left* side of the TV set, shown in Figure 4.1. Sides of an object have neighborhood relations. The *front* side is a neighbor of the *left* side and the *right* side; the *left* side is a neighbor of the *front* side and the *back* side; etc.

4.2 Spatial relations as spatial extensions

When Mr. Bertel's mother stood at the entrance door, she could recognize not only objects in the apartment, but also spatial relations among them. Imagine that she stood at the place and faced the apartment as shown in Figure 4.2 (a), she would recognize spatial relations between herself and objects, such as *she is nearer to the balloon than to the writing-desk*, *the balloon is in front of her*, and also spatial relations among objects, such as *the balloon is near and in front of the writing-desk*.

If you turn on a flashlight, it will emit a light beam. Imagine our eyes are such

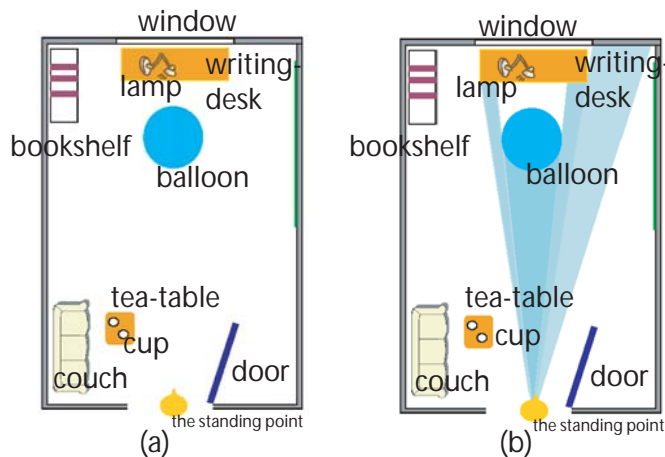


Figure 4.2: Spatial relations based on observation

a flashlight that they can emit a light beam and that we see the side of the object which blocks the light beam¹. Then, *that an object is in front of the observer* can be explained as follows: “If the observer faces to the object, there will be a light beam which is connected with both the eyes and the object”, shown in Figure 4.2 (b); *that an object is left to the observer* can be explained as: “If the observer turns to the left, there will be a light beam which is connected with both the eyes and the object”, or “if the observer faces forward, there will be a light beam which is connected with the left side of the observer’s face and the object” — The observer can prove this by turning to the left to see whether there is a light beam connecting the object and the eyes; *that the observer is nearer to object A than to object B* can be explained as: “The light beam which is connected with the observer and *object A* has a smaller size than the light beam which is connected with the observer and *object B*”.

Imagine the observer does not emit light beams, rather ultrasonic waves, like

¹This is the way that the ancient Greece explained how people see objects. Modern physics explains that it is not that eyes can emit light, rather eyes can receive reflection light from the objects.

blind bats, or imagine a blind man with a stick, they can also know whether there are obstacles in front of them and which obstacle is nearer to them than other objects, if they know the size of the emitted ultrasonic wave or the stretching degree of the stick. Light beams, ultrasonic waves, and sticks, etc. serve as extensions of the body space of the observer, no matter what these extension objects are, no matter these extension objects are created actively or received passively. Spatial relations between the observer and objects can be understood by making extension of the body space into the space surrounding the body.

By assigning extensions to objects, people can define new spatial relations between objects. As people experience that they can reach the writing-desk while sitting on the chair, they could assign the space of their body to the chair and indicate the spatial relation between the chair and the desk as *the chair is near the writing-desk*. If they notice that they reach the front side of the writing-desk while sitting on the chair, they would give the orientation relation between the chair and the desk as *the chair is in front of the writing-desk*.

Three natural extension objects relating to recognize spatial environments are: The body (or part of the body, like limbs), the step(s)², and the light beam (in physics it is called the *reflection light*). When Mr. Bertel's mother stood at the entrance door of her son's apartment, she recognized the objects using light beams, and knew the spatial relations among objects. If one object can be extended to the other by the body, then the two objects are near; if the extension needs several steps, the distance between them would be a bit far away; if the extension is only possible by the light beam, the two objects would be far away.

²Steps represent a sequence of body spaces such that successive body spaces are connected at legs.

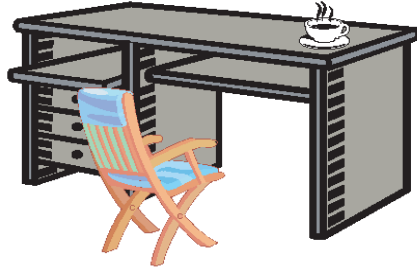


Figure 4.3: Qualitative orientation relations among the cup, the chair, and the desk will be described as *the cup is on the right side of the desk* and *the chair is in front of the desk*

4.2.1 Distances: The extension from one object to the other

When Mr. Bertel's mother observed that she was nearer to the chair than to the writing-desk, she just used the light beam as extension objects, and she found that the chair blocked part of the light beam which is connected with the writing-desk and the eye, shown in Figure 4.2 (b). That is, the light beam that is connected with her eyes and the chair has a smaller size than the light beam that is connected with the eyes and the writing-desk. This is also proved by the fact that if she walked to the writing-desk following the light beam which is connected with the writing-desk and the eye, she would reach the chair first. This time she used steps as the extension objects. Distances between two extended objects are the degree of the extension from one object to the other by certain extension objects.

4.2.2 Orientations: The extension to which side

If the observer sits in the chair in Figure 4.3, she/he could easily reach the front side of the desk. That is, the chair with the body of the observer as the extension is connected with the front side of the desk. The spatial relation between the chair

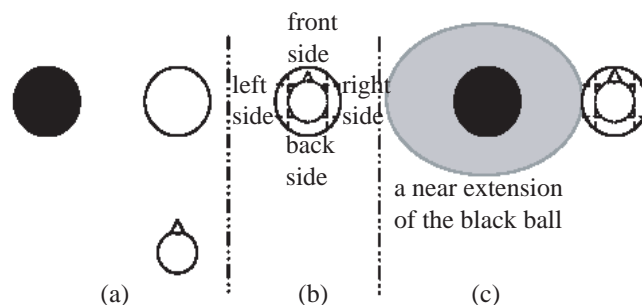


Figure 4.4: A *flat* projection of the observer to the white ball

and the desk is not only that *the chair is near the desk* but also *the chair is near to the front side of the desk, instead of its other sides*. That is, the chair is nearer to the front side of the desk than to its other sides. This is described as an orientation relation that *the chair is in front of the desk*. The orientation relation between two extended objects can be explained by the distance comparison between one object and sides of the other object.

Some objects are perceived the same from different perspectives, e.g., the black ball and the white ball in Figure 4.4 (a). An observer can describe the orientation relation between the two balls as follows: *If I stand at the place shown in Figure 4.4 (a), the black ball is at the left hand side of the white ball*. This statement assigns a name to the side of the white ball which is nearer to the black ball than its other sides. This side is named based on the standing place and the facing direction of the observer. The observer imagines that she/he were at the white ball while keeping the facing direction and names sides of the white ball with reference to names of her/his own sides, shown in Figure 4.4 (b). Then, the orientation relation between the black ball and the imagined herself/himself can be given: “The black ball is located left to the imagined her/him”. In the language description, the observer says: *“If I stand at the place shown in Figure 4.4 (a), the black ball is at the left hand side of the white ball.”* It

is such a mechanism as if the observer “projects” herself/himself to the white ball, and this mechanism is called “the *fiat* projection”. This mechanism requires the observer to know the standing place and the facing direction of herself/himself and to extend this knowledge to the space surrounding the body³. From the perspective of the *fiat* projection mechanism, the deictic orientation reference framework in the literature is a kind of naming the sides of an object⁴. In Figure 4.3, the orientation relation between the cup and the desk can be explained by the *fiat* projection as follows: The observer imagines herself/himself sitting in the chair, and projects her/his sides to the desk, then the cup is nearer to her/his right side. The linguistic description can be: “*The cup is on the right side of the desk*”.

4.3 The relative spaces

Space is simply the order or relation of things among themselves.

–*Leibnitz*

When Mr. Bertel stretches out his hand to the cup, his arm serves as the extension object of the body. He will say “the cup is near me”, if he can hold the cup. If he holds the cup and moves the arm around, all the locations of the cup are near him. Therefore, “the cup is near me” means that the cup is located in one of the

³This is consistent with the empirical findings of Franklin and Tversky (1990) and the functional spaces, i.e., the space of the body, the space around the body, in Tversky et al. (1999b) and Tversky (2005).

⁴Similarly, the absolute orientation reference framework can be explained as follows: The observer projects the imagined earth to the reference object, so that its sides could be named after the imagined earth: The north, the west, the south, and the east. The absolute orientation relation is therefore the result of the distance comparison between the location object and the four imagined sides of the reference object. In general, the *fiat* projection is described as follows: The observer projects an imagined object which has clear sides to the reference object. The orientation relation is the result of the distance comparison between the location object and the imaged sides of the reference object.

locations that all are “near” Mr. Bertel. If he stretches out the arm and holds a book, a cigarette, he will agree that the book or the cigarette is near him. Even he stretches out the arm and holds nothing, he will say, “there is nothing near me”. Therefore, the existence of the space delineated by “near me” is independent of the existences of the cup, the book, the cigarette, etc. It is a **relative space** (following Smith (2001) it is a *fiat* object) created by Mr. Bertel and the arm. It refers to the space that Mr. Bertel can reach by the arm. Mr. Bertel is called the anchor object of the relative space; the arm is called the extension object of this relative space. Mr. Bertel’s relative space of “near me” might have a different size from his mother’s relative space of “near me” in the sense that their arms might be of different sizes. Mr. Bertel’s relative space of “near me” is also changed along with the size of his arm. When he was a child, he had a smaller space of “near me” than the one that he now has.

In particular, a **connectedness relative space** is constructed by an anchor object such that any object in this relative space is connected with the anchor object. For example, the lamp is in the connectedness relative space of the writing-desk.

A **distance relative space** is a relative space constructed by an anchor object and one or more extension object(s) such that it spatially extends the anchor object to reach other objects by the extension object(s). For example, suppose that ‘near’ is interpreted as the distance relation such that two objects are ‘near’, if the body of the observer can connect the two objects, then ‘near the couch’ is a distance relative space such that any object in it can be connected with the observer’s body which is also connected with the couch.

An **orientation relative space** is a relative space constructed by an anchor object with its particular side such that any object inside this relative space is nearer

to this particular side than to its other sides. For example, ‘in front of the desk’ is an orientation relative space constructed by the desk and its front side such that if Mr. Bertel is in this relative space, he will be nearer to the front side of the desk than to its other sides.

Objects are recognized categorically. This leads to the fact that relative spaces are also recognized categorically. The category of a relative space is determined by the categories of the objects and relations that construct the relative space. In particular, the category of a connectedness relative space is determined by the connectedness relation and the category of its anchor object. The category of a distance relative space is determined by the distance relation and the category of its anchor object. The category of an orientation relative space is determined by the orientation relation and the category of its anchor object.

4.4 Knowledge of the relative stability

People have the common sense that the ground and the sky are like a motionless box and that other objects are inside, moving from here to there. This box with the ground as the bottom and the sky as sides and the cover is more stable than the objects inside, because when a box moves, the objects inside are moved along with it, however, when the objects inside move, the box does not move along with them. This implies the principle of commonsense reasoning of the relative stability between two objects as follows: *Object A is relatively more stable than object B, if object B moves along with the move of object A, and object A does not move along with the move of object B.* For example, a table is relatively more stable than a picture on the table, because the picture moves along with the move of the table, but not vice versa.

In the common sense, the earth is motionless and people frequently move here and there on the earth. Trees and lawns are planted in the ground; streets, plaza, and buildings are built on the ground. They are almost as motionless as the earth. For indoor spatial environments, floors, walls, and ceilings are motionless components of buildings. Objects that are affixed to these motionless components are as stable as them, e.g., sinks in the kitchen. All of the motionless objects along with objects that are affixed on them are called *rarely moved objects*. Big pieces of furniture, such as writing-desks, couches, shelves, etc., are moved into rooms. Therefore, they are relatively less stable than these *rarely moved objects*. On the other hand, people walk actively inside of rooms. Objects that are often held by people move along with them, such as newspapers, cups of tea, glasses of wine, books, pens, shoes, plates, etc. These objects are called *always moved objects*. Big pieces of furniture can not be held easily by people, therefore, people move themselves from one big piece of furniture to another along with *always moved objects*. For example, Mr. Bertel takes a book from the bookshelf, then walks to the desk; later, he walks to the couch and reads newspapers while drinking a cup of tea. Big pieces of furniture may have peripheral objects to support people or some *always moved objects*. Couches have tea-tables as the peripheral objects which support cups, glasses, apples, newspapers, etc.; desks and tables have chairs as the peripheral objects that support people. Peripheral objects, therefore, have relatively stable locations with corresponding big pieces of furniture. And, it is usual that they are moved by people for other usages. For example, Mr. Certel moved the balloon from near the desk to near the tea-table to chat with Mr. Bertel. Therefore, they are less stable than the big pieces of furniture. As a summary, the qualitative space of the relative stability is bounded by two poles —

the motionless pole (the earth) and the motion pole (people) and spanned according to the principle of commonsense reasoning of the relative stability as follows: *Rarely moved objects* are closest to the motionless pole, such as walls, floors, sinks in the kitchen, etc.; followed by the *seldom moved objects*, such as writing-desks, couches, etc. *Always moved objects* are closest to the motion pole (people), such as cups, shoes, newspapers; *often moved objects* are located between the *seldom moved objects* and *always moved objects*, such as chairs, tea-tables, etc. *Rarely moved objects* are relatively more stable than *seldom moved objects*; *seldom moved objects* are relatively more stable than *often moved objects*, and so on. The diagrammatical representation is shown in Figure 4.5.

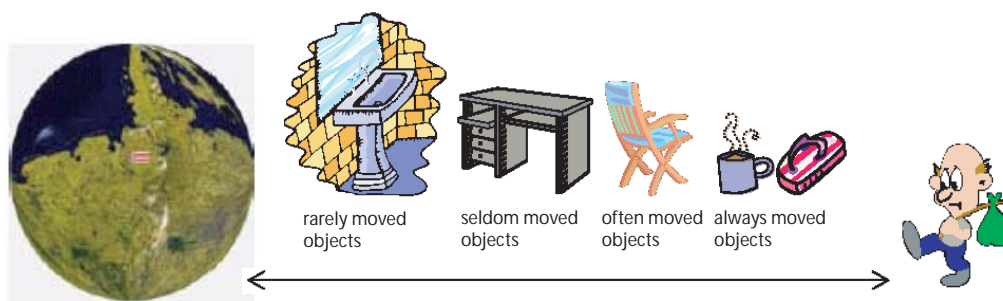


Figure 4.5: Four object classes of indoor spatial environments based on the relative stability

When you move your arm, when you open your mouth, when you close your eyes, your body may not move. The arm, the mouth, and the eyes are functional parts of the body. When a functional part of an object moves, the object itself may not move. However, when your front side moves, your body must move along with the front side. It is therefore assumed that the object and its sides have the same relative stability.

4.5 The reference ordering

4.5.1 The principle of reference in spatial linguistic descriptions

When a pilot has lost his location information, he expects the location information such as “*you are above the South Pole*” rather than “*you are in your plane*”, because the latter provides no information about the location of the plane. His relative location should be referenced to more stable objects. The commonsense knowledge of the relative stabilities affects the selecting of the reference object in spatial linguistic descriptions, in order to keep the description informative. For example, in describing relative locations of *the sun*, people prefer to saying that “*the sun is in the sky*” and “*the sun moves around the earth*” instead of “*the sky surrounds the sun*” or “*the earth moves around the sun*”⁵. People prefer to saying “*the bike is beside the tree*”, “*the bike is on the lawn*”, “*the car is on the plaza*”, “*the carpet is on the floor*”, “*the picture is on the wall*”, instead of “*the tree is beside the bike*”, “*the lawn is under the bike*”, “*the plaza is under the car*”, “*the floor is under the carpet*”, “*the wall is behind the picture*”. This is summarized as the first criterion of selecting reference objects in spatial linguistic description as follows: The reference object in spatial linguistic descriptions should be of the higher or the same relative stability than the object. This is called “the criterion of stability” which keeps the linguistic description informative.

According to the criterion of stability, the writing-desk in Figure 4.6 can be referenced to the door — it is ‘far away from the door’. However, ‘the door’ is normally not selected as the reference object, because it is farther away from ‘the writing-desk’ than ‘the window’ and ‘the front wall’ which also satisfy the criterion of stability. This

⁵In the history, people who insisted saying that *the earth moves around the sun* were even burnt to death.

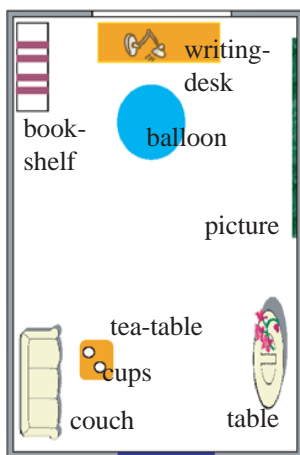


Figure 4.6: The cups are referenced to the tea-table; the balloon is referenced to the writing-desk; the picture is referenced to the walls; etc.

infers that among reference objects satisfying the criterion of stability, the nearer reference objects are preferred, because the farther away a reference object is, the more effort will be cost to make spatial extensions from the reference object to the object. This is called “the criterion of economics”. In Figure 4.6, there are many objects that are relatively more stable than the cups, however, ‘the tea-table’ is selected as the reference object for the cups, instead of ‘the couch’, ‘the floor’, ‘the ceiling’, because according to the criterion of economics, objects that are connected with the cups have the priority to be the reference objects. The tea-table is connected with the cups; and it is also of higher relative stability than cups, which satisfies the criterion of stability.

The principle of reference in spatial linguistic descriptions is summarized as follows: The reference object in spatial linguistic description is of higher or the same relative stability than the location object (the criterion of stability) and the nearer reference objects are preferred (the criterion of economics).

4.5.2 The cognitive reference objects

People have cognitive reference points (see 2.3.3) in spatial cognition. Following Rosch (1975)'s definition of "cognitive reference point"⁶, to be a "reference object" in a vista spatial environment, an object must be shown to be one which other objects are seen "in relation to". For the task of recalling and describing spatial environments, "in relation to" is taken to mean, in the spatial linguistic descriptions, that the relationship between reference objects and non-reference objects are asymmetrical; whereas relationships between two non-reference objects are symmetrical. This follows that an object is less stable than its cognitive reference object. For example, in Figure 4.6, 'the writing-desk' is the cognitive reference object of 'the lamp', because people would say that *the lamp is on the writing-desk*, rather than *the writing-desk in under the lamp*; 'the bookshelf' is not the cognitive reference object of 'the writing-desk', because people would say that *the writing-desk is near the bookshelf* and that *the bookshelf is near the writing-desk*. The principle of selecting cognitive reference objects can be derived from the principle of reference in spatial linguistic description as follows: The cognitive reference object is of higher relative stability than the location object and that the nearer objects are preferred to be cognitive reference objects.

A relative space is defined as *a location* of an object, if the anchor object of the relative space is one of the cognitive reference objects of the object. For example, *that the tea-table is near the couch* is interpreted as *a location* of the tea-table is *near the couch*, because the couch is one of the cognitive reference objects of the tea-table. *The*

⁶To be a "reference point" within a category, a stimuli must be shown to be one which other stimuli are seen "in relation to."... For purpose of the present research, "in relation to" was taken to mean, operationally, that there were judgment tasks in which the relationship between stimuli in the category and the reference stimulus were asymmetrical; . . . , relationships between two non-reference stimulus members of the category were symmetrical. (Rosch, 1975, p.532)

location of an object in a spatial environment is defined as the conjunction of all its *locations*. For example, *locations* of the tea-table in Figure 4.6 are *on the floor*, *near the couch*, *in front of the couch*, then *the location* of the tea-table is the conjunction of the locations as follows: *on the floor* and *in front of the couch* and *near the couch*. Locations of objects delineate ‘where’ objects are in spatial environments.

4.6 Cognitive spectrums of spatial environments

When a snapshot view of a spatial environment passes through people’s eyes, it is partitioned and understood as objects, and spatial relations, which are structured based on *the principle of selecting cognitive reference objects*, forming a cognitive spectrum of the spatial environment. For example, when a scene, shown in Figure 4.7, is perceived by an observer, it will be partitioned and understood as objects, such as a couch, a tea-table, a right wall, a front wall⁷, and a floor. The observer has commonsense knowledge of relative stabilities of objects as follows: The floor, the front wall⁸, and the walls are rarely moved objects, the couch is a seldom moved object, the tea-table is an often moved object. Based on the principle of selecting cognitive reference objects, objects shall be referenced to more stable objects nearby. Therefore, the cognitive reference objects of tea-table are the floor which is connected with the tea-table, and the couch which is near the tea-table; similarly, the cognitive reference objects of the couch are the floor, the right-wall, and the front-wall. As the floor and the walls are sides of the room, spatial relations among them are here

⁷The names of the walls are based on the *flat* projection of the observer who is facing the door. The right-wall is the side of the room on the right-hand side, the front-wall is the side of the room to which the observer faces.

⁸Although a door often turns on one side of the doorframe, the front wall with the door and the doorframe as a whole is a side of the room which is a rarely moved object.

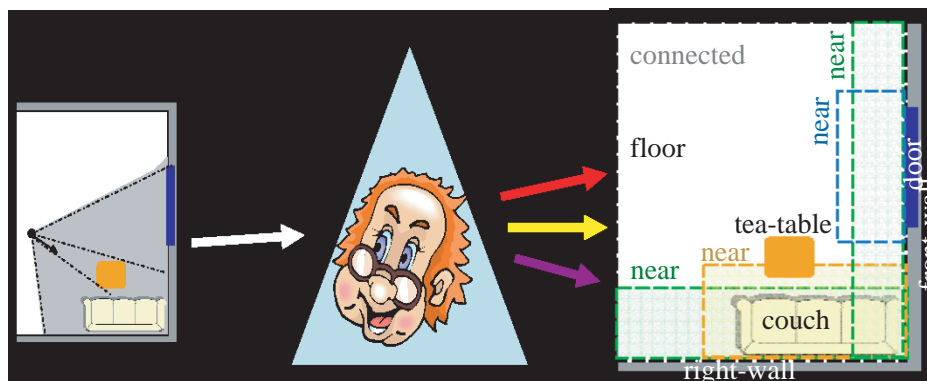


Figure 4.7: When a scene passes through a cognitive prism, a cognitive spectrum will be formed

beyond the scope. They are used as single objects, as evidenced in spatial linguistic descriptions.

4.6.1 A diagrammatic representation of cognitive spectrums

A cognitive spectrum can be diagrammatically represented by nested relative spaces. Colors (or textures) of objects represent relative stabilities. An object is connected with a relative space, if it is referenced to the anchor object of this relative space and it has the spatial relation to the anchor object which is delineated by this relative space. The cognitive spectrum in Figure 4.7 can be diagrammatically represented by the nested relative spaces shown in Figure 4.8. The gray (cotton), green (marble), and blue (grid) colors (textures) represent rarely moved objects; the sandy beige (nut tree) represents seldom moved objects; the amber (oak) represents often moved objects. Sandy beige (nut tree) objects can reference to gray (cotton), green (marble), and blue (grid) objects; amber (oak) objects can reference to Sandy beige (nut tree), gray (cotton), green (marble), and blue (grid) objects. A colored (textured) region with an object name represents the connectedness relative space of this object; a

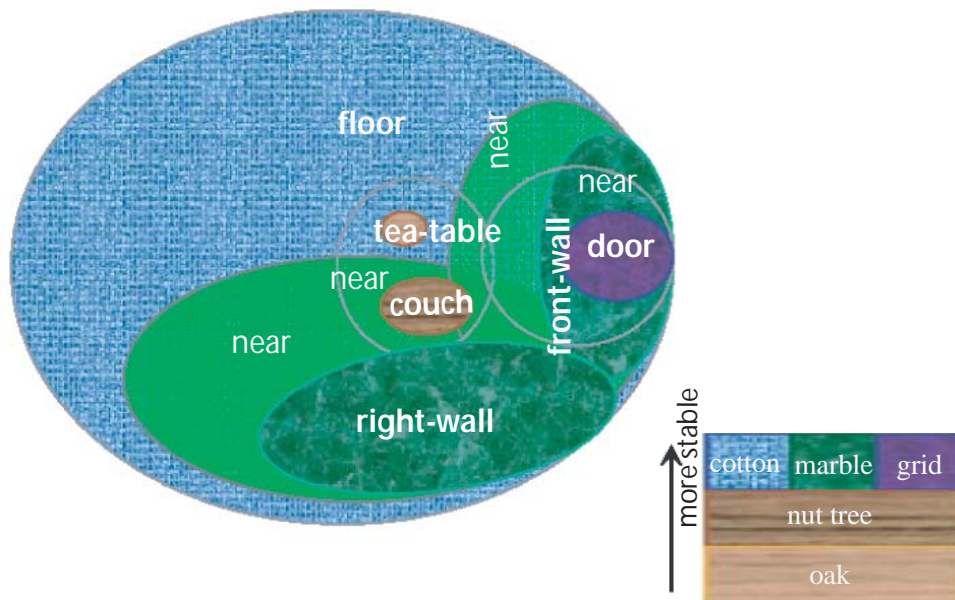


Figure 4.8: A diagrammatic representation of a cognitive spectrum. The gray (cotton), green (marble), and blue (grid) represent rarely moved objects; the sandy beige (nut tree) represents seldom moved objects; the amber color (oak) represents often moved objects

light colored region with a label ‘near’ around an object region represents a ‘near’ relative space such that objects connected with this relative space and disconnected with its anchor object are near the object region. Among the objects that the tea-table (the amber/oak region) can reference, the floor (the gray/cotton region) and the ‘near’ relative space of the couch (the sandy beige/nut tree region) are connected with the tea-table. Among the objects that the couch can reference, the floor, the ‘near’ relative space of the right-wall (the green region/marble), and the ‘near’ relative space of the front-wall (the green region/marble) are connected with the couch.

4.6.2 A symbolic representation of cognitive spectrums

Symbolically, a cognitive spectrum of an environment can be represented by two tables — the table of objects, which lists properties of objects in the environment, and the table of relative spaces, which lists locations of objects in the environment.

Table 4.1: The table of knowledge of objects

<i>Name</i>	<i>Category</i>	<i>Relative stability</i>
a room	$\langle \text{ROOM}_{Bertel} \rangle$	rarely moved
a couch	$\langle \text{COUCH}_{Bertel} \rangle$	seldom moved
a tea-table	$\langle \text{TEATABLE}_{Bertel} \rangle$	often moved

The table of objects collects object knowledge about the cognitive spectrum. The object knowledge includes the object name, its category, and its relative stability. For example, object knowledge of the scene in Figure 4.7 includes knowledge of a room, a couch, and a tea-table. The room is a rarely moved object; knowledge of the room characterizes a categorization $\langle \text{ROOM}_{Bertel} \rangle$ which means objects in this category are indistinguishable from the room through perception; the couch

is a seldom moved object; knowledge of the couch characterizes a categorization $\langle \text{COUCH}_{Bertel} \rangle$ which means objects in this category are indistinguishable from the couch through perception. The tea-table is an often moved object; knowledge of the tea-table characterizes a categorization $\langle \text{TEATABLE}_{Bertel} \rangle$ which means objects in this category are indistinguishable from the tea-table through perception.

Knowledge of object sides provides feature to recognize an object. Object sides can be named through the *fiat* projection of the observer, like the front side of the writing-desk, or the salient side of the object itself, like the floor, the ceiling, the wall with the door. The knowledge of object sides, as part of knowledge of object category, is not explicitly listed in the table of knowledge of objects, rather it is implied in the knowledge of the object category.

Table 4.2: The structure of the table of relative spaces

	a spatial relation, e.g., in front of
<i>an object, e.g., the couch</i>	objects located in this relative space, e.g., <i>the tea-table</i>

The table of relative spaces represents locations of objects in the environment. The left column is the list of cognitive reference objects; the top row of the right column is a cell for a spatial relation. Then a cognitive reference object and a spatial relation delineates a relative space which is the cell locating at the same column as the spatial relation and the same row as the cognitive reference object. Object names in this cell represent objects that are referenced to the cognitive reference object at this row and located in this relative space. The structure of the table of relative spaces is shown in Table 4.2.

An object in the cell of a connectedness relative space of its reference object represents the object such that it is relatively less stable than the anchor object of the connectedness relative space and that it is connected with the anchor object. For example, in Figure 4.8 objects in the connectedness relative space of the room are the couch and the tea-table, shown in Table 4.3, because they are less stable than the room and they are connected with the room.

Table 4.3: The table of connectedness relative spaces

	connected with
the room	the couch, the tea-table

An object in the cell of a distance relative space represents the object such that it is relatively less stable than the anchor object of the distance relative space and that it has a distance with the anchor object. For example, in Figure 4.8 objects in the near relative spaces are listed in Table 4.4.

Table 4.4: The table of distance relative spaces

	near
the couch	the tea-table
the right wall	the couch
the front wall	the couch

An object in the cell of an orientation relative space represents the object such that it is relatively less stable than the anchor object of the orientation relative space and that it is nearer to one side of the anchor object than to its other sides (denoted by the orientation relation). For example, the tea-table in front of the couch is located

in the orientation (front) relative space of the couch, shown in Table 4.5, which means the tea-table is nearer to the front side of the couch than to its other sides.

Table 4.5: The table of orientation relative spaces

	in front of
the couch	the tea-table

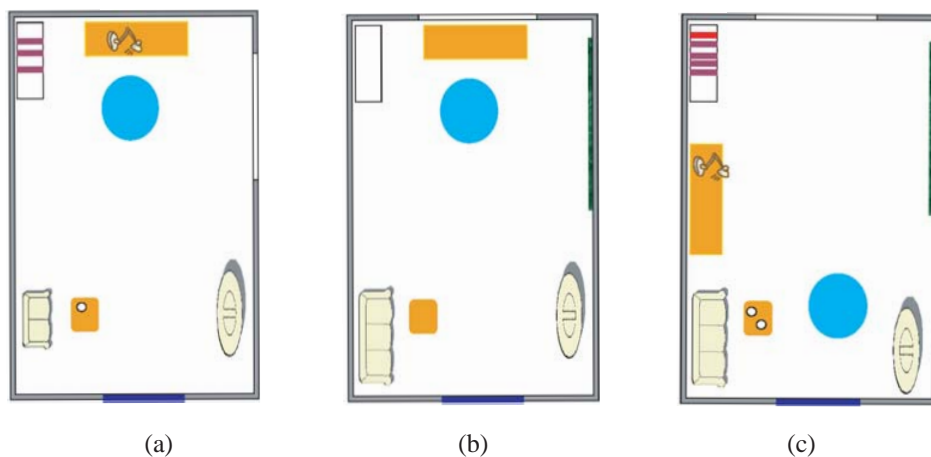


Figure 4.9: Mr. Bertel's mother looked at Mr. Certel's apartment (a) and *mapped* it with her target layout — Mr. Bertel's apartment (b). She concluded that it was not Mr. Bertel's apartment. At last she found her son's apartment (c)

4.7 Relations between two cognitive spectrums

Unfortunately, Mr. Bertel's mother was neither able to recognize Mr. Certel's apartment by recognizing Mr. Certel's objects, nor able to recognize his son's apartment by recognizing his object. The reason is that recognizing objects is only categorization. Two objects in different snapshots are indistinguishable through perception, if they are of the same category. When Mr. Bertel's mother visited her son's apartment for the third time (on the ninth day), she went into Mr. Certel's apartment by

mistake, Figure 4.9 (a), while Mr. Certel was not in the apartment. Although the layout was very similar to Mr. Bertel's, she knew that it was not her son's home and left the apartment. She knew this because she had a cognitive spectrum of Mr. Bertel's apartment in the mind. She knew what objects should be in the apartment and where the objects should be, etc. When she went to Mr. Certel's apartment, the cognitive spectrum of Mr. Bertel's apartment in the mind was used as the criterion to justify whether the perceived one was the target apartment. She expected a window in the front wall⁹, however, there is no window in the perceived front wall; she expected a picture on the right wall¹⁰, however, there is no picture but a window on the perceived right wall. She expected a big couch in the corner of the perceived apartment, however, the couch in the perceived apartment is smaller. She knew that sides of rooms, locations of windows, and sizes of couches can not be changed easily. This led to the doubt whether it was Mr. Bertel's apartment. When she came to the apartment shown in Figure 4.9 (c), she saw furniture that looked the same as those in Mr. Bertel's apartment; and she found that the window, the book-shelf, the couch and the picture were located in the expected places and that the writing-desk and the balloon were not located in the expected places. However, this environment was much more like Mr. Bertel's apartment, because she thought that all these differences could happen. Recognizing spatial environment is a judgement on the spatial differences between the cognitive spectrum of the currently perceived environment and the cognitive spectrum of the target environment. Following questions are raised: How shall two cognitive spectrums be compared? How shall the judgement be made?

⁹The names of the wall is based on the *fiat* projection of the observer who is standing at the door. The front-wall is the side of the room that the observer faces.

¹⁰The name of the wall is based on the same *fiat* projection as the above note. The right-wall is the side of the room on the right-hand side.

4.7.1 The categorical comparison

The starting point to compare two cognitive spectrums is that objects of the same category are indistinguishable in isolation through perception. The primitive relation between two cognitive spectrums is the *categorically the same* relation between two objects. Two objects are *categorically the same*, if they are of the same category. For example, Mr. Bertel's writing-desk and Mr. Certel's writing-desk are *categorically the same*, though they are different ones. However, Mr. Bertel's room and Mr. Certel's room are not *categorically the same*, because their windows are located in difference walls.

Two clusters of objects are *categorically the same*, if for each object category, there is the same number of objects in the two clusters. For example, the cluster of Mr. Bertel's writing-desk and his balloon and the cluster of Mr. Certel's writing-desk and his balloon are *categorically the same*, because for each object category there is one object in each cluster.

Two relative spaces are *categorically the same*, if they are of the same category. In particular, two connectedness relative spaces are *categorically the same*, if their anchor objects are *categorically the same*. For example, the connectedness relative space of Mr. Bertel's writing-desk and the connectedness relative space of Mr. Certel's writing-desk are *categorically the same*, because the two writing-desks are *categorically the same*. Two distances relative spaces are *categorically the same*, if their anchor objects are *categorically the same* and their distance relations are *categorically the same*. That two distance relations are *categorically the same* means that their extension objects are *categorically the same*. For example, the *near* relative space with Mr. Bertel's writing-desk as the anchor object and Mr. Bertel's mother as the extension object and

the *near* relative space with Mr. Certel's writing-desk as the anchor object and Mr. Bertel's mother as the extension object are *categorically the same*. Two orientation relative spaces are *categorically the same*, if their anchor objects are *categorically the same* and their orientation relations are *categorically the same*. That two orientation relations are *categorically the same* means that the two orientation relations have the same name¹¹. For example, the orientation relative space with Mr. Bertel's writing-desk as the anchor object and the front orientation relation and the orientation relative space with Mr. Certel's writing-desk as the anchor object and the front orientation relation are *categorically the same*.

Two clusters of relative spaces are *categorically the same*, if for each category of the relative spaces, there is the same number of relative spaces in the two clusters. Two conjunctions of relative spaces are *categorically the same*, if the two clusters of the components of the conjunctions are *categorically the same*. For example, let the first conjunction of relative spaces be *near the writing-desk in Mr. Bertel's room* and *on the floor in Mr. Bertel's room*, the second conjunction of relative spaces be *near the writing-desk in Mr. Certel's room* and *on the floor in Mr. Certel's room*, then the first conjunction and the second conjunction are *categorically the same*, because the two clusters of their components are *categorically the same*. That is, the cluster of *near the writing-desk in Mr. Bertel's room* and *on the floor in Mr. Bertel's room* is *categorically the same* as the cluster of *near the writing-desk in Mr. Certel's room* and *on the floor in Mr. Certel's room*.

¹¹It assumes that they are based on the same *fiat* projection (this makes the two anchor objects have the same side names).

4.7.2 The process of mapping cognitive spectrums

When Mr. Bertel's mother stood at the door of an apartment, judging whether it was her target apartment, she should map objects in the target cognitive spectrum with objects in the cognitive spectrum of the perceived environment. Suppose that in her target cognitive spectrum there was a writing-desk and that she perceived two writing-desks that are *categorically the same* with the target writing-desk. Which of the two should be *mapped* to the target one? This depends on the location of the target writing-desk. If it was located in front of the window, then the one in front of the window in the perceived environment, if existed, was *mapped* to it. However, this explanation is not total, as it assumes that the target window was *mapped* to the perceived window. That is, she should map the target window to a window in the perceived environment before mapping writing-desks. This also depends on locations of the windows. If the target window was in the front wall, then the perceived window in the front wall was *mapped* to it. And again, this requires to compare the target front wall with the perceived front wall. At last, it requires to compare the two rooms at the very beginning.

This leads to the conditions of object mapping and the principle of the sequence of mapping objects between the target cognitive spectrum and the perceived one as follows: Cognitive reference objects shall be *mapped* before the mapping of their location objects. That is, rarely moved objects are firstly compared and *mapped*, followed by seldom moved objects, and often moved objects. For indoor spatial environment, firstly the perceived room is compared with the target room. If they are not *categorically the same*, then the perceived environment shall not be the target environment,

else the two rooms are *mapped*¹² and each relative space with the perceived room or its side as the anchor object is *mapped* to a relative space with the target room or its side as the anchor object with the condition that the two relative spaces are *categorically the same*. For example, the room in Figure 4.9(a), namely room₁, and the room in Figure 4.9(b), namely room₂, are not *categorically the same*, because the window in the perceived room is located differently as that in the target room. So, the perceived cognitive spectrum is not the target cognitive spectrum. The room in Figure 4.9(c), namely room₃, and room₂ are *categorically the same*, therefore, room₃ and room₂ are *mapped*, and a relative space with room₃ as the anchor object will be *mapped* to a relative space with room₂ as the anchor object, if the two relative spaces are *categorically the same*. The connectedness relative space with room₃ as the anchor object is *mapped* to the connectedness relative space with room₂ as the anchor object; the near relative space with the front wall of room₃ as the anchor object is *mapped* to the near relative space with the front wall of room₂ as the anchor object; etc. For short, two rooms are *mapped*, if they are *categorically the same*. Two relative spaces are *mapped*, if their anchor objects are *mapped* and they are *categorically the same*. Two objects inside the rooms are *mapped*, if they are *categorically the same* and their locations are *mapped*. The process of object mapping proceeds by mapping objects in the *mapped* relative spaces as follows.

(Step 1) If the perceived room and the target room are *categorically the same*, then they will be *mapped* and put them as a pair into a queue; set the index to the first element of the queue.

¹²The location of a room is *mapped* when the observer is outside of the room, e.g., in the corridor. People's going into the room implies that the location of the room is *mapped* with the location of the target room and that the outer side of the perceived room is *categorically the same* as the outer side of the target room. It is therefore assumed that the locations of the rooms are *mapped*.

(Step 2) If the index points to a pair in the queue, then take the pointed pair of the queue and find their *mapped* relative spaces, else stop;

(step 3) For each *mapped* relative spaces, find *mapped* objects and put each *mapped* objects as a pair to the tail of the queue;

(step 4) increase the index by 1, and go to (Step 2).

The *mapping* process produces a queue of object pairs. Two objects in each pair are *categorically the same* and *mapped* in location.

4.7.3 The spatial difference

The spatial difference is the un-mapped objects after the process of object mapping. If Mr. Bertel's mother saw a washing-machine in the perceived environment (in the connectedness relative space of the perceived room), and on the other hand, there was no washing-machine in the cognitive spectrum of the target environment, then the existence of this washing-machine cannot be *mapped* to an object in the target environment; if Mr. Bertel moved his writing-desk to Mr. Certel's room, and Mr. Bertel's mother saw an environment with no writing-desk, on the other hand, there was a writing-desk in the cognitive spectrum of her target environment, then the writing-desk in the target cognitive spectrum cannot be *mapped* to an object in the cognitive spectrum of the perceived environment. In general, for each object category in the two *mapped* relative spaces, if there are more objects of this category in the perceived relative space than those of the same category in the target relative space, new objects of this category are perceived, this kind of difference is called an *appearance*; if there are more objects of this object category in the target relative space than those in the perceived relative space, some objects of this category are not perceived, this kind of difference is called a *disappearance*. For example, there is a small couch in

the perceived cognitive spectrum in Figure 4.9(a), and there is no such a small couch in the target cognitive spectrum in Figure 4.9(b). It is called *there is an appearance of a small couch in the perceived environment*.

4.7.4 The compatibility

If we recognize that the perceived cognitive spectrum as the target cognitive spectrum, spatial differences between the two cognitive spectrums will be explained as the result of the transformations of the un-mapped objects. The ease of transformations can be determined by the relative stabilities of the un-mapped objects. The compatibility between the cognitive spectrum of the perceived environment and the target cognitive spectrum qualitatively represents the ease of the transformations.

The cognitive spectrum of the perceived environment and the target cognitive spectrum are hardly compatible, if there is any spatial difference that happens to *rarely moved objects*, such as rooms (including windows, doors), sinks. For example, the cognitive spectrum of the environment shown in Figure 4.9(a) and the cognitive spectrum of the environment shown in Figure 4.9(b) is hardly compatible, because the two rooms are not *categorically the same*, as there is a window in the right wall of the perceived environment, and there is no windows in the right wall of the target environment.

The cognitive spectrum of the perceived environment and the target cognitive spectrum are possibly compatible, if there is no spatial difference that happens to *rarely moved objects* and there is spatial difference that happens to *seldom moved objects*, such as writing-desks, couches, bookshelves, etc. For example, there is no spatial difference that happens to rarely moved objects between the cognitive spectrums of the environments shown in Figure 4.9(b) and Figure 4.9(c), and there is an

appearance of a writing-desk in the near relative space of the left wall and a disappearance of a writing-desk in the near relative space of the front wall in the cognitive spectrum of the environment shown in Figure 4.9(c), writing-desks are seldom moved objects, therefore, the cognitive spectrum of the environment shown in Figure 4.9(b) and the cognitive spectrum of the environment shown in Figure 4.9(c) are possibly compatible.

The cognitive spectrum of the perceived environment and the target cognitive spectrum are compatible, if there is no spatial difference that happens to *rarely moved objects* and *seldom moved objects* and there is spatial difference that happens to *often moved objects*, such as chairs, tea-tables, sitting-balls, etc.

The cognitive spectrum of the perceived environment and the target cognitive spectrum are very compatible, if there is no spatial difference that happens to *rarely moved objects*, *seldom moved objects*, and *often moved objects* and there is spatial difference that happens to *always moved objects*, such as books, cups, pens, etc.

The cognitive spectrum of the perceived environment and the target cognitive spectrum are indeed compatible, if there is no spatial difference found.

4.7.5 Recognition as the judgment on the compatibility

Recognizing spatial environments is a judgment on the compatibility between the cognitive spectrum of the perceived environment and the cognitive spectrum of the target environment. In everyday life situation, people's activities in indoor spatial environments are limited to chatting, eating, playing, sleeping, walking, etc. These activities result in many spatial differences that happen to *always moved objects*, such as books, cups, pens, etc., some differences of *often moved objects*, such as chairs, sitting-balls, etc., a little bit differences of *seldom moved objects*, such as writing-desks,

couches, etc., and hardly differences of *rarely moved objects* such as door frames, window frames, floors, ceilings, sinks, etc. So, people can recognize the perceived environment as the expected one according to the degree of the compatibility between the cognitive spectrum of the perceived environment and the cognitive spectrum of the target environment.

No, it is hardly . . .

If the perceived environment is hardly compatible with the expected one, then in everyday life situation, the perceived environment will be hardly thought of as the expected one. For example, when Mr. Bertel's mother went to Mr. Certel's apartment, shown in Figure 4.9(a), and found that the window was located in a different wall, then she thought that it was hardly Mr. Bertel's apartment, and went out.

It might be . . .

If the perceived environment is possibly compatible with the expected one, then in everyday life situation, the perceived environment might be thought of as the expected one. In this situation, the observer would ask for reasons for the spatial differences. For example, after Mr. Bertel's mother went out from Mr. Certel's apartment, she went to Mr. Bertel's apartment which showed her the after party layout, shown in Figure 4.9(c). She found that the writing-desk was located between the bookshelf and the couch and that the table and the balloon were located differently as she expected, then she wondered for a while, before recognizing it as Mr. Bertel's apartment. When she met Mr. Bertel, she asked what happened last night.

It is ...

If the perceived environment is compatible with the expected one, then in everyday life situation, the perceived environment is thought of as the expected one. In this situation, the observer will take it for granted that it is the expected environment, and some observers may move these often moved objects back to the expected location. For example, when Mr. Bertel's mother went to Mr. Bertel's apartment for the second time (on the seventh day), she found that the perceived layout was compatible with the expected one and that the balloon was located differently. She mumbled why Mr. Bertel moved the balloon to the tea-table, and she moved the balloon back to the writing-desk.

Yes, it is exactly ...

If the perceived environment is very compatible with the expected one, then in everyday life situation, the perceived environment is thought of as exactly the expected one. In this situation, the observer will also take it for granted that it is the expected environment.

Yes, it is indeed ...

If the perceived environment is indeed compatible with the expected one, then in everyday life situation, the perceived environment is thought of as indeed the expected one. In this situation, the observer will certainly take it for granted that it is the expected environment. In most cases, such an indeed compatible layout will be changed by the activities of the people, for example, by reading newspapers people change the location of newspapers; by drinking tea people change the location of cups.

4.8 The Theory of Cognitive Prism

By answering questions (1)–(5) in Chapter 3, The Theory of Cognitive Prism is summarized as follows: When a cognitive system observes a spatial environment, it will select part of the objects in the environment, while neglecting others, subjectively re-arrange the selected objects based on the commonsense knowledge of relative stabilities, make spatial extensions from one object to others, and form a cognitive spectrum following the principle of selecting cognitive reference objects. To recognize a spatial environment, the cognitive system compares the currently perceived cognitive spectrum with the cognitive spectrum of the target environment in its memory following the stable-object-first order. Rooms are *mapped*, if they are *categorically the same*; objects inside rooms are *mapped*, if they are *categorically the same* and their locations are *mapped*. The compatibility between two cognitive spectrums is determined by the un-mapped objects in both cognitive spectrums and their relative stabilities. In everyday life situation, the recognition result is directly interpreted by their compatibility.

Chapter 5

The formalism: A region-based representation and reasoning of spatial environments

This chapter formalizes the commonsense knowledge represented in Chapter 4. Formulae have the form of Z notation, Woodcock and Davies (1996), “ $(\forall x|p \bullet q)$ ” (it is read as “for all x satisfying p , q holds”), “ $(\forall x \bullet q)$ ” (it is read as “for all x , q holds”), “ $(\exists x|p \bullet q)$ ” (it is read as “there is x satisfying p such that q ”), “ $(\exists x \bullet q)$ ” (it is read as “there is x such that q ”), and ‘ $\iota x(q)$ ’ (it is read as “the x that q ’s”). x is the bound variable, p is the constraint of x , and q is the predicate. ‘**true**’ stands for T and ‘**false**’ stands for F . The proofs of the theorems in this chapter are listed in the appendix.

5.1 The object region and its properties

An *object region* represents an imagined or recognized object. An object region has a *flat boundary* and its interior part. An object region belongs to the preferred category in which it is imagined or recognized; it has a degree of the relative stability. The preferred category of an object region corresponds to a collection of features which

each object region in this category has. An object region has sides. A *side* is a part of the *fiat* boundary of the object region that can be seen from a view point. It is called a *side region*. It is assumed in (4.1.2) that neighborhood sides are overlapped, therefore, neighborhood *side regions* are overlapped. It is assumed in (4.3) that an object moves along with its sides, therefore, an object region and any of its side regions have the same relative stability. The side of an object region belongs to a category, which corresponds to a subset of the features of the preferred category of the object region.

Object regions are denoted by *mathcal* capital letters, such as *DESK*, *CHAIR*, *DOOR*, Categories are denoted by *typewriter* capital letters, such as *DESK*, *CHAIR*, *DOOR*, Let *OBJ* represent an object region, and the preferred category of *OBJ* be *OBJ*, *OBJ.category* represents the preferred category of *OBJ*, *OBJ.category=OBJ*. *OBJ.stability* represents the relative stability of *OBJ*. For objects of indoor spatial environments, four degrees of relative stabilities are distinguished: *rarely moved*, *seldom moved*, *often moved*, and *always moved*. The four degrees are represented by *rarelyMoved*, *seldomMoved*, *oftenMoved*, and *alwaysMoved*, respectively; *OBJ.stability* can hold one value of the relative stabilities. The side of an object region is denoted by *OBJ.side*. In particular, *OBJ.front*, *OBJ.left*, *OBJ.back*, *OBJ.right*, *OBJ.upper*, and *OBJ.bottom* represent qualitative sides¹ of *OBJ*. *OBJ.side.stability* represents the relative stability of a side of *OBJ*, and *OBJ.side.stability=OBJ.stability*. The category of a side of an object region *OBJ.side* is written as *OBJ.side*, e.g., the category of *OBJ.front* is *OBJ.front*.

¹The side can be named by saliency of sides of object regions (the intrinsic reference framework) or by a *fiat* projection (the deictic reference framework, or the absolute reference framework).

5.2 Spatial relations between regions

The distance relation and the orientation relation between object regions can be formalized using the connectedness relation.

5.2.1 ‘Connectedness’ is primitive

A *region* refers to an *object region*, a *side region*, or a *constructed region* (defined later). The only primitive relation between regions is “connected with”: **C**. In the literature, *that two regions are connected* is interpreted as *their closures share a point² in common*, i.e. Randell et al. (1992). When two regions are connected, for any category there is an object region in this category such that it is connected with both of the two regions. This is equivalent to that if two regions are not connected, then there is a category such that all regions in this category are not connected with both of the two objects. Let the category be the length unit which is smaller than the minimal distance between two disconnected regions, then any region in this category is not connected with both of the two disconnected regions. An axiom that governs the connectedness relation is as follows.

Axiom 5.2.1. *All regions \mathcal{A} and \mathcal{B} , it holds that if one is connected with the other, then all category Z there is a region \mathcal{Z} satisfying that \mathcal{Z} is a member of Z such that \mathcal{Z} is connected with both \mathcal{A} and \mathcal{B} .*

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}, \mathcal{B}) \rightarrow \forall Z \exists \mathcal{Z} | \mathcal{Z} \in Z \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{Z})$$

Another axioms governing the connectedness relation, following Randell et al. (1992), are as follows.

Axiom 5.2.2. *All region \mathcal{A} , it holds that \mathcal{A} is connected with \mathcal{A} .*

$$\forall \mathcal{A} \bullet \mathbf{C}(\mathcal{A}, \mathcal{A})$$

²A *point* can be mereotopologically defined, see Eschenbach (1994).

Axiom 5.2.3. *All regions \mathcal{A} and \mathcal{B} , it holds that if \mathcal{A} is connected with \mathcal{B} , then \mathcal{B} is also connected with \mathcal{A} .*

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{B}, \mathcal{A})$$

The relation of parthood $\mathbf{P}(\mathcal{A}, \mathcal{B})$ can be defined immediately.

Definition 5.2.1. *Given two regions \mathcal{A} and \mathcal{B} , ‘ \mathcal{A} is part of \mathcal{B} ’ is defined as all region \mathcal{Z} , it holds that if \mathcal{Z} is connected with \mathcal{A} , then \mathcal{Z} is connected with \mathcal{B} .*

$$\mathbf{P}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \forall \mathcal{Z} \bullet \mathbf{C}(\mathcal{Z}, \mathcal{A}) \rightarrow \mathbf{C}(\mathcal{Z}, \mathcal{B})$$

\mathbf{P} is governed by the axiom as follows.

Axiom 5.2.4. *All regions \mathcal{A} and \mathcal{B} , it holds that if each one is the part of the other, then they are the same.*

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{P}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{P}(\mathcal{B}, \mathcal{A}) \rightarrow \mathcal{A} = \mathcal{B}$$

5.2.2 The representation of spatial extensions

When Mr. Bertel sits on the couch, he stretches out his arm to reach the cup on the tea-table and drinking tea. The relative space that Mr. Bertel can reach with the help of his arms is larger than the space of his body. It is a spatial extension of his body. The boundary of the extended space is such that for any object connecting with it, Mr. Bertel can stretch out his arm and be connected with this object. So, the extended space is the sum of all possible locations of Mr. Bertel’s arms including the space of his body.

Formally, let \mathcal{A} (category \mathbf{A}), \mathcal{X} (category \mathbf{X}) be two object regions. The spatial extension of \mathcal{A} by \mathcal{X} is called “near extension of \mathcal{A} by \mathcal{X} ” can be defined as the sum³ of all possible regions of category \mathbf{X} that are connected with the region \mathcal{A} and

³The definition of sum is a deviation from the sum definition in (Smith, 1996, p.289): *The sum of φ s can be defined as that entity y which is such that, given any entity w , w overlaps with y if and only if w overlaps with something that φ s. That is: $\sigma x(\varphi x) := \iota y(\forall w(wOy \equiv \exists v(\varphi v \wedge wOv)))$.*

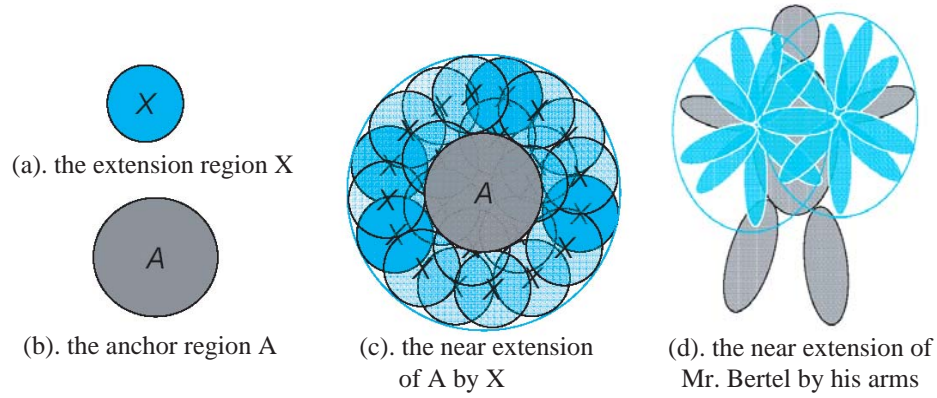


Figure 5.1: (a) The extension region \mathcal{X} ; (b) the anchor region \mathcal{A} ; (c) the *near* extension region of \mathcal{A} by \mathcal{X} ; (d) the near extension region of Mr. Bertel by his arms

is written as $\mathcal{A}^{\mathcal{X}}$. \mathcal{A} is called “the anchor region” and \mathcal{X} is called “the extension region”, shown in Figure 5.1.

Definition 5.2.2. *Given two object regions \mathcal{A} and \mathcal{X} (category \mathbf{X}), the near extension of \mathcal{A} by \mathcal{X} is defined as the \mathcal{Y} that all object region \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Y} , if and only if there is an object region \mathcal{V} satisfying that \mathcal{V} is of the same category as \mathcal{X} and connected with \mathcal{A} such that \mathcal{V} is connected with \mathcal{W} .*

$$\mathcal{A}^{\mathcal{X}} \stackrel{\text{def}}{=} \iota \mathcal{Y} (\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathcal{X} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V})))$$

The existence of $\mathcal{A}^{\mathcal{X}}$ is guaranteed by **Theorem 5.2.1**; the uniqueness of $\mathcal{A}^{\mathcal{X}}$ is guaranteed by **Axiom 5.2.4**. The near extension of \mathcal{A} by \mathcal{X} is called a *constructed region*. The category of $\mathcal{A}^{\mathcal{X}}$ is written as ‘ $\mathbf{A}^{\mathcal{X}}$ ’.

Theorem 5.2.1. *Given \mathcal{A} an object region, and \mathbf{X} be a category of an object region. There is \mathcal{X} satisfying that \mathcal{X} is a member of \mathbf{X} such that if \mathcal{X} is connected with \mathcal{A} , then there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{Y} is connected with \mathcal{W} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is a member of \mathbf{X} and connected with \mathcal{A} such that \mathcal{V} is connected with \mathcal{W} .*

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

For example, let \mathcal{BERTEL} be the object region of Mr. Bertel's body and \mathcal{ARM} (category ARM_{bio}) be the object region of his arm — the category ARM_{bio} has two features: (1) Elements are indistinguishable from one of Mr. Bertel's arms, (2) one end of each element is connected with one of Mr. Bertel's shoulder. Then the *near* extension of \mathcal{BERTEL} by \mathcal{ARM} ($\mathcal{BERTEL}^{\mathcal{ARM}}$) refers to the union of two object regions: (1) the object region of his body; and (2) the object region that his arm can reach, shown in Figure 5.1 (d).

Theorem 5.2.2. *All object regions \mathcal{A} and \mathcal{X} , it holds that \mathcal{A} is a part of the near extension of \mathcal{A} by \mathcal{X} .*

$$\forall \mathcal{A}, \mathcal{X} \bullet \mathbf{P}(\mathcal{A}, \mathcal{A}^{\mathcal{X}})$$

Theorem 5.2.3. *All object regions \mathcal{A} , \mathcal{B} and \mathcal{X} , it holds that the near extension of \mathcal{A} by \mathcal{X} is connected with \mathcal{B} , if and only if \mathcal{A} is connected with the near extension of \mathcal{B} by \mathcal{X} .*

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \equiv \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}})$$

A constructed region can have its near extension region. For example, Mr. Bertel walks five steps from the couch to the table. By his first step, the near extension of the couch by his body is made; by his second step, the first near extension region is further extended with the condition that the body spaces of his first step and his second step are connected;... The near extension of a constructed region by an object region is formalized as follows.

Definition 5.2.3. *Let \mathcal{C} be a constructed region, and \mathcal{X} be an object region (category \mathbf{X}), the near extension of \mathcal{C} by \mathcal{X} is defined as the \mathcal{Y} that all object region \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Y} , if and only if there is an object region \mathcal{V} satisfying that \mathcal{V} is of the same category as \mathcal{X} and connected with \mathcal{C} such that \mathcal{V} is connected with \mathcal{W} .*

$$\mathcal{C}^{\mathcal{X}} \stackrel{\text{def}}{=} \iota \mathcal{Y} (\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathcal{X} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{C}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V})))$$

The existence of $\mathcal{C}^{\mathcal{X}}$ is guaranteed by **Theorem 5.2.4**; the uniqueness of $\mathcal{A}^{\mathcal{X}}$ is guaranteed by **Axiom 5.2.4**. The near extension of \mathcal{C} by \mathcal{X} is a *constructed region*. The category of $\mathcal{C}^{\mathcal{X}}$ is written as ‘ $\mathcal{C}^{\mathcal{X}}$ ’.

Theorem 5.2.4. *Let \mathcal{C} be a constructed region, and \mathbf{X} be a category of an object region. There is \mathcal{X} satisfying that \mathcal{X} is a member of \mathbf{X} such that if \mathcal{X} is connected with \mathcal{C} , there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{Y} is connected with \mathcal{W} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is a member of \mathbf{X} and connected with \mathcal{C} such that \mathcal{V} is connected with \mathcal{W} .*

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{X}, \mathcal{C}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{C}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

When Mr. Bertel sits on the couch, he can stretch out his arm and reach the cup of tea on the tea-table, however, he cannot reach the books on the writing-desk, no matter how he moves his two arms around. The books are located outside of the near extension region of his body by his arms. This results in the distance comparison that the books on the writing-desk are further away to him than the cup of tea on the tea-table. Formally, given an object region \mathcal{A} , and object regions or side regions \mathcal{B} and \mathcal{C} , that \mathcal{A} is nearer to \mathcal{B} than to \mathcal{C} can be defined as there is an extension region \mathcal{X} such that the near extension of \mathcal{A} by \mathcal{X} is connected with \mathcal{B} and disconnected with \mathcal{C} .

Definition 5.2.4. *Let \mathcal{A} be an object region, and \mathcal{B} and \mathcal{C} be object regions or side regions, then that \mathcal{A} is nearer to \mathcal{B} than to \mathcal{C} is defined as there is an object region \mathcal{X} , such that the near extension of \mathcal{A} by \mathcal{X} is connected with \mathcal{B} and disconnected with \mathcal{C} .*

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{C} \bullet \text{nearer}(\mathcal{A}, \mathcal{B}, \mathcal{C}) \stackrel{\text{def}}{=} \exists \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{C})$$

5.2.3 Defining qualitative distances using *extension* regions

The distance between two disconnected objects can be defined by the amount of extension objects that are connected with both of them. For example, when Mr.

Bertel sits on the couch, it will take him seven steps to take the books on the writing-desk. The distance between Mr. Bertel sitting on the couch and the books on the writing-desk is “seven steps”.

Formally, the distance from object region \mathcal{A} to object region \mathcal{B} is represented by the minimal number of extension regions (all are of category \mathbf{X}) such that the near extension of \mathcal{A} by these regions is connected with \mathcal{B} . If the first *near* extension region is named as “1 \mathbf{X} ”, the second *near* extension region is named as “2 \mathbf{X} ”, the third *near* extension region is named as “3 \mathbf{X} ”, \dots , then a **naïve natural number system** for distance relation is created. Given two object regions \mathcal{A} and \mathcal{B} , “the distance from \mathcal{A} to \mathcal{B} is $n\mathbf{X}$ ” is interpreted as that \mathcal{B} is connected with the n^{th} *near* extension region \mathcal{X} of \mathcal{A} and disconnected with its $(n - 1)^{\text{th}}$ *near* extension region. The qualitative distance from \mathcal{A} to \mathcal{B} is, therefore, defined in the notion of the naïve natural number system as follows.

Definition 5.2.5. *Let \mathcal{A} , \mathcal{B} be two object regions and $\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n$ be n extension regions of category \mathbf{X} , the distance from \mathcal{A} to \mathcal{B} is defined as ‘ $n\mathbf{X}$ ’, if*

$$\neg \mathbf{C}(\left(\left(\left(\mathcal{A}^{\mathcal{X}_1}\right)^{\mathcal{X}_2}\right)^{\dots}\right)^{\mathcal{X}_{n-1}}, \mathcal{B}) \wedge \mathbf{C}(\left(\left(\left(\mathcal{A}^{\mathcal{X}_1}\right)^{\mathcal{X}_2}\right)^{\dots}\right)^{\mathcal{X}_n}, \mathcal{B})$$

5.2.4 Defining qualitative orientations using the *nearer* predicate

The orientation relation between two extended objects can be interpreted as the distance comparison between one extended object and the sides of the other object. For example, *that the balloon is in front of the writing-desk* can be interpreted as the balloon is nearer to the front side of the writing-desk than to its other sides. Formally, the orientation relation is formalized by the connectedness relation between a near extension of one object region and side regions of the other object region.

Definition 5.2.6. Let \mathcal{A} and \mathcal{B} be two object regions, and $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{front}$, $\mathcal{B}.\text{right}$, $\mathcal{B}.\text{back}$ be four side regions of \mathcal{B} . ‘ \mathcal{A} is in front of \mathcal{B} ’, written as $\mathbf{Front}(\mathcal{A}, \mathcal{B})$, is defined as \mathcal{A} is nearer to $\mathcal{B}.\text{front}$ than to $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{right}$, and $\mathcal{B}.\text{back}$.

$$\mathbf{Front}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\} \\ \bullet p \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, p)$$

Similarly, ‘ $\mathbf{Left}(\mathcal{A}, \mathcal{B})$ ’ stands for “ \mathcal{A} is left of \mathcal{B} ”; ‘ $\mathbf{Right}(\mathcal{A}, \mathcal{B})$ ’ for “ \mathcal{A} is right of \mathcal{B} ”; ‘ $\mathbf{Behind}(\mathcal{A}, \mathcal{B})$ ’ for “ \mathcal{A} is behind \mathcal{B} ”.

Definition 5.2.7. Let \mathcal{A} and \mathcal{B} be two object regions, and $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{front}$, $\mathcal{B}.\text{right}$, $\mathcal{B}.\text{back}$ be four side regions of \mathcal{B} . ‘ \mathcal{A} is left of \mathcal{B} ’, written as $\mathbf{Left}(\mathcal{A}, \mathcal{B})$, is defined as \mathcal{A} is nearer to $\mathcal{B}.\text{left}$ than to $\mathcal{B}.\text{front}$, $\mathcal{B}.\text{right}$, and $\mathcal{B}.\text{back}$.

$$\mathbf{Left}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\} \\ \bullet p \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, p)$$

Definition 5.2.8. Let \mathcal{A} and \mathcal{B} be two object regions, and $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{front}$, $\mathcal{B}.\text{right}$, $\mathcal{B}.\text{back}$ be four side regions of \mathcal{B} . ‘ \mathcal{A} is right of \mathcal{B} ’, written as $\mathbf{Right}(\mathcal{A}, \mathcal{B})$, is defined as \mathcal{A} is nearer to $\mathcal{B}.\text{right}$ than to $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{front}$, and $\mathcal{B}.\text{back}$.

$$\mathbf{Right}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\} \\ \bullet p \neq \mathcal{B}.\text{right} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{right}, p)$$

Definition 5.2.9. Let \mathcal{A} and \mathcal{B} be two object regions, and $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{front}$, $\mathcal{B}.\text{right}$, $\mathcal{B}.\text{back}$ be four side regions of \mathcal{B} . ‘ \mathcal{A} is behind \mathcal{B} ’, written as $\mathbf{Behind}(\mathcal{A}, \mathcal{B})$, is defined as \mathcal{A} is nearer to $\mathcal{B}.\text{back}$ than to $\mathcal{B}.\text{left}$, $\mathcal{B}.\text{right}$, and $\mathcal{B}.\text{front}$.

$$\mathbf{Behind}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\} \\ \bullet p \neq \mathcal{B}.\text{back} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{back}, p)$$

Theorem 5.2.5. Let \mathcal{A} and \mathcal{B} be two object regions, $\mathbf{Front}(\mathcal{A}, \mathcal{B})$, $\mathbf{Left}(\mathcal{A}, \mathcal{B})$, $\mathbf{Right}(\mathcal{A}, \mathcal{B})$ and $\mathbf{Behind}(\mathcal{A}, \mathcal{B})$ are pairwise disjoint.

$$\begin{aligned} \mathbf{Front}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Left}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \\ \mathbf{Left}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Behind}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \\ \mathbf{Behind}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Right}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \\ \mathbf{Right}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Front}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \\ \mathbf{Front}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Behind}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \\ \mathbf{Left}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Right}(\mathcal{A}, \mathcal{B}) &\equiv \text{false} \end{aligned}$$

5.3 *Fiat* containers: Formalizing relative spaces

Spatial relations between objects are interpreted as objects located in relative spaces in Chapter 4. For example, *that the cup is near Mr. Bertel* is interpreted as the cup is located in the relative space delineated by ‘near Mr. Bertel’. This relative space can be defined by the region such that any object connected with it can be reached by Mr. Bertel’s arm. Formally, this relative space *near Mr. Bertel* is called a **near fiat container**, *Mr. Bertel* is called the anchor of the *fiat* container, “be reached” can be formalized as “C”. The near *fiat* container is formalized as follows.

Definition 5.3.1. Let \mathcal{BERTEL} be the object region of Mr. Bertel’s body, \mathcal{ARM} (category $\mathbf{ARM}_{\text{bio}}$) be his arm, \mathcal{P} be a region, that \mathcal{P} is near \mathcal{BERTEL} , written as $\mathbf{NR}_{\mathcal{ARM}}(\mathcal{BERTEL}, \mathcal{P})$, is defined as that \mathcal{P} is disconnected with \mathcal{BERTEL} and connected with $\mathcal{BERTEL}^{\mathcal{ARM}}$.

$$\mathbf{NR}_{\mathcal{ARM}}(\mathcal{BERTEL}, \mathcal{P}) \stackrel{\text{def}}{=} \neg \mathbf{C}(\mathcal{BERTEL}, \mathcal{P}) \wedge \mathbf{C}(\mathcal{BERTEL}^{\mathcal{ARM}}, \mathcal{P})$$

Definition 5.3.2. Let \mathcal{BERTEL} be the object region of Mr. Bertel’s body, \mathcal{ARM} (category $\mathbf{ARM}_{\text{bio}}$) be his arm, then the **near fiat container**, $\mathfrak{S}_{\mathcal{BERTEL}}(\mathbf{NR}_{\mathcal{ARM}})$, is defined as the region \mathcal{Z} that all region \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Z} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is near \mathcal{BERTEL} such that \mathcal{V} is connected with \mathcal{W} .

$$\mathfrak{S}_{\mathcal{BERTEL}}(\mathbf{NR}_{\mathcal{ARM}}) \stackrel{\text{def}}{=} \iota \mathcal{Z} (\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Z}) \equiv \exists \mathcal{V} | \mathbf{NR}_{\mathcal{ARM}}(\mathcal{BERTEL}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V})))$$

The existence of $\mathfrak{S}_{\mathcal{BERTEL}}(\mathbf{NR}_{\mathcal{ARM}})$ is guaranteed by **Theorem 5.3.1**; the uniqueness of $\mathfrak{S}_{\mathcal{BERTEL}}(\mathbf{NR}_{\mathcal{ARM}})$ is guaranteed by **Axiom 5.2.4**.

Theorem 5.3.1. Given \mathcal{A} an object region. There is \mathcal{X} such that if \mathcal{X} is near \mathcal{A} , then there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Y} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is near \mathcal{A} such that \mathcal{W} is connected with \mathcal{V} .

$$\begin{aligned} \exists \mathcal{X} \bullet \mathbf{NR}_{\mathcal{ARM}}(\mathcal{A}, \mathcal{X}) \rightarrow \\ \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathbf{NR}_{\mathcal{ARM}}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V})) \end{aligned}$$

In general, a **fiat container** of an object region is defined as follows.

Definition 5.3.3. *Let \mathcal{A} be an object region, and \mathbf{rel} be a spatial property (related with \mathcal{A}), then the **rel fiat container**, written as $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel})$, is defined as the region \mathcal{Z} that all region \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Z} , if and only if there is \mathcal{V} satisfying the property \mathbf{rel} such that \mathcal{V} is connected⁴ with \mathcal{W} .*

$$\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}) \stackrel{\text{def}}{=} \iota_{\mathcal{Z}}(\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Z}) \equiv \exists \mathcal{V} | \mathbf{rel}(\mathcal{V}) \bullet \mathbf{C}(\mathcal{V}, \mathcal{W})))$$

The existence of $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel})$ is guaranteed by **Theorem 5.3.2**; the uniqueness of $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel})$ is guaranteed by **Axiom 5.2.4**. $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel})$ is a *constructed region*. Its category is written as ‘ $\mathbf{rel} * \mathcal{A}$ ’. That a region \mathcal{P} is connected with $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel})$ is read as “ \mathcal{P} **rel** \mathcal{A} ”.

Theorem 5.3.2. *Let \mathbf{rel} be a spatial property. There is \mathcal{X} such that if \mathcal{X} satisfies \mathbf{rel} , then there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Y} , if and only if there is \mathcal{V} satisfying \mathbf{rel} such that \mathcal{W} is connected with \mathcal{V} .*

$$\exists \mathcal{X} \bullet \mathbf{rel}(\mathcal{X}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathbf{rel}(\mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

Definition 5.3.4. *Let $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}})$ and $\mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$ be two fiat containers, \mathcal{V} be an object region such that $\mathbf{rel}_{\mathcal{A}}(\mathcal{V})$ and $\mathbf{rel}_{\mathcal{B}}(\mathcal{V})$, then the conjunction of $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}})$ and $\mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$, written as $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}}) \cap \mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$, is defined as the region \mathcal{Z} that all \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Z} , if and only if there is \mathcal{V} satisfying the property $\mathbf{rel}_{\mathcal{A}}$ and $\mathbf{rel}_{\mathcal{B}}$ such that \mathcal{V} is connected with \mathcal{W} .*

$$\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}}) \cap \mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}}) \stackrel{\text{def}}{=} \iota_{\mathcal{Z}}(\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Z}) \equiv \exists \mathcal{V} | \mathbf{rel}_{\mathcal{A}}(\mathcal{V}) \wedge \mathbf{rel}_{\mathcal{B}}(\mathcal{V}) \bullet \mathbf{C}(\mathcal{V}, \mathcal{W})))$$

The existence of $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}}) \cap \mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$ is guaranteed by **Theorem 5.3.3**; the uniqueness of $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}}) \cap \mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$ is guaranteed by **Axiom 5.2.4**. Its category is written as ‘ $\mathbf{rel}_{\mathcal{A}} * \mathcal{A} + \mathbf{rel}_{\mathcal{B}} * \mathcal{B}$ ’. That a region \mathcal{P} is connected with $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}_{\mathcal{A}}) \cap \mathfrak{S}_{\mathcal{B}}(\mathbf{rel}_{\mathcal{B}})$ is read as “ \mathcal{P} **rel** _{\mathcal{A}} \mathcal{A} and **rel** _{\mathcal{B}} \mathcal{B} ”.

⁴The connectedness relations between \mathcal{W} and \mathcal{Z} , and between \mathcal{V} and the \mathcal{W} here can be further generalized into a dyadic relation based on the connectedness relation, $f(\mathbf{C})$. A fiat container can, therefore, further generalized as: $\mathfrak{S}_{\mathcal{A}}(\mathbf{rel}) \stackrel{\text{def}}{=} \iota_{\mathcal{Z}}(\forall \mathcal{W} \bullet (f(\mathbf{C})(\mathcal{W}, \mathcal{Z}) \equiv \exists \mathcal{V} | \mathbf{rel}(\mathcal{V}) \bullet f(\mathbf{C})(\mathcal{V}, \mathcal{W})))$.

Theorem 5.3.3. *Let \mathbf{rel}_A and \mathbf{rel}_B be two spatial properties that are related with regions A and B , respectively. There is a region V such that if V satisfies $\mathbf{rel}_A(V)$ and $\mathbf{rel}_B(V)$, then there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Y} , if and only if there is V satisfying \mathbf{rel}_A and \mathbf{rel}_B such that \mathcal{W} is connected with V .*

$$\exists \mathcal{X} \bullet \mathbf{rel}_A(\mathcal{X}) \wedge \mathbf{rel}_B(\mathcal{X}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists V | \mathbf{rel}_A(V) \wedge \mathbf{rel}_B(V) \bullet \mathbf{C}(\mathcal{W}, V))$$

Let “.anchor” and “.relation” be two operators of a *fiat* container that return the anchor region and the relation of the *fiat* container, respectively. For example, $\mathfrak{S}_{COUCH}(\mathbf{NR}_{ARM}).\mathbf{anchor} = COUCH$ and $\mathfrak{S}_{COUCH}(\mathbf{NR}_{ARM}).\mathbf{relation} = \mathbf{NR}_{ARM}$; and let “.objects” be the operator of a *fiat* container that returns the set of object regions such that elements in the set are connected with this *fiat* container. These elements are called “located” in the *fiat* containers. For example, *that the tea-table is near the couch* is formally represented as ‘ $TEATABLE \in \mathfrak{S}_{COUCH}(\mathbf{NR}_{ARM}).\mathbf{objects}$ ’.

Connectedness *fiat* containers

*That an object region \mathcal{P} is located in a connectedness fiat container of an object region \mathcal{O} , $\mathfrak{S}_{\mathcal{O}}(\mathbf{C})$, represents that \mathcal{P} is connected with \mathcal{O} . This depicts a trivial distance relation between \mathcal{P} and \mathcal{O} – they are connected. The category of a connectedness *fiat* container $\mathfrak{S}_{\mathcal{O}}(\mathbf{C})$ is written as $\mathbf{C} * \mathcal{O}$, where \mathbf{C} represents the connectedness relation, and \mathcal{O} represents the category of the anchor object. For example, let $COUCH$ and $TEATABLE$ be object regions representing Mr. Bertel’s couch and his tea-table, $ROOM$ be the object region representing Mr. Bertel’ room, then *that the couch and the tea-table are in Mr. Bertel’s room* is formalized as*

$$\mathfrak{S}_{ROOM}(\mathbf{C}).\mathbf{objects} = \{COUCH, TEATABLE\}$$

The category of $\mathfrak{S}_{ROOM}(\mathbf{C})$ is $\mathbf{C} * ROOM$. The sides of the room are used as objects, rather than object sides. *That the couch and the tea-table stand on the floor* is

formalized as follows.

$$\mathfrak{S}_{\text{FLOOR}}(\mathbf{C}).\text{objects} = \{\text{COUCH}, \text{TEATABLE}\}$$

The category of $\mathfrak{S}_{\text{FLOOR}}(\mathbf{C})$ is $\mathbf{C} * \text{FLOOR}$.

Distance *flat* containers

For disconnected object regions, their spatial relations can be specified by distance relations. Formally, one object region near the other object region can be defined in the notion of *flat* containers as follows: For two disconnected object regions \mathcal{A} and \mathcal{B} , if \mathcal{B} is near \mathcal{A} , then \mathcal{B} is located in the *near flat* container of \mathcal{A} . Let \mathcal{X} be the extension region. This near relation $\mathbf{NR}_{\mathcal{X}}$ is defined as follows.

$$\mathbf{NR}_{\mathcal{X}}(\mathcal{A}, \mathcal{B}) \stackrel{\text{def}}{=} \neg \mathbf{C}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B})$$

That an object region \mathcal{P} is located in a near flat container of an object region \mathcal{O} , $\mathfrak{S}_{\mathcal{O}}(\mathbf{NR}_{\mathcal{X}})$, represents that \mathcal{P} is disconnected with \mathcal{O} and connected with $\mathcal{O}^{\mathcal{X}}$. This depicts a distance relation between \mathcal{P} and \mathcal{O} – \mathcal{P} is near \mathcal{O} . The category of the *near flat* container $\mathfrak{S}_{\mathcal{O}}(\mathbf{NR}_{\mathcal{X}})$ is written as $\mathbf{NR}_{\mathcal{X}} * \mathbf{0}$, where $\mathbf{NR}_{\mathcal{X}}$ represents the near relation, and $\mathbf{0}$ represents the category of the anchor object. For example, Mr. Bertel's mother observed that the balloon was near the writing-desk and the tea-table was near the couch, as she could reach the writing-desk while sitting on the balloon and she could reach the tea-table while sitting on the couch (The extension object was her body, BODY_m). Let WRITINGDESK , COUCH , BALLOON and TEATABLE be the object regions representing the writing-desk, the couch, the balloon, and the tea-table,

respectively, the distance relations are formalized as follows.

$$\begin{aligned}\mathfrak{S}_{\text{WRITINGDESK}}(\mathbf{NR}_{\text{BODY}_m}).\text{objects} &= \{\text{BALLOON}\} \\ \mathfrak{S}_{\text{COUCH}}(\mathbf{NR}_{\text{BODY}_m}).\text{objects} &= \{\text{TEATABLE}\}\end{aligned}$$

The category of $\mathfrak{S}_{\text{WRITINGDESK}}(\mathbf{NR}_{\text{BODY}_m})$ is $\mathbf{NR}_{\text{BODY}_m} * \text{WRITINGDESK}$; and the category of $\mathfrak{S}_{\text{COUCH}}(\mathbf{NR}_{\text{BODY}_m})$ is $\mathbf{NR}_{\text{BODY}_m} * \text{COUCH}$. *That the couch is in the corner* can be interpreted as the couch is near the two walls. Let *FRONTWALL* and *RIGHTWALL* be the front wall and the right wall. The location of the couch is formalized as follows.

$$\begin{aligned}\mathfrak{S}_{\text{FRONTWALL}}(\mathbf{NR}_{\text{BODY}_m}).\text{objects} &= \{\text{COUCH}\} \\ \mathfrak{S}_{\text{RIGHTWALL}}(\mathbf{NR}_{\text{BODY}_m}).\text{objects} &= \{\text{COUCH}\}\end{aligned}$$

Orientation *fiat* containers

Orientation relations are relations on distance comparison between an object region and sides of the other object region, for example, *that object region \mathcal{A} is in front of object region \mathcal{B}* means that \mathcal{A} is nearer to $\mathcal{B}.\text{front}$ than to other sides of \mathcal{B} ($\mathcal{B}.\text{left}$, $\mathcal{B}.\text{right}$, and $\mathcal{B}.\text{back}$). Formally, it is written as

$$\begin{aligned}\forall p|p \in \{\mathcal{B}.\text{front}, \mathcal{B}.\text{left}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\} \\ \bullet p \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, p)\end{aligned}$$

This is defined as ‘**Front**(\mathcal{A}, \mathcal{B})’ in 5.2.4. (page 91). The region \mathcal{Z} that is delineated by ‘in front of \mathcal{B} ’ is such that all object region \mathcal{W} , it holds that \mathcal{W} is connected with \mathcal{Z} , if and only if there is an object region \mathcal{V} satisfying that it is located ‘in front of \mathcal{B} ’ such that \mathcal{V} is connected with \mathcal{W} . The region \mathcal{Z} is formalized as follows.

$$\mathfrak{S}_{\mathcal{B}}(\mathbf{Front}) \stackrel{\text{def}}{=} {}_l\mathcal{Z}(\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Z}) \equiv \exists \mathcal{V} | \mathbf{Front}(\mathcal{V}, \mathcal{B}) \bullet \mathbf{C}(\mathcal{V}, \mathcal{W})))$$

The category of $\mathfrak{S}_{\mathcal{B}}(\mathbf{Front})$ is $\mathbf{Front} * \mathcal{B}$. In general, the category of an orientation *fiat* container $\mathfrak{S}_{\mathcal{O}}(\mathbf{Ori})$ is written as $\mathbf{Ori} * \mathcal{O}$, where \mathbf{Ori} represents an orientation relation, and \mathcal{O} represents the category of the anchor object. For example, *that the balloon is in front of the writing-desk* and *that the tea-table is in front of the couch* are formalized as follows.

$$\mathfrak{S}_{\mathit{WRITINGDESK}}(\mathbf{Front}).\mathit{objects} = \{\mathit{BALLOON}\}$$

$$\mathfrak{S}_{\mathit{COUCH}}(\mathbf{Front}).\mathit{objects} = \{\mathit{TEATABLE}\}$$

The category $\mathfrak{S}_{\mathit{WRITINGDESK}}(\mathbf{Front})$ is written as $\mathbf{Front} * \mathit{WRITINGDESK}$; the category $\mathfrak{S}_{\mathit{COUCH}}(\mathbf{Front})$ is written as $\mathbf{Front} * \mathit{COUCH}$.

5.4 The principle of selecting *fiat* containers

The principle of selecting cognitive reference objects can be stated in the notion of *fiat* container and called the principle of selecting *fiat* containers as follows: Let \mathcal{A} , \mathcal{B}_i ($\forall i \bullet 1 \leq i \leq n$) be object regions, \mathcal{A} is referenced to connectedness, distance, or orientation *fiat* containers of $\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_n$, then (1) $\mathcal{A}.\mathit{stability} < \mathcal{B}_i.\mathit{stability}$ ($\forall i \bullet 1 \leq i \leq n$) and (2) for any object region \mathcal{C} such that it is relatively more stable than region \mathcal{A} , it holds that $\mathit{nearer}(\mathcal{A}, \mathcal{B}_i, \mathcal{C})$ ($\forall i \bullet 1 \leq i \leq n$). The first statement corresponds to the criterion of stability; the second statement corresponds to the criterion of economics.

5.5 Formalization of cognitive spectrums: \mathfrak{C}

A cognitive spectrum of indoor spatial environment can be symbolically represented by a table of objects and a table of relative spaces. The table of objects can be formalized by property operators of object regions, such as *.category*, *.side*, *.stability*,

etc. Relative spaces are formalized by *fiat* containers. A cognitive spectrum \mathfrak{C} of an indoor spatial environment can be formalized by a pair

$$\mathfrak{C} = \langle \langle \mathfrak{S} \rangle, ROOM \rangle$$

$\langle \mathfrak{S} \rangle$ represents set of *fiat* containers and $ROOM$ is the object region of the room of this indoor environment. For example, a cognitive spectrum of the indoor spatial environment $\mathfrak{C}_{F4.7}$, shown in Figure 4.7, can be formalized as follows.

$$\mathfrak{C}_{F4.7} = \langle \langle \mathfrak{S} \rangle, ROOM \rangle$$

$$\langle \mathfrak{S} \rangle = \{ \mathfrak{S}_{ROOM}(\mathbf{C}), \mathfrak{S}_{FLOOR}(\mathbf{C}), \mathfrak{S}_{FRONTWALL}(\mathbf{NR}_{BODY_m})$$

$$\mathfrak{S}_{RIGHTWALL}(\mathbf{NR}_{BODY_m}), \mathfrak{S}_{COUCH}(\mathbf{NR}_{BODY_m}), \mathfrak{S}_{COUCH}(\mathbf{Front}) \}$$

$$\mathfrak{S}_{ROOM}(\mathbf{C}).\text{objects} = \{ COUCH, TEATABLE \}$$

$$\mathfrak{S}_{FLOOR}(\mathbf{C}).\text{objects} = \{ COUCH, TEATABLE \}$$

$$\mathfrak{S}_{FRONTWALL}(\mathbf{NR}_{BODY_m}).\text{objects} = \{ COUCH \}$$

$$\mathfrak{S}_{RIGHTWALL}(\mathbf{NR}_{BODY_m}).\text{objects} = \{ COUCH \}$$

$$\mathfrak{S}_{COUCH}(\mathbf{NR}_{BODY_m}).\text{objects} = \{ TEATABLE \}$$

$$\mathfrak{S}_{COUCH}(\mathbf{Front}).\text{objects} = \{ TEATABLE \}$$

5.6 A location and the location

Definition 5.6.1. *Let A be an object region, and \mathfrak{C} be a cognitive spectrum, \mathfrak{S} be a fiat container of \mathfrak{C} , then \mathfrak{S} is a location of A in \mathfrak{C} , if $A \in \mathfrak{S}.\text{objects}$. The location of A in \mathfrak{C} is the conjunction of all locations of A in \mathfrak{C} , written as ' $A.\text{Loc}|_{\mathfrak{C}}$ ' or ' $A.\text{Loc}$ ' for short, if \mathfrak{C} is clear.*

For example, Mr. Bertel's balloon is located on the floor, near the writing-desk and in front of the writing-desk. Let $BALLOON_B$, $WRITINGDESK_B$ and $FLOOR_B$

be object regions representing Mr. Bertel's balloon, his writing-desk, and the floor of his room, respectively, then *the location* of the balloon is formalized as follows.

$$\begin{aligned} \mathit{BALLOON}_{B.\text{Loc}} &= \mathit{WRITINGDESK}_B(\mathbf{Front}) \\ &\cap \mathit{WRITINGDESK}_B(\mathbf{NR}_{\text{BODY}_m}) \\ &\cap \mathit{FLOOR}_B(\mathbf{C}) \end{aligned}$$

Suppose that there are n object regions in a cognitive spectrum \mathfrak{C} , then there will be $C * n$ fiat containers in \mathfrak{C} , where C is a constant; and the computational complexity of .Loc would be no higher than $\mathcal{O}(n)$.

Definition 5.6.2. Let $\mathfrak{S}_1, \dots, \mathfrak{S}_n$ be n fiat containers, $\mathfrak{S} = \mathfrak{S}_1 \cap \dots \cap \mathfrak{S}_n$, written as $\bigcap_{i=1}^n \mathfrak{S}_i$, then $\mathfrak{S}.\text{Components} = \{\mathfrak{S}_1, \dots, \mathfrak{S}_n\}$

For example,

$$\begin{aligned} \mathit{BALLOON}_{B.\text{Loc}.\text{Components}} &= \{\mathit{WRITINGDESK}_B(\mathbf{Front}), \\ &\quad \mathit{WRITINGDESK}_B(\mathbf{NR}_{\text{BODY}_m}), \\ &\quad \mathit{FLOOR}_B(\mathbf{C})\} \end{aligned}$$

Each component of the conjunction is a *location* of the balloon.

5.7 Relations between two \mathfrak{C} s

5.7.1 The primitive relation

The primitive relation between two cognitive spectrums is the *categorically the same* relation between two object regions. Other relations are defined based on this primitive relation.

Definition 5.7.1. Let OBJ_1 and OBJ_2 be two object regions, then OBJ_1 and OBJ_2 are categorically the same, written as $OBJ_1 \stackrel{\text{cat}}{=} OBJ_2$, if and only if

$$OBJ_1.\text{category} = OBJ_2.\text{category}$$

$OBJ_1 \stackrel{\text{cat}}{\neq} OBJ_2$ denotes OBJ_1 and OBJ_2 are not categorically the same.

For example, Let $ROOM_B$, $BOOKSHELF_B$ be object regions representing Mr. Bertel's room and his bookshelf, and $ROOM_C$, $BOOKSHELF_C$ be object regions representing Mr. Certel's room and his bookshelf, then Mr. Bertel's bookshelf and Mr. Certel's bookshelf are *categorically the same*, $BOOKSHELF_B \stackrel{\text{cat}}{=} BOOKSHELF_C$. Mr. Certel's room $ROOM_C$ and Mr. Bertel's room $ROOM_B$ are not *categorically the same*, $ROOM_C \stackrel{\text{cat}}{\neq} ROOM_B$.

5.7.2 The relations between two sets of object regions

Definition 5.7.2. Let \mathcal{S} be a set of object regions and OBJ be an object region, if there is an object region $\mathcal{O} \in \mathcal{S}$ such that $\mathcal{O} \stackrel{\text{cat}}{=} OBJ$, then that \mathcal{S} categorically minuses OBJ , written as ' $\mathcal{S} \ominus OBJ$ ', is defined as $\mathcal{S} - \{\mathcal{O}\}$.

$$\mathcal{S} \ominus OBJ \stackrel{\text{def}}{=} \begin{cases} \mathcal{S} - \{\mathcal{O}\} & \text{if } \exists \mathcal{O} | \mathcal{O} \in \mathcal{S} \bullet \mathcal{O} \stackrel{\text{cat}}{=} OBJ \\ \mathcal{S} & \text{otherwise} \end{cases}$$

Suppose that \mathcal{S} has n object regions, then the computational complexity of $\mathcal{S} \ominus OBJ$ would be no higher than $\mathcal{O}(n)$.

For example, let $ROOM_B$, $BOOKSHELF_B$ and $WRITINGDESK_B$ be Mr. Bertel's room, his bookshelf, and his writing-desk, and $BOOKSHELF_C$ be Mr. Certel's bookshelf, $\mathcal{S} = \{ROOM_B, BOOKSHELF_B, WRITINGDESK_B\}$, then $\mathcal{S} \ominus BOOKSHELF_C = \{ROOM_B, WRITINGDESK_B\}$.

Definition 5.7.3. Let \mathcal{S}_1 and \mathcal{S}_2 be two sets of object regions, then \mathcal{S}_1 categorically minuses \mathcal{S}_2 , written as ' $\mathcal{S}_1 \ominus \mathcal{S}_2$ ', is defined recursively as follows.

$$\mathcal{S}_1 \ominus \mathcal{S}_2 \stackrel{\text{def}}{=} \begin{cases} \mathcal{S}_1 & \text{if } \mathcal{S}_2 = \emptyset \\ (\mathcal{S}_1 \ominus \mathcal{O}') \ominus (\mathcal{S}_2 - \{\mathcal{O}'\}) & \exists \mathcal{O}' \bullet \mathcal{O}' \in \mathcal{S}_2 \end{cases}$$

Suppose that each of \mathcal{S}_1 and \mathcal{S}_2 has at most n object regions, then the computational complexity of $\mathcal{S}_1 \ominus \mathcal{S}_2$ would be no higher than $\mathcal{O}(n^2)$.

Definition 5.7.4. Let \mathcal{S}_1 and \mathcal{S}_2 be two sets of object regions, then \mathcal{S}_1 and \mathcal{S}_2 are categorically the same, written as ' $\mathcal{S}_1 \stackrel{\text{cat}}{=} \mathcal{S}_2$ ' if and only if

$$\mathcal{S}_1 \ominus \mathcal{S}_2 = \mathcal{S}_2 \ominus \mathcal{S}_1 = \emptyset$$

For example, let BOOKSHELF_B and WRITINGDESK_B be Mr. Bertel's bookshelf and his writing-desk, BOOKSHELF_C and WRITINGDESK_C be Mr. Certel's bookshelf and his writing-desk, $\mathcal{S}_B = \{\text{BOOKSHELF}_B, \text{WRITINGDESK}_B\}$ and $\mathcal{S}_C = \{\text{BOOKSHELF}_C, \text{WRITINGDESK}_C\}$, then

$$\begin{aligned} \mathcal{S}_B \ominus \mathcal{S}_C &= \{\text{BOOKSHELF}_B, \text{WRITINGDESK}_B\} \\ &\quad \ominus \{\text{BOOKSHELF}_C, \text{WRITINGDESK}_C\} \\ &= (\{\text{BOOKSHELF}_B, \text{WRITINGDESK}_B\} \ominus \text{BOOKSHELF}_C) \\ &\quad \ominus (\{\text{BOOKSHELF}_C, \text{WRITINGDESK}_C\} - \{\text{BOOKSHELF}_C\}) \\ &= \{\text{WRITINGDESK}_B\} \ominus \{\text{WRITINGDESK}_C\} \\ &= \emptyset \end{aligned}$$

$$\begin{aligned} \mathcal{S}_C \ominus \mathcal{S}_B &= \{\text{BOOKSHELF}_C, \text{WRITINGDESK}_C\} \\ &\quad \ominus \{\text{BOOKSHELF}_B, \text{WRITINGDESK}_B\} \\ &= (\{\text{BOOKSHELF}_C, \text{WRITINGDESK}_C\} \ominus \text{BOOKSHELF}_B) \\ &\quad \ominus (\{\text{BOOKSHELF}_B, \text{WRITINGDESK}_B\} - \{\text{BOOKSHELF}_B\}) \\ &= \{\text{WRITINGDESK}_C\} \ominus \{\text{WRITINGDESK}_B\} \\ &= \emptyset \end{aligned}$$

Therefore, \mathcal{S}_B and \mathcal{S}_C are categorically the same: $\mathcal{S}_B \stackrel{\text{cat}}{=} \mathcal{S}_C$.

Note that the result of \ominus is unique with respect to the *categorically the same* relation: $\stackrel{\text{cat}}{=}$, rather than with respect to the classic equal relation. For example, let \mathcal{S} be a set of object regions, $\mathcal{S} = \{\text{BOOKSHELF}_1, \text{BOOKSHELF}_2\}$, such that BOOKSHELF_1 and BOOKSHELF_2 are of the same category, then the result of $\mathcal{S} \ominus \{\text{BOOKSHELF}_1\}$ can be $\mathcal{R}_1 = \{\text{BOOKSHELF}_1\}$ or $\mathcal{R}_2 = \{\text{BOOKSHELF}_2\}$. It holds that $\mathcal{R}_1 \stackrel{\text{cat}}{=} \mathcal{R}_2$, but not $\mathcal{R}_1 = \mathcal{R}_2$.

5.7.3 The relation between *fiat* containers

Definition 5.7.5. *Let \mathfrak{S}_1 and \mathfrak{S}_2 be two fiat containers, if they are of the same category, then they are categorically the same, written as ' $\mathfrak{S}_1 \stackrel{\text{cat}}{=} \mathfrak{S}_2$ '.*

For example, let WRITINGDESK_B and WRITINGDESK_C be Mr. Bertel's writing-desk and Mr. Certel's writing-desk, respectively, the category of $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{C})$ is $\mathbf{C} * \text{WRITINGDESK}$, the category of $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{C})$ is $\mathbf{C} * \text{WRITINGDESK}$, therefore, $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{C})$ and $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{C})$ are *categorically the same*. The category of $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{NR}_{\text{BODY}_m})$ is $\mathbf{NR}_{\text{BODY}_m} * \text{WRITINGDESK}$, the category of $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{NR}_{\text{BODY}_m})$ is $\mathbf{NR}_{\text{BODY}_m} * \text{WRITINGDESK}$, therefore, the distance *fiat* containers $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{NR}_{\text{BODY}_m})$ and $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{NR}_{\text{BODY}_m})$ are *categorically the same*. The category of $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{Front})$ is $\mathbf{Front} * \text{WRITINGDESK}$, the category of $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{Front})$ is $\mathbf{Front} * \text{WRITINGDESK}$, therefore, the orientation *fiat* containers $\mathfrak{S}_{\text{WRITINGDESK}_B}(\mathbf{Front})$ and $\mathfrak{S}_{\text{WRITINGDESK}_C}(\mathbf{Front})$ are *categorically the same*.

5.7.4 The relation between two sets of *fiat* containers

Definition 5.7.6. *Let $\langle \mathfrak{S} \rangle$ be a set of fiat containers and \mathfrak{S}' be a fiat container, if there is a fiat container $\mathfrak{S}_1 \in \langle \mathfrak{S} \rangle$ such that $\mathfrak{S}_1 \stackrel{\text{cat}}{=} \mathfrak{S}'$, then $\langle \mathfrak{S} \rangle$ categorically minuses*

\mathfrak{S}' , written as ' $\langle \mathfrak{S} \rangle \ominus \mathfrak{S}'$ ', is defined as $\langle \mathfrak{S} \rangle - \{\mathfrak{S}_1\}$

$$\langle \mathfrak{S} \rangle \ominus \mathfrak{S}' \stackrel{\text{def}}{=} \begin{cases} \langle \mathfrak{S} \rangle - \{\mathfrak{S}_1\} & \text{if } \exists \mathfrak{S}_1 | \mathfrak{S}_1 \in \langle \mathfrak{S} \rangle \bullet \mathfrak{S}_1 \stackrel{\text{cat}}{=} \mathfrak{S}' \\ \langle \mathfrak{S} \rangle & \text{otherwise} \end{cases}$$

Suppose that $\langle \mathfrak{S} \rangle$ has n fiat containers, then the computational complexity of $\langle \mathfrak{S} \rangle \ominus \mathfrak{S}$ would be no higher than $\mathcal{O}(n)$.

Definition 5.7.7. Let $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be two sets of fiat containers, then $\langle \mathfrak{S} \rangle_1$ categorically minuses $\langle \mathfrak{S} \rangle_2$, written as ' $\langle \mathfrak{S} \rangle_1 \ominus \langle \mathfrak{S} \rangle_2$ ', is defined recursively as follows

$$\langle \mathfrak{S} \rangle_1 \ominus \langle \mathfrak{S} \rangle_2 \stackrel{\text{def}}{=} \begin{cases} \langle \mathfrak{S} \rangle_1 & \text{if } \langle \mathfrak{S} \rangle_2 = \emptyset \\ (\langle \mathfrak{S} \rangle_1 \ominus \mathfrak{S}') \ominus (\langle \mathfrak{S} \rangle_2 - \{\mathfrak{S}'\}) & \exists \mathfrak{S}' \bullet \mathfrak{S}' \in \langle \mathfrak{S} \rangle_2 \end{cases}$$

Suppose that each of $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ has at most n fiat containers, then the computational complexity of $\langle \mathfrak{S} \rangle_1 \ominus \langle \mathfrak{S} \rangle_2$ would be no higher than $\mathcal{O}(n^2)$.

Definition 5.7.8. Let $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be two sets of fiat containers, then $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ are categorically the same, written as ' $\langle \mathfrak{S} \rangle_1 \stackrel{\text{cat}}{=} \langle \mathfrak{S} \rangle_2$ ' if and only if

$$\langle \mathfrak{S} \rangle_1 \ominus \langle \mathfrak{S} \rangle_2 = \langle \mathfrak{S} \rangle_2 \ominus \langle \mathfrak{S} \rangle_1 = \emptyset$$

Definition 5.7.9. Let \mathcal{A} , \mathcal{B} be two object regions, then their locations $\mathcal{A}.\text{Loc}$ and $\mathcal{B}.\text{Loc}$ are categorically the same, if $\mathcal{A}.\text{Loc}.\text{Components} \stackrel{\text{cat}}{=} \mathcal{B}.\text{Loc}.\text{Components}$, written as ' $\mathcal{A}.\text{Loc} \stackrel{\text{cat}}{=} \mathcal{B}.\text{Loc}$ '.

5.7.5 Mapping object regions and mapping fiat containers

Definition 5.7.10. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathfrak{C}_1 = \langle \langle \mathfrak{S} \rangle_1, \text{ROOM}_1 \rangle$, $\mathfrak{C}_2 = \langle \langle \mathfrak{S} \rangle_2, \text{ROOM}_2 \rangle$, ROOM_1 and ROOM_2 are mapped, ' $\text{ROOM}_1 \stackrel{\text{map}}{=} \text{ROOM}_2$ ', if and only if they are categorically the same.

$$\text{ROOM}_1 \stackrel{\text{map}}{=} \text{ROOM}_2 \stackrel{\text{def}}{=} \text{ROOM}_1 \stackrel{\text{cat}}{=} \text{ROOM}_2$$

Definition 5.7.11. Let $\langle \mathfrak{S} \rangle$ be a set of fiat containers, \mathcal{A} be an object region, then $\mathcal{A}.\text{fiatContainers}$ is the subset of $\langle \mathfrak{S} \rangle$ such that the anchor object of each fiat container in the subset is \mathcal{A} or one side of \mathcal{A} .

For example, let $\langle \mathfrak{S} \rangle$ be the set of fiat containers as follows.

$$\begin{aligned} \langle \mathfrak{S} \rangle = & \{ \mathfrak{S}_{\text{ROOM}_B}(\mathbf{C}), \mathfrak{S}_{\text{FLOOR}_B}(\mathbf{C}), \mathfrak{S}_{\text{FRONTWALL}_B}(\mathbf{NR}_{\text{BODY}_m}), \\ & \mathfrak{S}_{\text{RIGHTWALL}_B}(\mathbf{NR}_{\text{BODY}_m}), \mathfrak{S}_{\text{COUCH}_B}(\mathbf{Front}) \} \end{aligned}$$

Then, $ROOM_B.fiatContainers$ of $\langle \mathfrak{S} \rangle$ is the subset as follows.

$$\begin{aligned} ROOM_B.fiatContainers &= \{ \mathfrak{S}_{ROOM_B}(\mathbf{C}), \mathfrak{S}_{FLOOR_B}(\mathbf{C}), \\ &\quad \mathfrak{S}_{FRONTWALL_B}(\mathbf{NR}_{BODY_m}), \\ &\quad \mathfrak{S}_{RIGHTWALL_B}(\mathbf{NR}_{BODY_m}) \} \end{aligned}$$

Suppose that $\langle \mathfrak{S} \rangle$ has n elements, then the computational complexity of $.fiatContainers$ is no higher than $\mathcal{O}(n)$.

Definition 5.7.12. *Two fiat containers \mathfrak{S}_1 and \mathfrak{S}_2 are mapped, written as ' $\mathfrak{S}_1 \stackrel{\text{map}}{=} \mathfrak{S}_2$ ', if they are categorically the same and their anchor objects are mapped.*

$$\mathfrak{S}_1 \stackrel{\text{map}}{=} \mathfrak{S}_2 \stackrel{\text{def}}{=} (\mathfrak{S}_1 \stackrel{\text{cat}}{=} \mathfrak{S}_2) \wedge (\mathfrak{S}_1.\text{anchor} \stackrel{\text{map}}{=} \mathfrak{S}_2.\text{anchor})$$

Definition 5.7.13. *That two conjunctions of fiat containers $\bigcap_{i=1}^n \mathfrak{S}_{1i}$ and $\bigcap_{j=1}^m \mathfrak{S}_{2j}$ are mapped, written as ' $\bigcap_{i=1}^n \mathfrak{S}_{1i} \stackrel{\text{map}}{=} \bigcap_{j=1}^m \mathfrak{S}_{2j}$ ', is recursively defined as follows.*

$$\bigcap_{i=1}^n \mathfrak{S}_{1i} \stackrel{\text{map}}{=} \bigcap_{j=1}^m \mathfrak{S}_{2j} \stackrel{\text{def}}{=} \begin{cases} \mathfrak{S}_{11} \stackrel{\text{map}}{=} \mathfrak{S}_{21} & n = m = 1 \\ (\mathfrak{S}_{11} \stackrel{\text{map}}{=} \mathfrak{S}_{2k}) \wedge (\bigcap_{i=2}^n \mathfrak{S}_{1i} \stackrel{\text{map}}{=} \bigcap_{j=1, j \neq k}^m \mathfrak{S}_{2j}) & n = m > 1 \\ \text{false} & n \neq m \end{cases}$$

Suppose that each of the conjunctions of *fiat* containers $\bigcap_{i=1}^n \mathfrak{S}_{1i}$ and $\bigcap_{j=1}^m \mathfrak{S}_{2j}$ has at most n components, then the computational complexity of $\bigcap_{i=1}^n \mathfrak{S}_{1i} \stackrel{\text{map}}{=} \bigcap_{j=1}^m \mathfrak{S}_{2j}$ is no higher than $\mathcal{O}(n^2)$.

Definition 5.7.14. *Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathfrak{C}_1 = \langle \langle \mathfrak{S} \rangle_1, ROOM_1 \rangle$, $\mathfrak{C}_2 = \langle \langle \mathfrak{S} \rangle_2, ROOM_2 \rangle$, \mathcal{A} and \mathcal{B} be two object regions in $\mathfrak{S}_{ROOM_1}(\mathbf{C})$ and $\mathfrak{S}_{ROOM_2}(\mathbf{C})$ respectively. \mathcal{A} and \mathcal{B} are mapped, written as ' $\mathcal{A} \stackrel{\text{map}}{=} \mathcal{B}$ ', if and only if they are categorically the same and their locations in \mathfrak{C}_1 and \mathfrak{C}_2 are mapped.*

$$\mathcal{A} \stackrel{\text{map}}{=} \mathcal{B} \stackrel{\text{def}}{=} (\mathcal{A} \stackrel{\text{cat}}{=} \mathcal{B}) \wedge (\mathcal{A}.\text{Loc}|_{\mathfrak{C}_1} \stackrel{\text{map}}{=} \mathcal{B}.\text{Loc}|_{\mathfrak{C}_2})$$

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, then each of them has at most $C * n$ *fiat* containers, where C is a constant, the computational complexity of $\mathcal{A} \stackrel{\text{map}}{=} \mathcal{B}$ is no higher than the multiplication of the computational complexity of $.Loc$,

$\mathcal{O}(n)$, and the computational complexity of $\mathcal{A}.\text{Loc}|_{\mathfrak{c}_1} \stackrel{\text{map}}{=} \mathcal{B}.\text{Loc}|_{\mathfrak{c}_2}$, $\mathcal{O}(n^2)$. That is, the computational complexity of $\mathcal{A} \stackrel{\text{map}}{=} \mathcal{B}$ is no higher than $\mathcal{O}(n^3)$.

At his mother's first and second visits, Mr. Bertel's couch is located in the corner. Let ROOM_1 , COUCH_1 , FRONTWALL_1 , RIGHTWALL_1 and FLOOR_1 be object regions representing Mr. Bertel's room, the couch, the front wall, the right wall⁵, and the floor at his mother's first visit; ROOM_2 , COUCH_2 , FRONTWALL_2 , RIGHTWALL_2 and FLOOR_2 be object regions representing Mr. Bertel's room, the couch, the front wall, the right wall⁶, and the floor at his mother's second visit, $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be two sets of *fiat* containers defined as follows.

$$\begin{aligned} \langle \mathfrak{S} \rangle_1 &= \{ \mathfrak{S}_{\text{ROOM}_1}(\mathbf{C}), \mathfrak{S}_{\text{FRONTWALL}_1}(\mathbf{NR}_{\text{BODY}_m}), \\ &\quad \mathfrak{S}_{\text{RIGHTWALL}_1}(\mathbf{NR}_{\text{BODY}_m}), \mathfrak{S}_{\text{FLOOR}_1}(\mathbf{C}) \} \\ \langle \mathfrak{S} \rangle_2 &= \{ \mathfrak{S}_{\text{ROOM}_2}(\mathbf{C}), \mathfrak{S}_{\text{FRONTWALL}_2}(\mathbf{NR}_{\text{BODY}_m}), \\ &\quad \mathfrak{S}_{\text{RIGHTWALL}_2}(\mathbf{NR}_{\text{BODY}_m}), \mathfrak{S}_{\text{FLOOR}_2}(\mathbf{C}) \} \end{aligned}$$

The location of COUCH_1 and the location of COUCH_2 are

$$\begin{aligned} \text{COUCH}_1.\text{Loc} &= \mathfrak{S}_{\text{ROOM}_1}(\mathbf{C}) \cap \mathfrak{S}_{\text{FRONTWALL}_1}(\mathbf{NR}_{\text{BODY}_m}) \\ &\quad \cap \mathfrak{S}_{\text{RIGHTWALL}_1}(\mathbf{NR}_{\text{BODY}_m}) \cap \mathfrak{S}_{\text{FLOOR}_1}(\mathbf{C}) \\ \text{COUCH}_2.\text{Loc} &= \mathfrak{S}_{\text{ROOM}_2}(\mathbf{C}) \cap \mathfrak{S}_{\text{FRONTWALL}_2}(\mathbf{NR}_{\text{BODY}_m}) \\ &\quad \cap \mathfrak{S}_{\text{RIGHTWALL}_2}(\mathbf{NR}_{\text{BODY}_m}) \cap \mathfrak{S}_{\text{FLOOR}_2}(\mathbf{C}) \end{aligned}$$

⁵The front wall and the right wall are named by the same *fiat* projection as that in Figure 4.7, see note 7, on page 64.

⁶see the above note.

Then, COUCH_1 and COUCH_2 are mapped, $\mathit{COUCH}_1 \stackrel{\text{map}}{=} \mathit{COUCH}_2$, if

$$(1) \mathit{COUCH}_1 \stackrel{\text{cat}}{=} \mathit{COUCH}_2$$

$$(2) \mathit{COUCH}_1.\text{Loc} \stackrel{\text{map}}{=} \mathit{COUCH}_2.\text{Loc}$$

Definition 5.7.15. Let $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be two sets of fiat containers, the mapping between $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$, written as $\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)$, results in a queue⁷ of pairs such that two components of each pair are mapped and that they are members of $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$, respectively.

$$\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2) \stackrel{\text{def}}{=} [(\mathfrak{S}_{1i}, \mathfrak{S}_{2j}) | \mathfrak{S}_{1i} \in \langle \mathfrak{S} \rangle_1 \wedge \mathfrak{S}_{2j} \in \langle \mathfrak{S} \rangle_2 \wedge \mathfrak{S}_{1i} \stackrel{\text{map}}{=} \mathfrak{S}_{2j}]$$

Suppose that each of $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ has at most n fiat containers, then the computational complexity of $\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)$ is no higher than $\mathcal{O}(n^2)$.

For example, let COUCH_1 and COUCH_2 be two object regions representing Mr. Bertel's couch of his mother's first and second visits, and $\mathit{COUCH}_1 \stackrel{\text{map}}{=} \mathit{COUCH}_2$; let $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be sets of fiat containers as follows.

$$\langle \mathfrak{S} \rangle_1 = \{\mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{Front}), \mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{NR}_{\text{BODY}_m})\}$$

$$\langle \mathfrak{S} \rangle_2 = \{\mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{Front}), \mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{NR}_{\text{BODY}_m})\}$$

Then, $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ are mapped as follows.

$$\begin{aligned} \text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2) &= [(\mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{Front}), \mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{Front})), \\ &\quad (\mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{NR}_{\text{BODY}_m}), \mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{NR}_{\text{BODY}_m}))] \end{aligned}$$

$$\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)[0] = (\mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{Front}), \mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{Front}))$$

$$\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)[1] = (\mathfrak{S}_{\mathit{COUCH}_1}(\mathbf{NR}_{\text{BODY}_m}), \mathfrak{S}_{\mathit{COUCH}_2}(\mathbf{NR}_{\text{BODY}_m}))$$

⁷A queue \mathbb{A} is written as $[a_0, a_1, \dots, a_n]$, $\mathbb{A}[i] = a_i$; that a is a component of \mathbb{A} is written as $a \in \mathbb{A}$.

Definition 5.7.16. Let $\langle \mathfrak{S} \rangle_1$ and $\langle \mathfrak{S} \rangle_2$ be two sets of fiat containers, \mathbb{Q} be the result of $\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)$, then $\mathbb{Q}.\text{first}$ be the set of the first component of each pair in \mathbb{Q} , $\mathbb{Q}.\text{second}$ be the set of the second component of each pair in \mathbb{Q} .

$$\begin{aligned}\mathbb{Q}.\text{first} &= \{\mathfrak{S} | (\mathfrak{S}, \mathfrak{S}') \in \mathbb{Q}\} \\ \mathbb{Q}.\text{second} &= \{\mathfrak{S}' | (\mathfrak{S}, \mathfrak{S}') \in \mathbb{Q}\}\end{aligned}$$

For example,

$$\begin{aligned}\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2).\text{first} &= \{\mathfrak{S}_{\text{COUCH}_1}(\mathbf{Front}), \mathfrak{S}_{\text{COUCH}_1}(\mathbf{NR}_{\text{BODY}_m})\} \\ \text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2).\text{second} &= \{\mathfrak{S}_{\text{COUCH}_2}(\mathbf{Front}), \mathfrak{S}_{\text{COUCH}_2}(\mathbf{NR}_{\text{BODY}_m})\}\end{aligned}$$

Let $(\mathcal{A}, \mathcal{B})$ be a pair, then $(\mathcal{A}, \mathcal{B}).\text{first} = \mathcal{A}$, $(\mathcal{A}, \mathcal{B}).\text{second} = \mathcal{B}$. For example,

$$\begin{aligned}\text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)[0].\text{first} &= \mathfrak{S}_{\text{COUCH}_1}(\mathbf{Front}) \\ \text{mapping}_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)[0].\text{second} &= \mathfrak{S}_{\text{COUCH}_1}(\mathbf{NR}_{\text{BODY}_m})\end{aligned}$$

Definition 5.7.17. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathfrak{C}_1 = \langle \langle \mathfrak{S} \rangle_1, \text{ROOM}_1 \rangle$, $\mathfrak{C}_2 = \langle \langle \mathfrak{S} \rangle_2, \text{ROOM}_2 \rangle$, \mathfrak{S}_1 and \mathfrak{S}_2 be two mapped fiat containers in the two cognitive spectrums, respectively. Objects in \mathfrak{S}_1 and \mathfrak{S}_2 are mapped, if they are categorically the same and their locations in \mathfrak{C}_1 and \mathfrak{C}_2 are mapped. The object mapping process, written as $\text{mapping}_o(\mathfrak{S}_1, \mathfrak{S}_2)$, results in a queue of object region pairs such that two object regions in each pair are mapped.

$$\begin{aligned}\text{mapping}_o(\mathfrak{S}_1, \mathfrak{S}_2) &\stackrel{\text{def}}{=} [(\mathcal{O}_1, \mathcal{O}_2) | \mathcal{O}_1 \in \mathfrak{S}_1.\text{objects} \wedge \mathcal{O}_2 \in \mathfrak{S}_2.\text{objects} \\ &\quad \wedge \mathcal{O}_1 \stackrel{\text{map}}{=} \mathcal{O}_2]\end{aligned}$$

Two objects cannot be *mapped*, before their locations are *mapped*. Their locations which are conjunctions of *fiat* containers cannot be *mapped*, before the anchor objects of the *fiat* containers are mapped. Let \mathbb{Q} be the currently queue of the mapped *fiat* containers. The issue of whether two objects \mathcal{O}_1 and \mathcal{O}_2 are *mapped* can be addressed, if the components of their locations are subsets of $\mathbb{Q}.\text{first}$ and $\mathbb{Q}.\text{second}$,

respectively. That is,

$$\mathcal{O}_1.\text{Loc}|_{\mathfrak{C}_1}.\text{Components} \subseteq \mathbb{Q}.\text{first}$$

$$\mathcal{O}_2.\text{Loc}|_{\mathfrak{C}_2}.\text{Components} \subseteq \mathbb{Q}.\text{second}$$

The process of $\text{mapping}_o(\mathfrak{S}_1, \mathfrak{S}_2)$ is formalized as follows.

Input: Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums; let \mathfrak{S}_1 and \mathfrak{S}_2 be two *flat* containers, and \mathbb{Q} be the queue of the mapped *flat* containers.

Output: Let \mathbb{O} be the queue of mapped objects.

Initial state: $\mathbb{O} \leftarrow \emptyset$.

Process:

```

for each object region region1i in  $\mathfrak{S}_1$ .objects do
  for each object region region2j in  $\mathfrak{S}_2$ .objects do
    if region1icat = region2j and
      region1i.Loc| $\mathfrak{C}_1$   $\subseteq$   $\mathbb{Q}.$ first and
      region2j.Loc| $\mathfrak{C}_2$   $\subseteq$   $\mathbb{Q}.$ second and
      region1i.Loc| $\mathfrak{C}_1$   $\stackrel{\text{map}}{=}$  region2j.Loc| $\mathfrak{C}_2$ 
    then append (region1i, region2j) to  $\mathbb{O}$ 

```

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, then the computational complexity of $\text{mapping}_o(\mathfrak{S}_1, \mathfrak{S}_2)$ would be no higher than $\mathcal{O}(n) * \mathcal{O}(n) * (\mathcal{O}(n^2) + \mathcal{O}(n^2) + \mathcal{O}(n^3)) = \mathcal{O}(n^5)$.

Mapping cognitive spectrums

Input: Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums such that

$$\mathfrak{C}_1 = \langle \langle \mathfrak{S} \rangle_1, ROOM_1 \rangle$$

$$\mathfrak{C}_2 = \langle \langle \mathfrak{S} \rangle_2, ROOM_2 \rangle$$

Let \mathbb{O} be the queue of mapped objects, \mathbb{Q} be the queue of the mapped *fiat* containers, $Index_o$ be the index of the queue \mathbb{O} , and $Index_f$ be the index of the queue \mathbb{Q} .

Output: Let \mathbb{O} be the queue of mapped objects.

Initial state: $\mathbb{O} \leftarrow \emptyset$, $\mathbb{Q} \leftarrow \emptyset$, $Index_f \leftarrow 0$, $Index_o \leftarrow 0$

Process:

- (1) if $ROOM_1 \stackrel{cat}{=} ROOM_2$
 - then $\mathbb{O} \leftarrow [(ROOM_1, ROOM_2)]$
 - $\mathbb{Q} \leftarrow mapping_f(ROOM_1.fiatContainers, ROOM_2.fiatContainers)$
 - else go to (5)
- (2) while $Index_f$ points to a pair in \mathbb{Q} , do
 - (2.1) Append the result of $mapping_o \mathbb{Q}[Index_f]$ to \mathbb{O}
 - (2.2) $Index_f \leftarrow Index_f + 1$
- (3) while $Index_o$ points to a pair in \mathbb{O} , do
 - (3.1) $\langle \mathfrak{S} \rangle_1 \leftarrow \mathbb{O}[Index_o].first.fiatContainers$
 - (3.2) $\langle \mathfrak{S} \rangle_2 \leftarrow \mathbb{O}[Index_o].second.fiatContainers$
 - (3.3) Concatenate the result of $mapping_f(\langle \mathfrak{S} \rangle_1, \langle \mathfrak{S} \rangle_2)$ to \mathbb{Q}
 - (3.4) $Index_o \leftarrow Index_o + 1$
- (4) if $Index_f$ points to a pair in \mathbb{Q} , then go to (2)

(5) stop

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, the length of the \mathbb{Q} would be $C * n$, where C is a constant, the length of the \mathbb{O} would be n^2 , the computational complexity of step (1) would be no higher than $\mathcal{O}(n) * \mathcal{O}(n^2) = \mathcal{O}(n^3)$, the computational complexity of step (2) would be no higher than $\mathcal{O}(n^5) * C * n = \mathcal{O}(n^6)$, the computational complexity of step (3) would be no higher than $(\mathcal{O}(n) + \mathcal{O}(n) + \mathcal{O}(n^2)) * \mathcal{O}(n^2) = \mathcal{O}(n^4)$, the computational complexity of the whole process is $\mathcal{O}(n^3) + (\mathcal{O}(n^6) + \mathcal{O}(n^4)) * \mathcal{O}(n) = \mathcal{O}(n^7)$.

5.7.6 The judgement process

Recognizing the perceived environment is the judgement of whether the cognitive spectrum of the perceived environment is compatible with the cognitive spectrum of the target environment. The compatibility is determined by the spatial difference between the two cognitive spectrums and the commonsense knowledge of relative stabilities of related object regions.

The spatial differences

Definition 5.7.18. *Let $\mathbb{O} = [(\mathcal{O}_{10}, \mathcal{O}_{20}), \dots, (\mathcal{O}_{1n}, \mathcal{O}_{2n})]$ be a queue of object region pairs, then $\mathbb{O}.\text{first}$ be the set of the first component of each pair in \mathbb{O} , $\mathbb{O}.\text{second}$ be the set of the second component of each pair in \mathbb{O} .*

$$\begin{aligned}\mathbb{O}.\text{first} &= \{\mathcal{O}_{1i} | (\mathcal{O}_{1i}, \mathcal{O}_{2i}) \in \mathbb{O}\} \\ \mathbb{O}.\text{second} &= \{\mathcal{O}_{2j} | (\mathcal{O}_{1j}, \mathcal{O}_{2j}) \in \mathbb{O}\}\end{aligned}$$

Suppose that \mathbb{O} has n elements, then the computational complexity of .first and .second on \mathbb{O} are no higher than $\mathcal{O}(n)$.

For example, let

$$\mathbb{O} = [(\mathit{ROOM}_B, \mathit{ROOM}_C), (\mathit{BOOKSHELF}_B, \mathit{BOOKSHELF}_C), \\ (\mathit{BOOKS}_B, \mathit{BOOKS}_C)]$$

$$\mathbb{O}.\mathit{first} = \{\mathit{ROOM}_B, \mathit{BOOKSHELF}_B, \mathit{BOOKS}_B\}$$

$$\mathbb{O}.\mathit{second} = \{\mathit{ROOM}_C, \mathit{BOOKSHELF}_C, \mathit{BOOKS}_C\}$$

Definition 5.7.19. Let $\mathfrak{C} = \langle \mathfrak{S}, \mathit{ROOM} \rangle$ be a cognitive spectrum, $\mathfrak{C}.\mathit{basicObjects}$ be the union of the set $\{\mathit{ROOM}\}$ and the set of object regions that are located in $\mathfrak{S}_{\mathit{ROOM}}(\mathfrak{C})$.

$$\mathfrak{C}.\mathit{basicObjects} = \{\mathit{ROOM}\} \cup \mathfrak{S}_{\mathit{ROOM}}(\mathfrak{C}).\mathit{Objects}$$

The computational complexity of $\mathit{basicObjects}$ is a constant.

Definition 5.7.20. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathbb{O} be the queue of the mapped object regions. The spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 , written as $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$, is the set of object regions that are not mapped.

$$\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} (\mathfrak{C}_1.\mathit{basicObjects} - \mathbb{O}.\mathit{first}) \cup (\mathfrak{C}_2.\mathit{basicObjects} - \mathbb{O}.\mathit{second})$$

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, then the computational complexity of $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ would be no higher than $\mathcal{O}(n^7) * \mathcal{O}(n) = \mathcal{O}(n^8)$.

The degrees of the compatibility

Definition 5.7.21. Let \mathcal{S} be a set of object regions, then $\mathcal{S}.\mathit{rarelyMoved}$, $\mathcal{S}.\mathit{seldomMoved}$, $\mathcal{S}.\mathit{oftenMoved}$, and $\mathcal{S}.\mathit{alwaysMoved}$ are subsets of \mathcal{S} which collect rarely moved, seldom moved, often moved, and always moved object regions, respectively.

$$\mathcal{S}.\mathit{rarelyMoved} = \{\mathcal{O} \mid \mathcal{O} \in \mathcal{S} \wedge \mathcal{O}.\mathit{category} = \mathit{rarelyMoved}\}$$

$$\mathcal{S}.\mathit{seldomMoved} = \{\mathcal{O} \mid \mathcal{O} \in \mathcal{S} \wedge \mathcal{O}.\mathit{category} = \mathit{seldomMoved}\}$$

$$\mathcal{S}.\mathit{oftenMoved} = \{\mathcal{O} \mid \mathcal{O} \in \mathcal{S} \wedge \mathcal{O}.\mathit{category} = \mathit{oftenMoved}\}$$

$$\mathcal{S}.\mathit{alwaysMoved} = \{\mathcal{O} \mid \mathcal{O} \in \mathcal{S} \wedge \mathcal{O}.\mathit{category} = \mathit{alwaysMoved}\}$$

Suppose that \mathcal{S} has n elements, then the computational complexities of $\mathit{rarelyMoved}$, $\mathit{seldomMoved}$, $\mathit{oftenMoved}$, and $\mathit{alwaysMoved}$ are $\mathcal{O}(n)$.

Definition 5.7.22. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ be the spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 . If there are rarely moved objects in $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$, then \mathfrak{C}_1 and \mathfrak{C}_2 are hardly compatible, written as `hardlyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`.

$$\begin{aligned}\mathcal{H}_1 &= \mathfrak{C}_1.\text{basicObjects.rarelyMoved} \\ \mathcal{H}_2 &= \mathfrak{C}_2.\text{basicObjects.rarelyMoved} \\ \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) &\stackrel{\text{def}}{=} (\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2) \cap (\mathcal{H}_1 \cup \mathcal{H}_2)) \neq \emptyset\end{aligned}$$

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, then the computational complexity of `hardlyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)` would be no higher than the multiplication of the computational complexity of $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ and that of `.rarelyMoved`. That is, the computational complexity is no higher than $\mathcal{O}(n^8) * \mathcal{O}(n) = \mathcal{O}(n^9)$.

Definition 5.7.23. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ be the spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 . If \mathfrak{C}_1 and \mathfrak{C}_2 are not hardly compatible and there are seldom moved objects in $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$, then \mathfrak{C}_1 and \mathfrak{C}_2 are possibly compatible, written as `possiblyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`.

$$\begin{aligned}\mathcal{P}_1 &= \mathfrak{C}_1.\text{basicObjects.seldomMoved} \\ \mathcal{P}_2 &= \mathfrak{C}_2.\text{basicObjects.seldomMoved} \\ \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) &\stackrel{\text{def}}{=} \neg \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge (\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2) \cap (\mathcal{P}_1 \cup \mathcal{P}_2)) \neq \emptyset\end{aligned}$$

Definition 5.7.24. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ be the spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 . If \mathfrak{C}_1 and \mathfrak{C}_2 are neither hardly compatible nor possibly compatible and there are often moved objects in $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$, then \mathfrak{C}_1 and \mathfrak{C}_2 are compatible, written as `Compatible($\mathfrak{C}_1, \mathfrak{C}_2$)`.

$$\begin{aligned}\mathcal{F}_1 &= \mathfrak{C}_1.\text{basicObjects.oftenMoved} \\ \mathcal{F}_2 &= \mathfrak{C}_2.\text{basicObjects.oftenMoved} \\ \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) &\stackrel{\text{def}}{=} \neg \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge (\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2) \cap (\mathcal{F}_1 \cup \mathcal{F}_2)) \neq \emptyset\end{aligned}$$

Definition 5.7.25. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ be the spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 . If \mathfrak{C}_1 and \mathfrak{C}_2 are neither hardly compatible, nor possibly

compatible, nor compatible, and there are always moved objects in $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$, then \mathfrak{C}_1 and \mathfrak{C}_2 are very compatible, written as ‘`veryCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`’.

$$\begin{aligned} Q_1 &= \mathfrak{C}_1.\text{basicObjects.alwaysMoved} \\ Q_2 &= \mathfrak{C}_2.\text{basicObjects.alwaysMoved} \\ \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) &\stackrel{\text{def}}{=} \neg \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge (\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2) \cap (Q_1 \cup Q_2)) \neq \emptyset \end{aligned}$$

Definition 5.7.26. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, $\mathcal{D}(\mathfrak{C}_1, \mathfrak{C}_2)$ be the spatial difference between \mathfrak{C}_1 and \mathfrak{C}_2 . If \mathfrak{C}_1 and \mathfrak{C}_2 are neither hardly compatible, nor possibly compatible, nor compatible, nor very compatible, then \mathfrak{C}_1 and \mathfrak{C}_2 are indeed compatible, written as ‘`indeedCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`’.

$$\begin{aligned} \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) &\stackrel{\text{def}}{=} \neg \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \wedge \neg \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \end{aligned}$$

Suppose that each of \mathfrak{C}_1 and \mathfrak{C}_2 has at most n object regions, the computational complexities of `possiblyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, `Compatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, `veryCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, and `indeedCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)` are all no higher than $\mathcal{O}(n^9)$.

Theorem 5.7.1. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, their five relations be as follows `hardlyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, `possiblyCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, `Compatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, `veryCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, and `indeedCompatible($\mathfrak{C}_1, \mathfrak{C}_2$)`, then the five relations are jointly exhaustive and pairwise disjoint.

$$\begin{aligned} &\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \vee \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \vee \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) \\ &\quad \vee \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \vee \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{true} \\ &\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false} \\ &\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false} \\ &\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false} \\ &\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false} \\ &\text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false} \end{aligned}$$

$\text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false}$
 $\text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false}$
 $\text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false}$
 $\text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false}$
 $\text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) \wedge \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2) = \text{false}$

Making a judgement in everyday life

Definition 5.7.27. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathfrak{C}_1 is rarely recognized as \mathfrak{C}_2 , written as ‘ $\text{hardlyIs}(\mathfrak{C}_1, \mathfrak{C}_2)$ ’, if $\text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$.

$$\text{hardlyIs}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} \text{hardlyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$$

Definition 5.7.28. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathfrak{C}_1 might be recognized as \mathfrak{C}_2 , written as ‘ $\text{mightBe}(\mathfrak{C}_1, \mathfrak{C}_2)$ ’, if $\text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$.

$$\text{mightBe}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} \text{possiblyCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$$

Definition 5.7.29. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathfrak{C}_1 is recognized as \mathfrak{C}_2 , written as ‘ $\text{Is}(\mathfrak{C}_1, \mathfrak{C}_2)$ ’, if $\text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2)$.

$$\text{Is}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} \text{Compatible}(\mathfrak{C}_1, \mathfrak{C}_2)$$

Definition 5.7.30. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathfrak{C}_1 is exactly \mathfrak{C}_2 , written as ‘ $\text{exactlyIs}(\mathfrak{C}_1, \mathfrak{C}_2)$ ’, if $\text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$.

$$\text{exactlyIs}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} \text{veryCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$$

Definition 5.7.31. Let \mathfrak{C}_1 and \mathfrak{C}_2 be two cognitive spectrums, \mathfrak{C}_1 is indeed \mathfrak{C}_2 , written as ‘ $\text{indeedIs}(\mathfrak{C}_1, \mathfrak{C}_2)$ ’, if $\text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$.

$$\text{indeedIs}(\mathfrak{C}_1, \mathfrak{C}_2) \stackrel{\text{def}}{=} \text{indeedCompatible}(\mathfrak{C}_1, \mathfrak{C}_2)$$

5.8 The mereotopological formalism of The Theory of Cognitive Prism

This chapter formalized The Theory of Cognitive Prism in the notions of *regions* (including *object regions*, *side regions*, and *constructed regions*), and *fiat containers*.

The *fiat* container is constructed through the connectedness relation. *Locations* of object regions are defined in the notion of *fiat* containers based on the principle of selecting *fiat* containers. The cognitive spectrum of an indoor spatial environment is formalized by the pair of a set of *fiat* containers and an object region representing the room of the indoor environment. Relations between two cognitive spectrums are addressed based on the *categorically the same* relation between *object regions*. Two rooms in two cognitive spectrums are *mapped*, if they are *categorically the same*; objects inside of room are *mapped*, if they are *categorically the same* and their locations are *mapped*. The compatibility between two cognitive spectrums are determined by the un-mapped object regions and their relative stabilities. Recognizing variable spatial environments in the everyday-life situation is interpreted as the compatibility between the cognitive spectrum of the perceived environment and that of the target environment.

Chapter 6

A List representation of recognizing Indoor Vista spatial Environments: The LIVE model

You know my methods. Apply them!

— *Sherlock Holmes*

This chapter introduces a symbolic representation system, the LIVE model. Section 6.1 briefly introduces the general structure of the LIVE model; section 6.2 introduces the usage of the LIVE model; section 6.3-6.6 presents the list representation of indoor vista spatial environment that pertains to the formalism in Chapter 5; section 6.7 examines of the relationship between the principle of reference in spatial linguistic description and the principle of selecting cognitive reference objects; section 6.8 presents the representation of the recognition process; section 6.9 presents the simulation result of Mr. Bertel's apartment scenario of Chapter 1 in the LIVE model.

6.1 The general architecture of the LIVE model

The general architecture of the LIVE model is shown in Figure 6.1. It has five sub-models: The furniture system, the drawing system, the configuration files, the view system, and the comparison system.

The furniture system stores all the furniture information that the LIVE model represents. The configuration files store all the symbolic representations of configurations. The view system provides a graphical interface for the symbolic representation of configurations. The drawing system provides a graphical interface to create or modify configurations. The comparison system judges the compatibility of two selected configurations.

The furniture system is the basic system. It provides furniture class information for the view system, the drawing system, and the comparison system. The view system provides an easy way for the draw system to create new configurations by modifying existing one.

6.2 How to start the LIVE model?

The LIVE model has been implemented in LispWorks4.2 both on the Linux Susie 6.3 platform and on the Windows XP professional platform.

To start the LIVE model, the “`loader.lisp`” should be loaded firstly; then, type “`(in-package "LIVE-PACKAGE")`”, shown in Figure 6.2; thirdly, type “`(load-live)`” to load all the “.fsl” files of the LIVE model; at last, type “`(start-live-demo)`” and you will see the main menu of the LIVE model, shown in Figure 6.3.

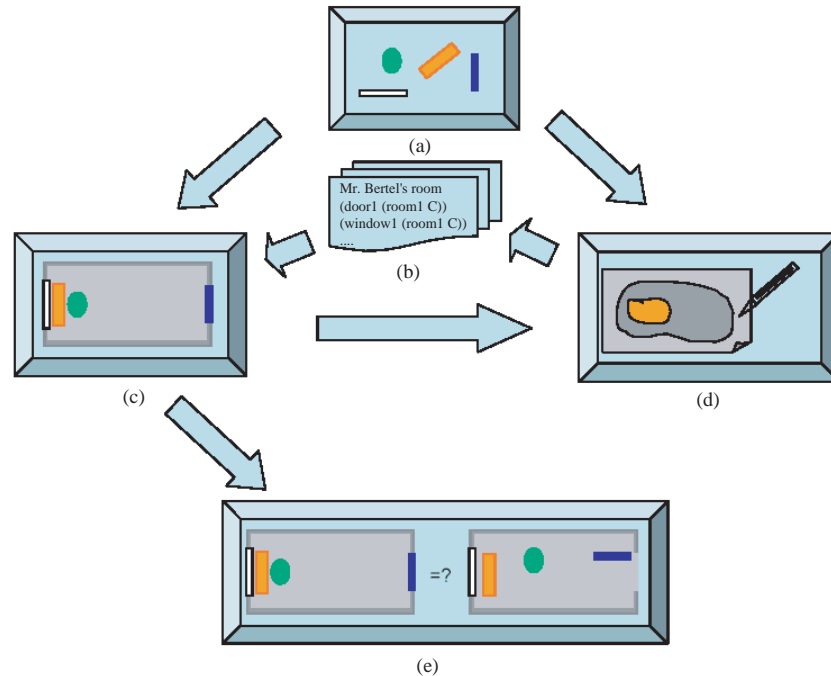


Figure 6.1: The general architecture of the LIVE model: (a) The furniture system, (b) configuration files, (c) the view system, (d) the drawing system, (e) the comparison system. Arrows represent information flow

6.3 The furniture system

If you click onto “furniture” on the main menu and then click on “view”, there will be a furniture view window prompted. This window displays all the furniture (e.g., doors, windows, couches, etc.) in the LIVE model both symbolically and diagrammatically, as shown in Figure 6.4.

In the LIVE model, a piece of furniture is represented by an instance of a class. The class has the category knowledge about the object, such as the name of this category, the default values of the degree of the relative stability, sides, etc.

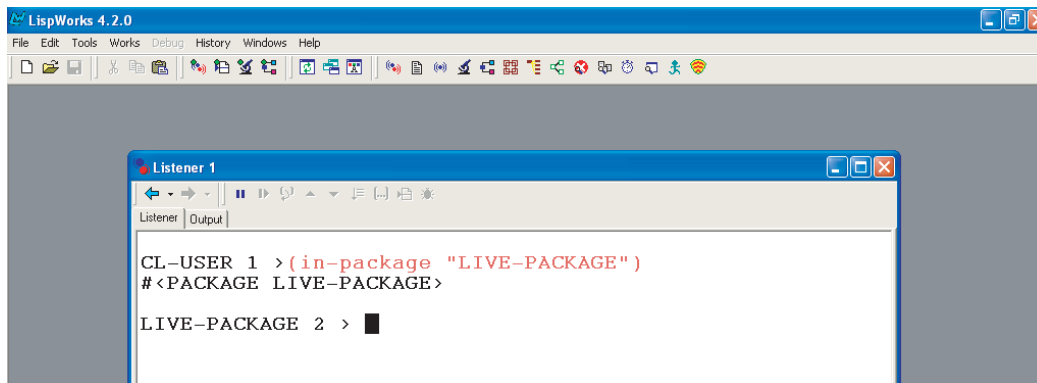


Figure 6.2: Before starting the LIVE model, you had better go into the “LIVE-PACKAGE”

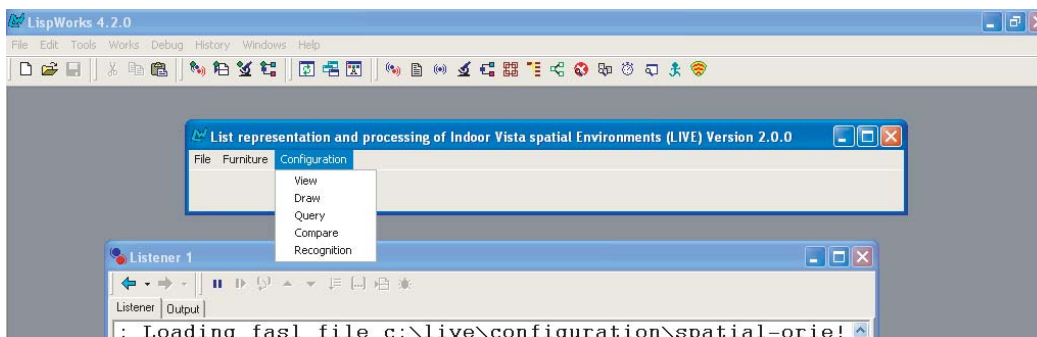


Figure 6.3: The main menu of the LIVE model

Class names

In the LIVE model, class names are constructed by adding `c_` before the name in natural language. For example, the object class name of the room is `c_room`. The LIVE model has 15 classes representing 15 preferable categories: ROOM (the `c_room` class), WINDOW (the `c_window` class), DOOR (the `c_door` class), WRITINGDESK (the `c_desk` class), BOOKSHELF (the `c_shelf` class), COUCH (the `c_couch` class), SMALLCOUCH (the `c_smallcouch` class), BALLOON (the `c_balloon` class), TEATABLE (the `c_table` class), CHAIR (the `c_chair` class), TABLE (the `c_table` class), BOOK (the `c_book` class), CUP (the `c_cup` class), PICTURE (the `c_picture` class), FLOWER (the `c_flower` class).

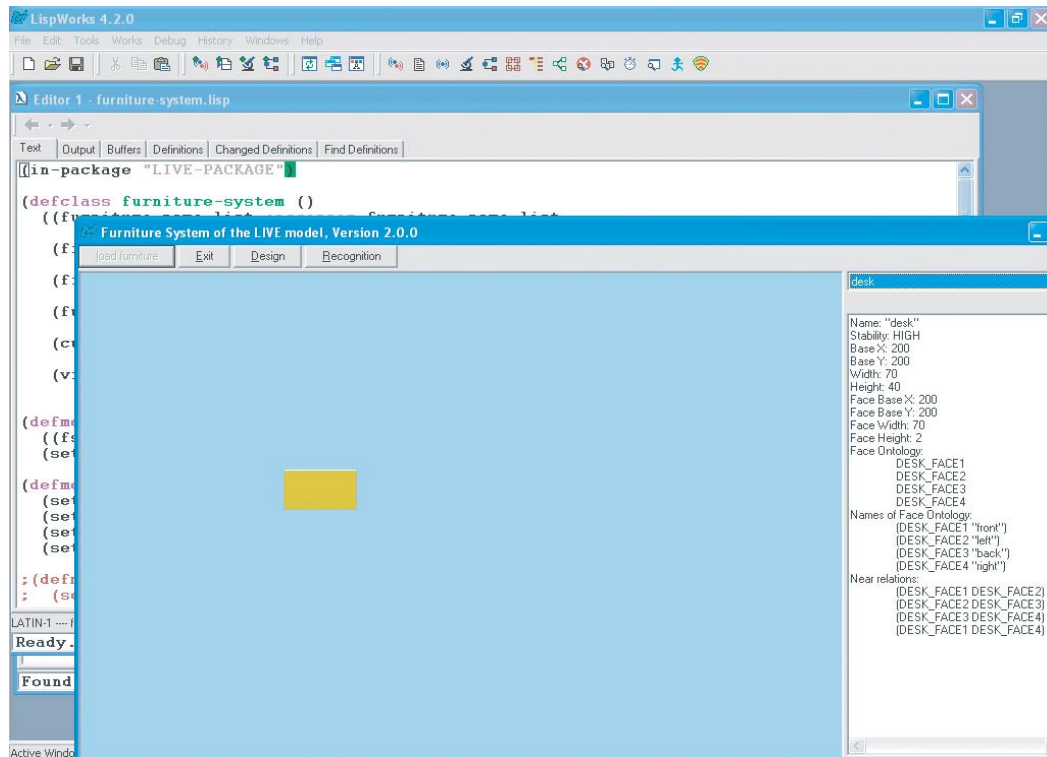


Figure 6.4: The view window of the LIVE furniture system

For the convenience of creating new environments, the `c_room` class in the LIVE model does not represent the `ROOM` category in Chapter 4. The `c_room` class does not include doors and windows, so that a new empty room environment can be easily created by putting doors and windows into difference sides of a `c_room` instance.

Object names

In the LIVE model, an object name has two parts: The first part is the name in English, the second part is a number which is used to distinguish different instances of the same class in one configuration. For example, names of windows can be `window1`, `window2`.

The relative stability

The LIVE model distinguishes four levels of stability: `highest`, `high`, `low`, and `lowest`. The `highest` stability represents the `rarelyMoved` stability; the `high` stability represents the `seldomMoved` stability; the `low` stability represents the `oftenMoved` stability; and the `lowest` stability represents the `alwayslyMoved` stability.

Face information

As the LIVE model does not represent observers, it does not represent the *fiat* projection mechanism. It distinguishes two kinds of objects: Objects that have an intrinsic reference framework and objects that do not have an intrinsic reference framework. Objects with an intrinsic reference framework have four faces: `<category name>_face1`, `<category name>_face2`, `<category name>_face3`, and `<category name>_face4`. Objects that do not have an intrinsic reference framework only have one face: `<category name>_face1`. For example, main slots of `c_window` are as follows:

```
(defclass c_window (c_furniture_rectangle)
  %this class is inherited from the c_furniture_rectangle class
  (name :accessor name
        :initform "window1") % the default name of a window
  (stability :accessor stability
             :initform 'highest) % a window is rarely moved
  (faces :accessor faces
         :initform '(window_face1 window_face2 window_face3 window_face4))
  % windows have intrinsic orientation reference frameworks
```

```

(face-names :accessor face-names
            :initform '((window_face1 "front")      % the linguistic
                        (window_face2 "left")      % descriptions of
                        (window_face3 "outside")    % each face of
                        (window_face4 "right")))    % a window

```

In the LIVE model, all objects are stored in a hash-table structure which is indexed by the name of the object.

6.4 Configurations in the LIVE model

In LIVE model a *configuration* refers to a set of list representation of the spatial linguistic description of an indoor spatial environment.

6.4.1 Spatial relations in the LIVE model

The Object-Object relation and the Object-Face relation in the LIVE model represent distance (including connectedness) relations and orientation relations, respectively.

The Object-Object relation

The connectedness or distance *fiat* container is symbolically represented by a two-element list. The first element is the name of the anchor object of the *fiat* container, the second element is a symbol representing the connectedness relation or a distance relation. Let C^1 be the symbol for the connectedness relation, NR be the symbol for

¹In the current version of the LIVE model, the connectedness relation C is specified into seven of the RCC-8 relations, namely, EC, PO, TPP, NTPP, EQ, TPP^{-1} , and $NTPP^{-1}$.

the distance relation of *near*, FR be the symbol for the distance relation of *far*, then (bookshelf1 NR) represents the *near fiat* container with bookshelf1 as the anchor object.

The distance location(including the connectedness relation) of the object is symbolically represented by an Object-Object-Relation list. It has two elements. The first element is the name of the object. The second element is the list whose element is the list representing a connectedness or distance *fiat* container. For example, (desk1 ((room1 C) (window1 NR) (shelf1 NR))) is the Object-Object-Relation list of the object desk1. It represents that desk1 is located in the *fiat* containers of (room1 C), (window1 NR), and (shelf1 NR).

The Object-Face relation

The orientation *fiat* container is symbolically represented by a three-element list. The first element is the name of the anchor object of the *fiat* container, the second element is the side of the anchor object such that any object located in the *fiat* container is nearer to this side, the third element NRR represents the nearer relation. For example, the front *fiat* container of the bookshelf bookshelf1 is symbolically represented by (bookshelf1 BOOKSHELF_FACE1 NRR).

An object's orientation location is represented by the Object-Face-Relation list. It has two elements. The first element is the name of an object. The second element is a list. Each element of this list is also a list that represents an orientation *fiat* container in which the object is located. For example, the Object-Face-Relation list (balloon1 ((bookshelf1 BOOKSHELF_FACE1 NRR))) represents balloon1's orientation. The second element represents that balloon1 is located nearer to the side

BOOKSHELF_FACE1 of bookshelf1 than to its other sides.

6.4.2 The drawing system

The drawing system provides graphical interfaces to create new configurations or modify already existing configurations. The drawing system has three sub-systems: The furniture draw panel sub-system, the furniture rotation sub-system, and the spatial relation sub-system. The furniture draw panel is an interface for adding or removing furniture; the furniture rotation panel is an interface for rotating furniture 90°, 180°, or 270°, the qualitative spatial panel is an interface for setting qualitative spatial relations between pieces of furniture, as shown in Figure 6.5.

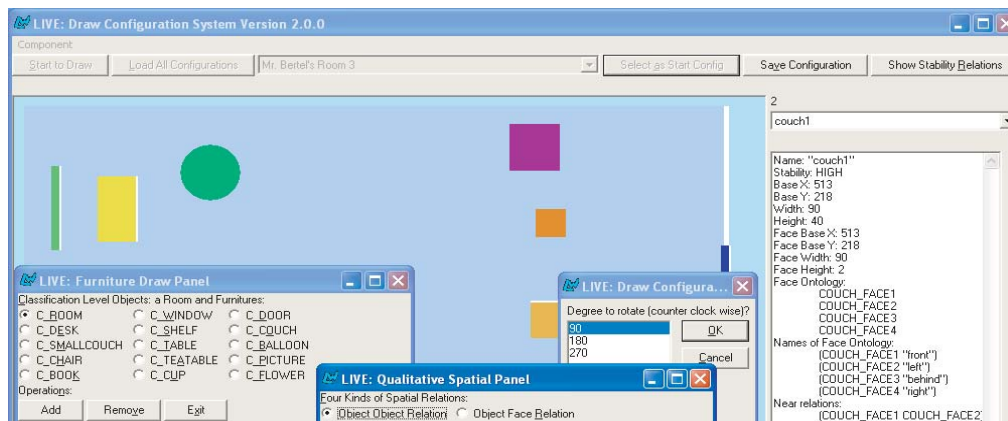


Figure 6.5: The drawing system creates a new configuration

6.5 The configuration file

Symbolic representations of configurations are stored in a file, which includes symbolic representations of furniture (shape, color, sides, etc.) and their spatial relations. For

example, the list representation of Mr. Bertel's apartment with no furniture is shown as follows.

```
(OBJECT-OBJECT-RELATION (|window1| ((|room1| C) (|door1| FR)))
                          (|door1| ((|room1| C) (|window1| FR))))
(OBJECT-FACE-RELATION (|window1| ((|room1| ROOM_FACE3 NRR)))
                      (|door1| ((|room1| ROOM_FACE1 NRR))))
```

6.6 The view system

The view system displays a configuration file diagrammatically. For example, Mr. Bertel's decorated apartment is shown in Figure 6.6.

6.7 Testing the principle of selecting *fiat* containers

The principle of selecting *fiat* containers is tested in the LIVE model by checking the diagram of the reference relation between the location object and the reference object. The method is as follows: If an object `obj1` is referenced to an object `obj2` and `obj2` is not reference to `obj1`, then there will be a line drawn between `obj1` and `obj2` and `obj2` is located a bit higher than `obj1`. The line represents the reference relation between two objects; the location in height represents the relative stability among objects — the higher an object is located, the more stable it is. If an object `obj1` is referenced to an object `obj2`, and `obj2` is also reference to `obj1`, then `obj2` is located at the same height as `obj1`, and there is no line between them.

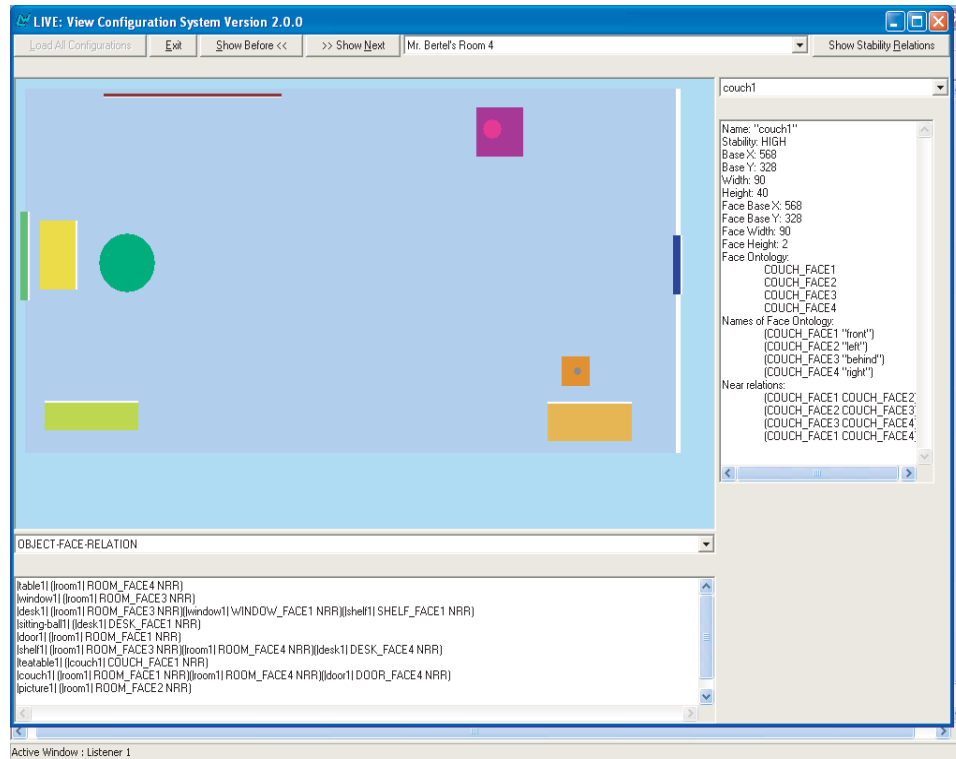


Figure 6.6: The diagrammatical representation of Mr. Bertel's apartment with full furniture and decoration

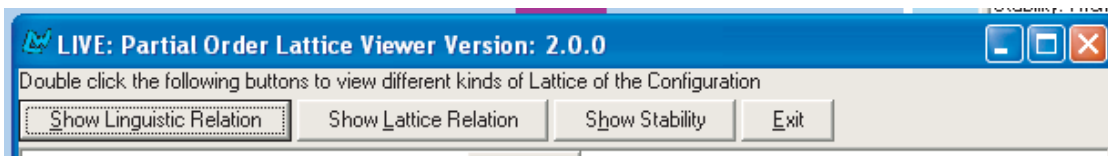


Figure 6.7: The window framework of three hierarchical structures

The diagram including all the reference relations of a configuration is a partial hierarchical structure, called “the linguistic reference hierarchy”. The view system displays the linguistic reference hierarchy diagrammatically. Click on “**Show Stability Relation**” button in the window of Figure 6.7, and a window to view three hierarchical structures relating to this configuration will be prompted, as shown in Figure 6.7. For example, click the button of “**Show Linguistic Relation**”, and the linguistic references of the configuration in Figure 6.7 will be shown in Figure 6.8.

A linguistic reference hierarchy may have triangles. For example, an object `obj1` is referenced to the second object `obj2`, a third object `obj3` is referenced to `obj2`, `obj1` is referenced to `obj3`, and `obj3` is located lower than `obj2` and higher than `obj1`, then there will be a triangle among `obj1`, `obj2` and `obj3`. If an object `obj1` is referenced to `obj2` and there is no third object `obj3` locating lower than `obj2` and higher than `obj1` (This guarantees that `obj3` is not referenced to `obj1` and that `obj2` is not referenced to `obj3`.) such that `obj1` is referenced to `obj3` and `obj3` is referenced to `obj2`, then the reference between `obj1` and `obj2` is called “a direct reference”, or else it is called “an indirect reference”. Removing all the indirect reference relations from a linguistic reference hierarchy results a diagram called “the hierarchical structure of reference objects”. A click on the button of “**Show Lattice Relation**” will prompt the hierarchical structure of direct reference relations of the configuration in Figure 6.6, shown in Figure 6.9.

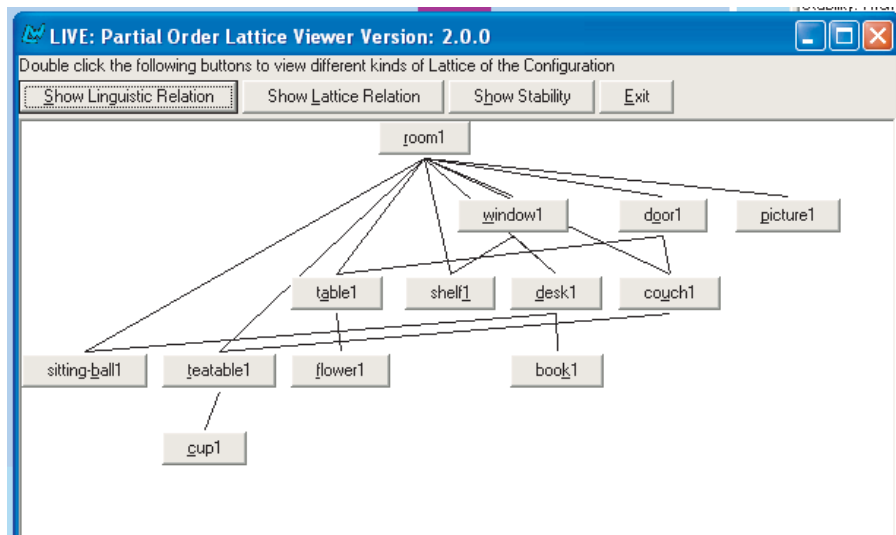


Figure 6.8: The diagrammatic representation of linguistic reference relations of objects in Mr. Bertel's apartment

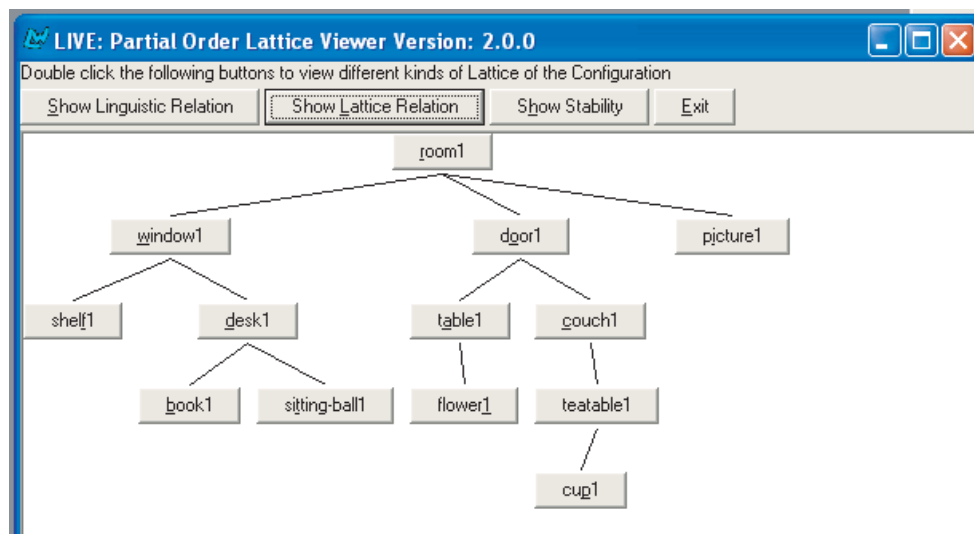


Figure 6.9: The diagrammatic representation of partial hierarchical structure of reference objects of Mr. Bertel's apartment

The result of the diagram in this example is consistent with the principle of selecting *fiat* containers as follows: The direct reference object is nearer to the location object than indirect reference objects; always moved objects (except `picture1`) are located lower in the diagram than the often moved objects; often moved objects are located lower than the seldom moved objects; seldom moved objects are located lower than the rarely moved objects. The result of the diagram in this example is inconsistent with the principle of selecting relative spaces as follows: Although `room1`, `window1` and `door1` are rarely moved objects, `room1` is located higher than `window1` and `door1`; although `picture1` is an always moved object², it is located at the same height as `window1` and `door1` which are rarely moved objects.

The reason for the inconsistency among `room1`, `window1`, and `door1` is that although `room1`, `window1` and `door1` are all rarely moved objects, the location change of `room1` is even much harder than that of `window1` and `door1`. That is, the pre-assumed degree of “rarely moved” granularity is a bit too coarse for `room1`, `window1` and `door1`. To meet the pre-assumed degree of “rarely moved” granularity, the partial hierarchical structure of reference objects is modified as follows: The rarely moved objects of a configuration are set at the same height. The original root is replaced by the name of the configuration. Click the button of “**Show stability**” to prompt the hierarchical structure of stability of the configuration in Figure 6.6 (shown in Figure 6.10). The modified structure is called “the partial order lattice of stabilities”. The inconsistency of `picture1` lies in the fact that it is hung on the wall, which promotes its relative stability. In the partial order lattice of stabilities, the picture hung on the wall is of the same relative stability as the shelf, the desk, the table, and the couch.

²As in the LIVE model, it is not assumed that pictures must be hung on the wall, therefore, the relative stability of a picture is set to always moved.

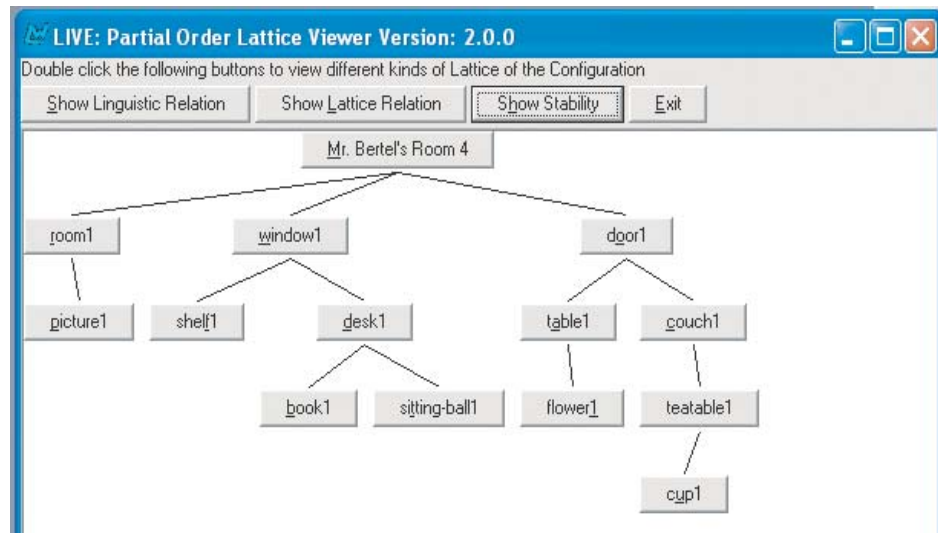


Figure 6.10: The diagrammatic representation of partial order lattice of relative stability of Mr. Bertel's apartment

6.8 The comparison system

The comparison system compares the two selected configurations. The first selected configuration represents the configuration in the mind; the second selected configuration represents the current perceived environment. The comparison system makes a judgment on to which degree the second configuration is recognized as the first one. Figure 6.11 shows the window frame of the comparison configuration system in the LIVE model. The functions of the five buttons are as follows: **Select the first configuration** and **Select the second configuration** buttons are used to select two configurations. After clicking each of them, the view configuration system will generate a configuration view interface for the user to select a configuration. Clicking the **Compare two configurations** button will start the recognition process. It first finds all the spatial differences between the two configurations. Then, it gives a judgment on whether it is the target one based on spatial differences and the relative

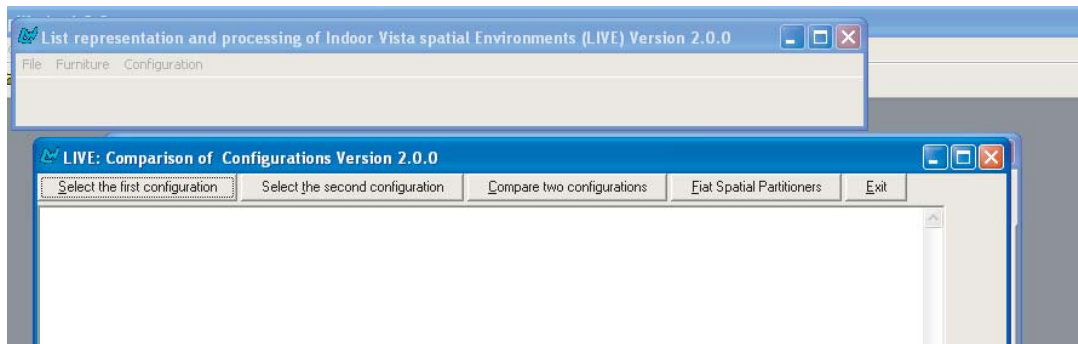


Figure 6.11: The comparison system in the LIVE model

stabilities of objects.

6.8.1 The main structure

The comparison system is the key system in the LIVE model. The three main classes of this sub-system are `compare-configuration-sys` class, `select-config-viewer`, and `compare-viewer` class.

```
(defclass compare-configuration-sys ()
  ((config-1 :accessor config-1
             :initform nil)
   (lattice-1 :accessor lattice-1
              :initform nil)
   (config-view-1 :accessor config-view-1
                  :initform nil)
   (config-2 :accessor config-2
             :initform nil)
   (lattice-2 :accessor lattice-2
              :initform nil))
```

```

(config-view-2 :accessor config-view-2
              :initform nil)

(fiat-spatial-partition :accessor fiat-spatial-partition
                       :initform nil)

(fiat-spatial-partition-viewer :accessor fiat-spatial-partition-viewer
                               :initform nil)

(result :accessor result
        :initform nil)

(log-str :accessor log-str
         :initform nil)))

```

The `compare-configuration-sys` class stores all the information that is needed for the comparison of two configurations. The `config-1` and the `config-2` slots store the first and the second configurations, respectively; the `lattice-1` and the `lattice-2` store the partial order lattice of stability in the first configuration and the second configuration respectively. They are constructed based on spatial relations of the two configurations and represent the hierarchy of the cognitive spectrum in Chapter 5. The `config-view-1` and the `config-view-2` slots are the graphical views of the first and the second configurations, respectively. The `fiat-spatial-partition` slot stores the mapped *fiat* containers hierarchically. This structure is created in the recognition process. The `fiat-spatial-partition-view` slot refers to the graphical view of the mapped *fiat* containers. The `result` stores the result of the comparison process. The `log-str` stores structured linguistic expressions about the comparison process.

6.8.2 The structure of mapped *fiat* containers

In the LIVE model, the mapped *fiat* containers are hierarchically constructed in the recognition process. An instance of `fiat-spatial-partition` class represents a mapped *fiat* container. The slots of `fiat-spatial-partition` class are as follows.

```
(defclass fiat-spatial-partition ()
  ((root-node-rem :accessor root-node-rem
                  :initform nil)
   (root-node-per :accessor root-node-per
                  :initform nil)
   (loc-partition :accessor loc-partition
                  :initform nil)
   (children :accessor children
             :initform nil)))
```

The `root-node-rem` slot and the `root-node-per` slot store objects in mind and objects perceived such that they are located in mapped *fiat* containers. If two rooms are categorically the same, then they are mapped and `root-node-rem` holds the object list (`room1`) of the remembered configuration, `root-node-per` holds the object list (`room1`) of the perceived configuration. The `loc-partition` slot is a hash-table of the distance (including connectedness) *fiat* container that uses distance relations as indexes. The mapped object and the distance relation represent a mapped distance *fiat* container which is an instance of the class `LocationObject`, whose slots of this class are shown as follows.

```
(defclass LocationObject ())
```



```

(all-rem :accessor all-rem
         :initform nil)

(all-per :accessor all-per
         :initform nil)

(orien-partition :accessor orien-partition
                 :initform nil)

;hashtable faces-->PostureObject ))

```

The `all-rem` and the `all-per` slots in `LocationObject` holds objects in the remembered configuration and objects in perceived configuration that are located in mapped distance (including connectedness) *fiat* containers, respectively. Their locations are specified by the orientation *fiat* containers. The `orien-partition` slot holds a hash table of mapped orientation *fiat* containers. This hash table uses side names of the mapped object in `root-node-rem` and `root-node-per` as indexes and creates an instance of the `PostureObject`³ class.

All object list pairs `all-rem` and `all-per` are checked to see whether there are mapped objects. Instances of `fiat-spatial-partition` class will be created for mapped objects in the earlier created instances of `fiat-spatial-partition` class. The newly created instance will be appended to the `children` slot of the mother instance. Figure 6.12 shows the instance of `fiat-spatial-partition` of recognizing Mr. Bertel's apartment. The connectedness relation `C` is specified into `PO`, `TPP`, `PP` representing *partially overlapped*, *tangential proper part*, *proper part* relations. `ORO.faceX` represents an orientation *fiat* container such that objects in it are nearer to `faceX` of

³The posture relation is explained in (Dong, 2005, p.354). Discussing the `PostureObject` class is beyond the scope here.

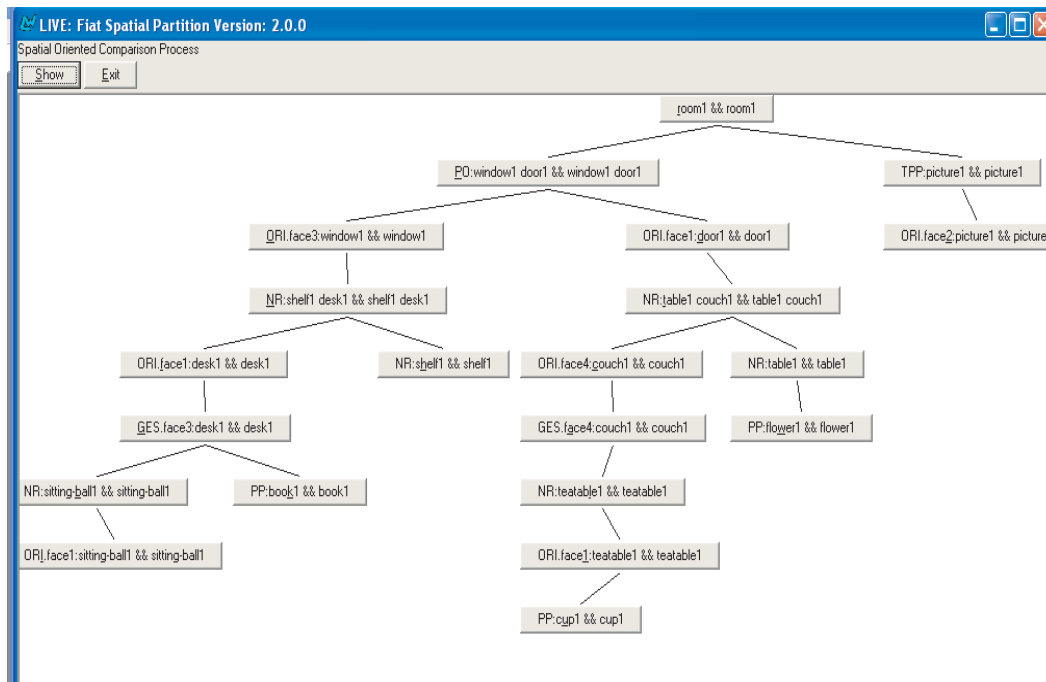


Figure 6.12: The mapped objects of Mr. Bertel’s apartment in mind and his apartment perceived are listed in the mapped *fiat* containers. Objects left and right of “&&” are *mapped*

the reference object than to its other sides. `GES.faceX` represents a posture relation⁴.

6.9 The simulation of recognizing Mr. Bertel’s apartment

This section presents the simulation results of the Mr. Bertel’s new home scenario in Chapter 1.

When Mr. Bertel’s mother came to see her son’s new home for the first time, she

⁴see note 3, on page 135.

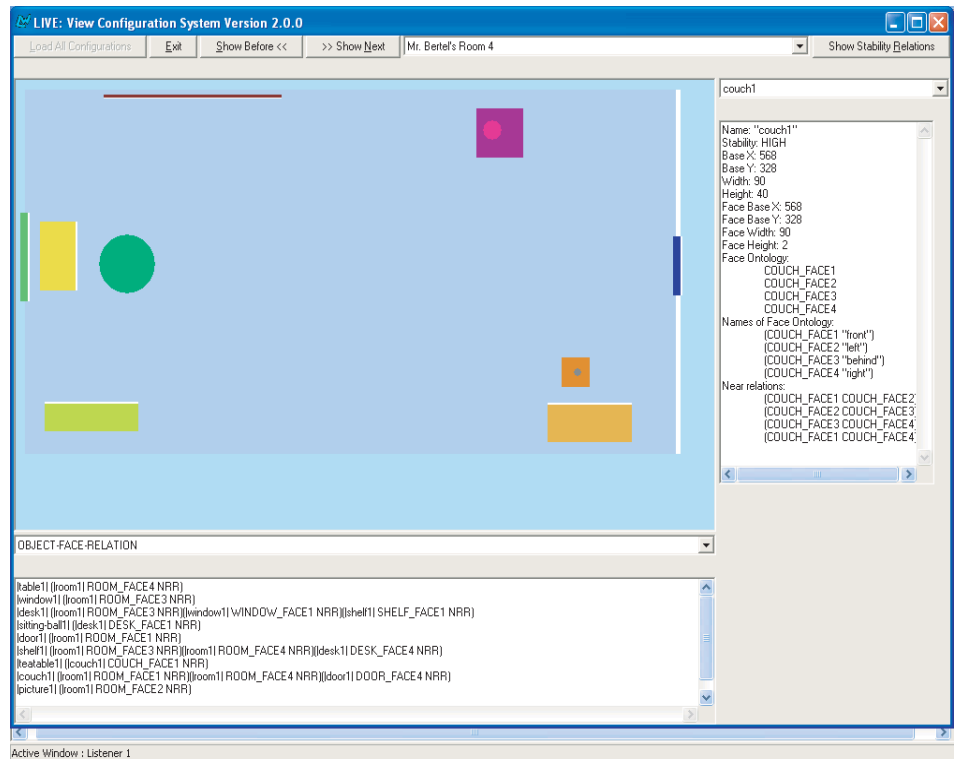


Figure 6.13: The view system shows the diagrammatic representation of Mr. Bertel's apartment with full furniture and decoration

saw the apartment with full furniture and decoration. In LIVE model this configuration is shown in Figure 6.13.

On the next day after Mr. Certel's visiting, Mr. Bertel's mother came to her son's apartment again. She noticed the difference of the location of the balloon, shown in Figure 6.14. However, she still recognized it her son's apartment, she just mumbled: Why did her son put the balloon to the tea-table? The comparison system of the LIVE model compares the above two configurations and reports "The two environments are the same!", shown in Figure 6.15.

On the eighth day the flowers are withered and thrown away. All books are put on the bookshelf. In the evening Mr. Bertel moves his writing-desk to the right side

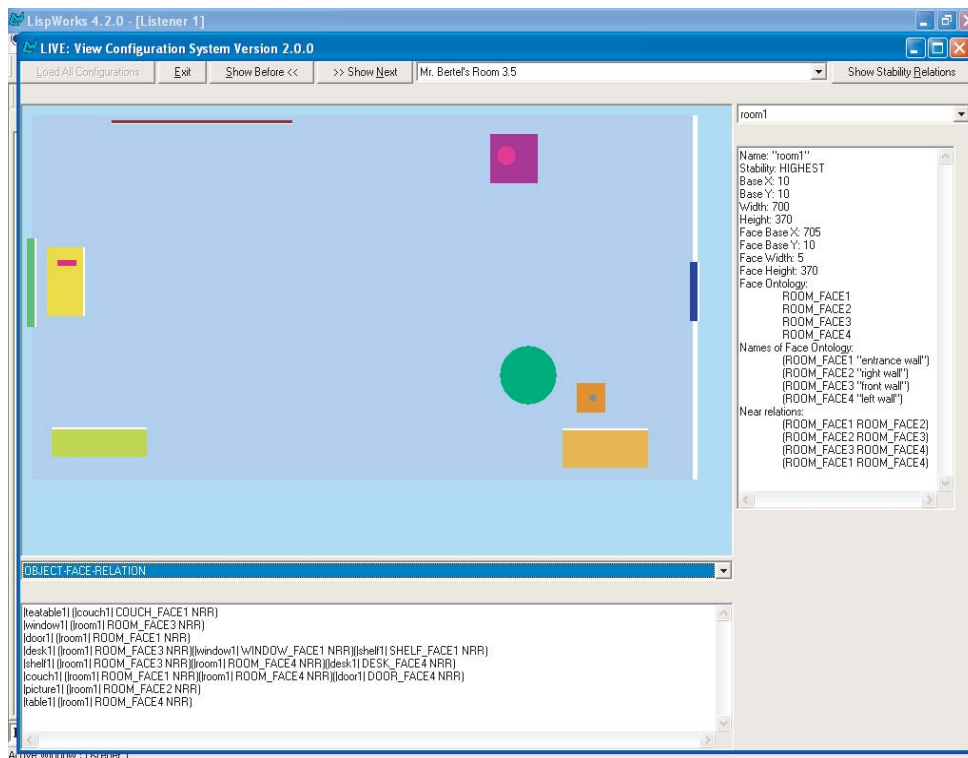


Figure 6.14: The view system shows the diagrammatic representation of Mr. Bertel's apartment after Mr. Certel's first visiting who moved the balloon to the tea-table

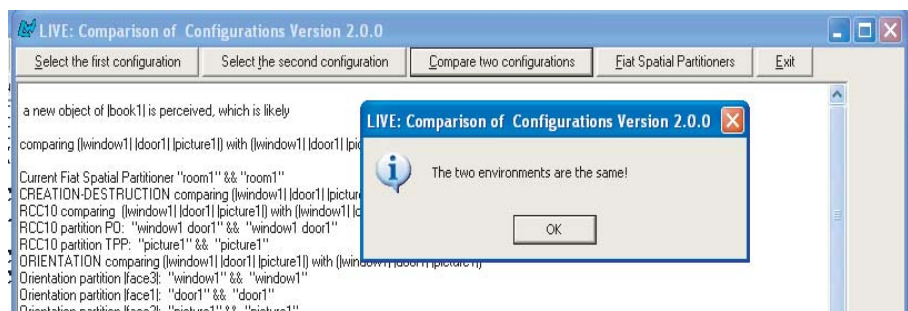


Figure 6.15: The comparison system compares two configurations, and only finds the spatial difference of the balloon. It believes that the perceived configuration is the target

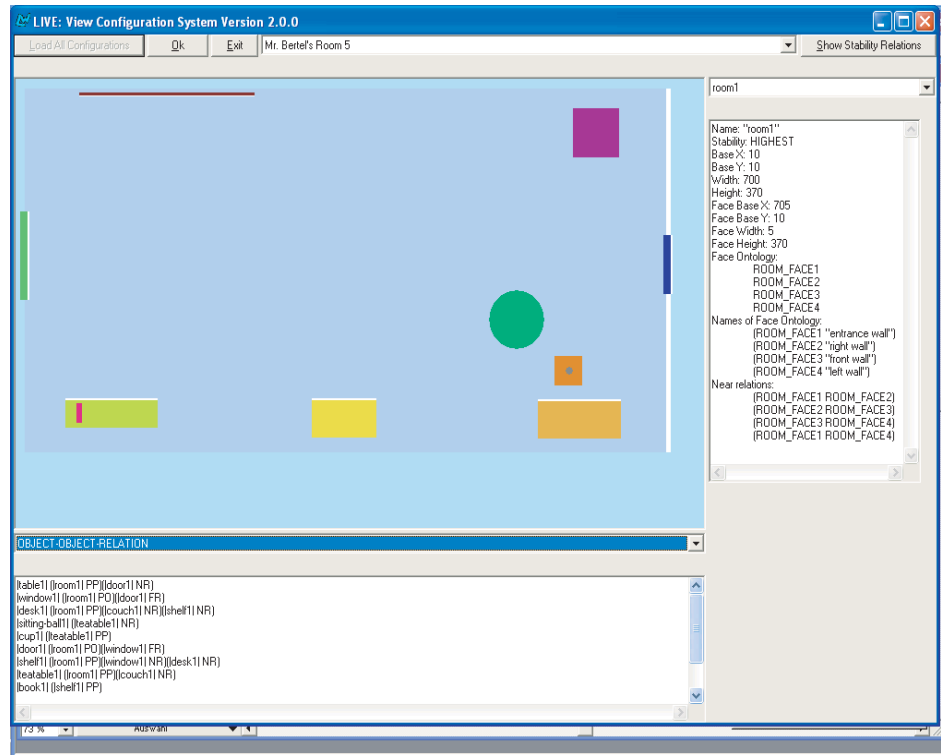


Figure 6.16: The view system shows the diagrammatic representation of Mr. Bertel's apartment with the after-party layout

of the bookshelf to make room for a party. The balloon is put near the tea-table, which is now at the right hand side of the door at the corner. In the LIVE model, Mr. Bertel's apartment after the party is shown in Figure 6.16. On the ninth day Mr. Bertel's mother came again. This time she went into Mr. Certel's apartment by mistake. In LIVE model the configuration of Mr. Certel's apartment is shown in Figure 6.17. Although Mr. Certel's apartment was very similar to Mr. Bertel's apartment, Mr. Bertel's mother noticed that she was not in her son's home and left the apartment. The comparison system of the LIVE model compares the two configurations and reports "The two environments are **HARDLY** the same!", shown in Figure 6.18. When she went to Mr. Bertel's apartment, She found that the

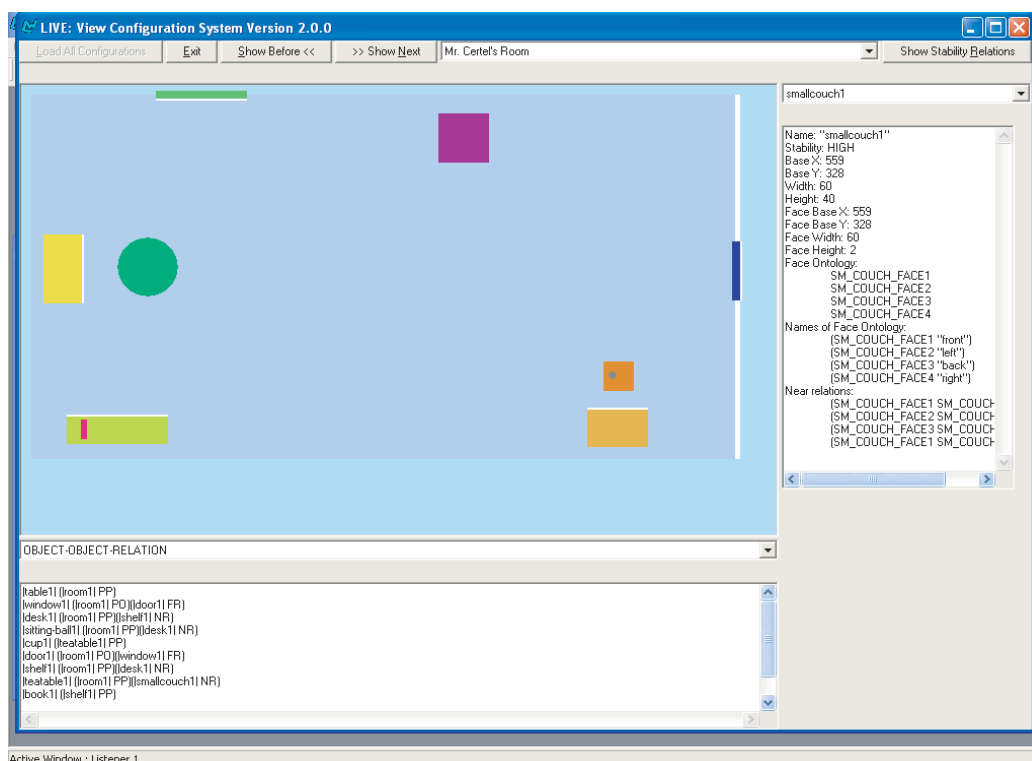


Figure 6.17: The view system shows the diagrammatic representation of Mr. Certel's apartment

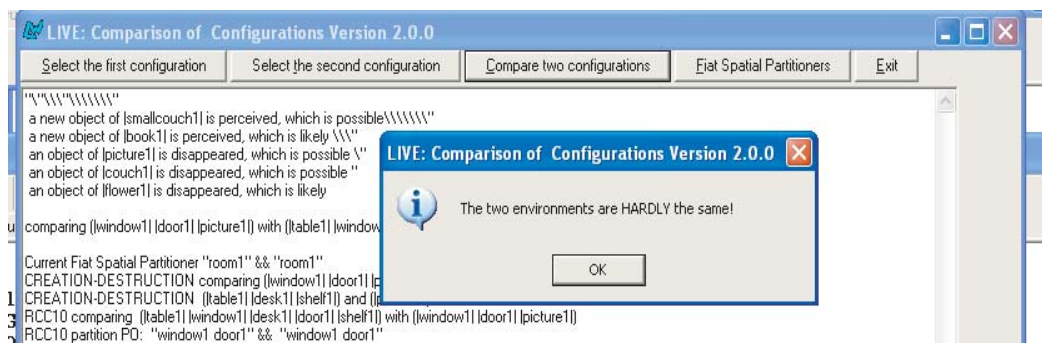


Figure 6.18: The comparison system of the LIVE model judges that Mr. Certel's apartment and Mr. Bertel's apartment are HARDLY the same

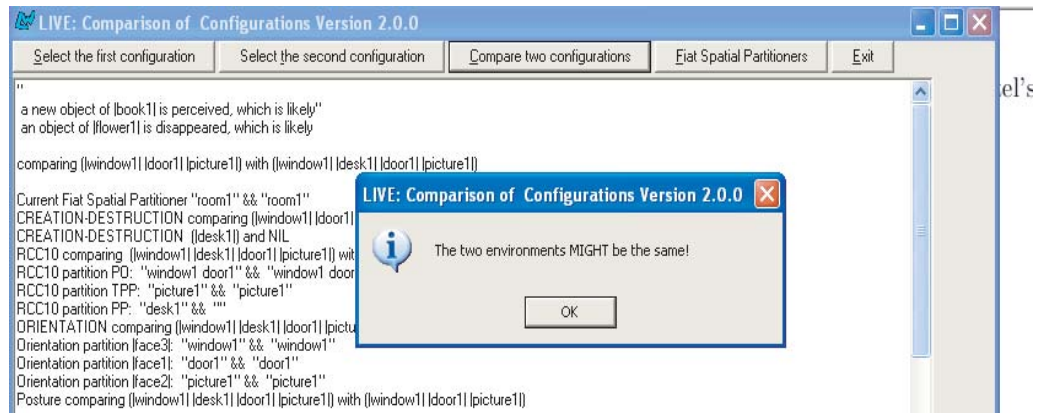


Figure 6.19: The comparison system of the LIVE model judges that Mr. Bertel's apartment with a after-party layout might be Mr. Bertel's apartment

writing-desk was located between the bookshelf and the couch and that the table and the balloon were located differently as she expected, she wondered for a while and accepted it as her son's apartment. The comparison system of the LIVE model results "The two environments MIGHT BE the same!", shown in Figure 6.19.

Chapter 7

Conclusions, evaluations, discussions, and future work

“道可道，非常道；名可名，非常名。”
老子·《道德经》

*"The way that can be told of is not an unvarying way;
The names that can be named are not unvarying names"*
Lao Tzu << Tao Te Ching >>

7.1 Conclusions

This work presented a computational theory of recognizing spatial environments — The Theory of Cognitive Prism. The theory assumed that people are able to categorize spatial objects through perception. It proposed the commonsense understanding of distance relations and orientation relations between extended objects and the commonsense knowledge of relative stabilities of extended objects. The knowledge of relative stability is used in the reference ordering between two extended objects following the principle of selecting cognitive reference objects. A snapshot view in the

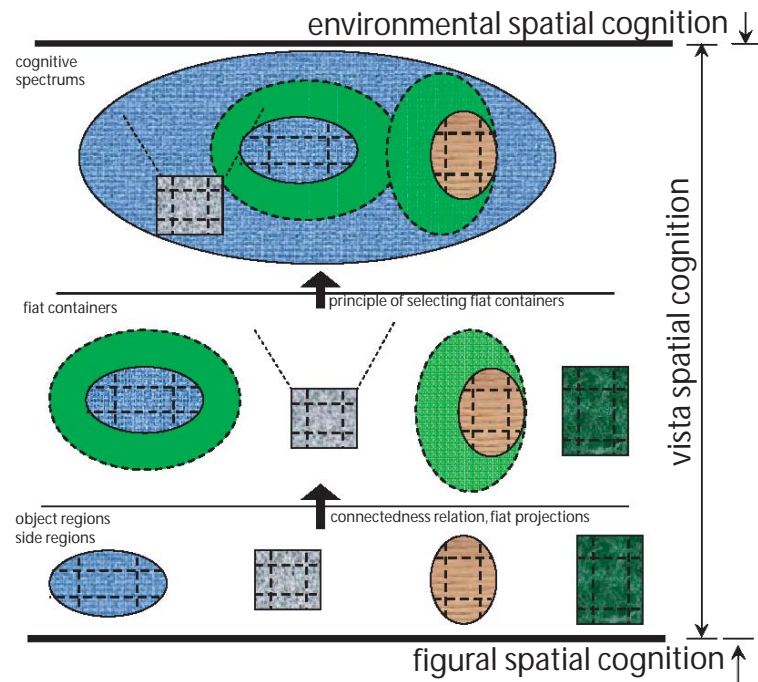


Figure 7.1: The framework of vista spatial cognition

mind, “cognitive spectrum”, is represented by nested relative spaces such that relative spaces of less stable objects are nested in relative spaces of more stable objects. The compatibility relationship between two cognitive spectrums is interpreted as the ease of the transformation from one cognitive spectrum to the other. The compatibility is determined by the relative stabilities of the un-mapped objects between two cognitive spectrums. The task of recognizing spatial environments in everyday life is interpreted as the judgement on the compatibility between the perceived environment and the target environment. The theory is mereotopologically formalized. The LIVE model symbolically implemented the main part of The Theory of Cognitive Prism. It tested the principle of selecting *fiat* containers, and symbolically simulated the Mr. Bertel’s apartment scenario. The Theory of Cognitive Prism outlines the research field of *vista spatial cognition* which bridges the research field of figural spatial cognition and

environmental spatial cognition, shown in Figure 7.1. Vista spatial cognition starts from the result of figural spatial cognition – knowledge of extended objects, researches relations (e.g., spatial relations, temporal relations, reference relations, etc.) among extended objects, and results in properties (e.g., existence, change, etc.) of vista spatial environments which are primitive for the environmental spatial cognition.

7.2 Evaluation on the representation and the reasoning

7.2.1 Obtainable by cognitive agents

The first criterion of a spatial representation that can be used by a cognitive agent is that the representation shall be obtainable by the sensors of the cognitive agent. It is assumed in the Theory of Cognitive Prism that a cognitive agent is able to recognize extended objects, make spatial extensions, and have the commonsense knowledge of relative stabilities of objects. With these three conditions, it is described in detail in Chapter 4 how the representation of a spatial environment, called “the cognitive spectrum”, is constructed.

7.2.2 Meaningful to languages

Cognitive agents need to make communications. The second criterion of a spatial representation is that the representation shall provide a systematical way to give meanings of the spatial linguistic descriptions, so that cognitive agents can exchange spatial information through language.

A piece of spatial linguistic description is a linear order of a location object, a reference object, and a spatial relation such that objects of less relative stability appear first. For example, *that the cup is on the table* is a linear order of *the cup*, *the table*, and the relation *on*; *the cup* is less stable than *the table*, therefore, in linguistic description *the cup* appears before *the table* as follows: *the cup is on the table*. In the Theory of Cognitive Prism an object which is located in a relative space has the meaning as follows: This object is relatively less stable than the anchor object of this relative space; this object is relative to the anchor object by this relative space.

In the Theory of Cognitive Prism three kinds of spatial relations (topological, distance, and orientation) are explained based on the connectedness relation. If there are only these three kinds of spatial relations for people (as claimed by Piaget and Inhelder (1948)), it would be a piece of future work for linguists to find out explanations for spatial relations in natural languages that are based on the connectedness relation.

7.2.3 Representing distortions

Cognitive agents acquire spatial knowledge by interacting with spatial environments using their sensors. Their knowledge of spatial environment inevitably has distortions. The third criterion of a spatial representation is that the representation shall be able to represent spatial distortions. In the Theory of Cognitive Prism objects are categorically recognized. That is, objects of the same category are equivalent and indistinguishable in isolation. Therefore, objects of the same category are believed to be of the same size (the mereotological formalism of object size is presented in Appendix A).

The distortion in spatial distance can result from subjectively putting objects of different sizes into the same category. For example, Sadalla and Magel (1980) reported that the qualitative distance of a route is positively related to the number of right-angle turns distributed along the route and that a route with more turns is estimated as longer than a route of equivalent length with fewer turns. In this case route segments between two consecutive right-angle turns are believed of the same category, and therefore route segments are believed of the same length. Consequently, the subjective distance of a route is positively related to the number of such route segments. A systematical research on the relationship between object categorizations and spatial distortions would be a future work for cognitive psychologists.

7.2.4 Computable

A computational theory shall be computable, so that it can be used by cognitive agents efficiently. It is roughly estimated in Chapter 5 that the computational complexity of the process of recognizing spatial environments in the Theory of Cognitive Prism is ‘P’ (polynomial to the input size).

7.3 Discussions and future work

7.3.1 The granularity of object categorizations

One of the starting points of The Theory of Cognitive Prism is that objects have been categorized. Given the scene of a spatial environment, the cognitive agent has recognized objects in the scene and categorized them at a certain granularity, such as a writing-desk, a table, a couch, etc. The granularity of object categorizations for

the recognition of spatial environments might be sometimes a bit different from the granularity of categorizations for the object recognition. For example, in the task of recognizing spatial environment, books in the bookshelf are grouped as one entity: “some books”. That is, for the configuration of the spatial environment, people will group all the books in the bookshelf, and treat them as a unit for the component of the configuration. On the other hand, people may use object parts as single objects. For example, people use sides of a room as single objects, e.g., floors, ceilings, walls, etc. It is a piece of future work on the granularity of objects that is suitable for the primitive components for cognitive tasks in vista spatial cognition.

7.3.2 Object categories recognized by different sensors

Blinded bats could fly, avoid obstacles, land on walls and ceiling, and survive in nature as well as bats with sight.

– *L. Spallanzani*

As visual creatures, humans have great difficulty imagining how animals that use sound, ultra-wave, smell, or magnetics orient in complex environments. At least nature tells us that vision is not the only possible way to “see” the world. Bats use sound as perceptive medium, rattle snakes use infrared ray as medium, pigeons use their magnetic sensors, ants use their noses, etc. It is not important what kinds of sensors are used, rather how to recognize the external world with the stimuli from the sensors. A sensor receives features of the external world from certain perspective. Different sensors can receive completely different features. Features obtained through visions are different from features obtained through noses. This leads to a research topic of *object recognition through a given sensors and preferred categories*

of the given sensors: What is a possible taxonomy of categories through a sensor? Given a taxonomy acquired by certain sensor, what can be the possible taxonomy of categories? Which level of abstraction in the taxonomy is the level at which categories carry the most information, possess the highest cue validity, and are, thus, the most differentiated from one another?

7.3.3 Detecting objects' movements

In The Theory of Cognitive Prism object mapping is restricted to the mapped *fiat* containers of two spatial environments. After a spatial environment is recognized, cognitive agents can further address the task of movement detection, which is a general object mapping task between the two sequential snapshots of a spatial environment. To detect object's movement is to explore principles of object mapping between unmapped *fiat* containers of two sequential snapshots of a spatial environment.

7.3.4 Humor: A window to the commonsense knowledge

One of the mainstays of The Theory of Cognitive Prism is the commonsense knowledge of relative stability, which is explored from commonsense understanding and evidenced by linguistic spatial descriptions. People have commonsense knowledge, but it is quite difficult for them to describe their commonsense knowledge explicitly, though they use it in their everyday life. People laugh, when they hear something against their commonsense. Therefore, it is possible to explore the commonsense knowledge through their laughter. This may link humor research with the commonsense research. For example, people laugh, when they hear *how do five Poles fix a lamp on the ceiling?— one stands on the table holding the lamp, and the four turning*

the table round and round. We know that these people have the common sense that *to fix a lamp on the ceiling, one should stand on the table turning the lamp.*

7.3.5 Situations of spatial environments

The process of judgment on whether the perceived environment is the target is separated from the process of finding the compatibility between the perceived environment and the target one. Although in everyday life situation, the more compatible they are, the higher degree the perceived environment is believed as the target one, there are situations that such a direct relation might not be held. For example, if it is the earthquake situation, then the perceived environment still can be the target one, even there are spatial differences of rarely moved objects.

When we navigate in spatial environments, we see a sequence of spatial environments—from one spatial environment into the next one. When we go to our offices from home, we have already entered and left lots of spatial environments, e.g., the spatial environments of home, street, university, inside of office building, corridor, etc. In each spatial environment we believe that it is the target environment, so that we are already sure that it is our office, even before entering the room and look at it. In this case the spatial differences in the perceived environment provide information for situation detection—what happened in this environment? For example, if there are neither spatial differences of rarely moved objects, nor spatial differences of seldom moved objects, then the situation can be the everyday life situation. If there are more new objects in the perceived environment and all the remembered objects are there, it can be a moving-in situation. If no new objects appear in the perceived environment and some objects disappear in the remembered environment, it can be a moving-out

situation. If there are spatial differences of locations of big furniture and there are neither new objects appearing nor remembered objects disappearing, it can be an after-a-party situation. If there are spatial differences of non-movable objects, it can even be an earthquake situation. A future work is to explore qualitative structure of situations.

Appendix A

Relations between the sizes of regions

When you sit on the couch, you can see the books on the writing-desk, although you cannot reach them by your arms. This implies that the light beam which is connected with the books and your eyes has larger size than your arm which cannot be connected with the books and the eyes. Sizes of regions can be compared as follows: Given two regions \mathcal{A} and \mathcal{B} , if region \mathcal{X}_1 can be moved to such a place that it is connected with both \mathcal{A} and \mathcal{B} , while for region \mathcal{X}_0 , there is no such a place, then the size of \mathcal{X}_0 is smaller than the size of \mathcal{X}_1 , de Laguna (1922), shown in Figure A.1. In the notion of category, sizes of regions are compared as follows: Given two regions \mathcal{A} and \mathcal{B} , if there is a region of the same category as region \mathcal{X}_1 which is connected with both \mathcal{A} and \mathcal{B} , while for region \mathcal{X}_0 , there is no region of the same category as \mathcal{X}_0 which is connected with \mathcal{A} and \mathcal{B} , then the size of \mathcal{X}_0 is smaller than the size of \mathcal{X}_1 . Two regions \mathcal{X}_0 and \mathcal{X}_1 of the same size is explained that for any two regions \mathcal{A} and \mathcal{B} , if both \mathcal{X}_0 and \mathcal{X}_1 can be connected with \mathcal{A} and \mathcal{B} or neither of \mathcal{X}_0 and \mathcal{X}_1 can be

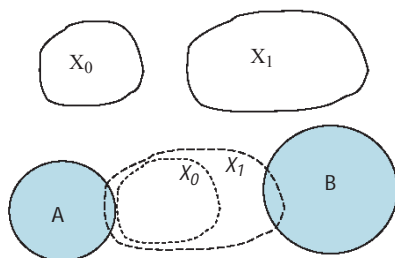


Figure A.1: The fact that region \mathcal{X}_0 is smaller than region \mathcal{X}_1 can be tested by given two regions \mathcal{A} and \mathcal{B} as follows: If \mathcal{X}_1 can be moved to such a place that it is connected both with \mathcal{A} and \mathcal{B} , while for \mathcal{X}_0 there is no such a place

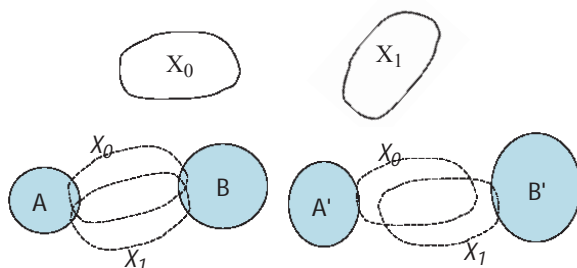


Figure A.2: The fact that region \mathcal{X}_0 is of the same size as region \mathcal{X}_1 can be tested as follows: For any two regions \mathcal{A} and \mathcal{B} , if \mathcal{X}_1 can be moved to such a place that it is connected both with \mathcal{A} and \mathcal{B} , so can \mathcal{X}_0 , and there is no such a place for \mathcal{X}_1 that it is connected both with \mathcal{A} and \mathcal{B} , neither is \mathcal{X}_0

connected with \mathcal{A} and \mathcal{B} , shown in Figure A.2. In the notion of category, two regions being of the same size is explained as follows: for any two regions \mathcal{A} and \mathcal{B} , if there is a region of the same category as region \mathcal{X}_1 which is connected with both \mathcal{A} and \mathcal{B} , then there is a region of the same category as region \mathcal{X}_0 which is connected with both \mathcal{A} and \mathcal{B} , and if there is a region of the same category as region \mathcal{X}_0 which is connected with both \mathcal{A} and \mathcal{B} , then there is a region of the same category as region \mathcal{X}_1 which is connected with both \mathcal{A} and \mathcal{B} , then the size of \mathcal{X}_0 is the same as the size of \mathcal{X}_1 . The basic relations of sizes are *smaller than* (symbolically represented by ' $<_s$ ', ' $_s$ ' represents 'size') and *equal* (symbolically represented by ' $=_s$ ').

Definition A.0.1. Given two regions \mathcal{X}_0 and \mathcal{X}_1 , then “ \mathcal{X}_0 is smaller than \mathcal{X}_1 ” is defined as that there are two regions \mathcal{A} and \mathcal{B} such that the near extension of \mathcal{A} by \mathcal{X}_0 is disconnected with \mathcal{B} and the near extension of \mathcal{A} by \mathcal{X}_1 is connected with \mathcal{B} .

$$\mathcal{X}_0 <_s \mathcal{X}_1 \stackrel{\text{def}}{=} \exists \mathcal{A} \exists \mathcal{B} \bullet \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B})$$

Definition A.0.2. Given two regions \mathcal{X}_0 and \mathcal{X}_1 , then “ \mathcal{X}_0 is of the same size as \mathcal{X}_1 ” is defined as all regions \mathcal{A} and \mathcal{B} , it holds that either the near extensions of \mathcal{A} by \mathcal{X}_0 and \mathcal{X}_1 are connected with both \mathcal{A} and \mathcal{B} , or the near extensions of \mathcal{A} by \mathcal{X}_0 and \mathcal{X}_1 are not connected with both \mathcal{A} and \mathcal{B} .

$$\begin{aligned} \mathcal{X}_0 =_s \mathcal{X}_1 \stackrel{\text{def}}{=} & \forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \\ & \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \end{aligned}$$

Theorem A.0.1. Given two regions, \mathcal{X}_0 (of category \mathbf{X}_0) and \mathcal{X}_1 (of category \mathbf{X}_1), all regions \mathcal{A} and \mathcal{B} , it holds that if $\mathcal{X}_0 <_s \mathcal{X}_1$, then if $\mathcal{A}^{\mathcal{X}_0}$ is connected with \mathcal{B} , then $\mathcal{A}^{\mathcal{X}_1}$ is also connected with \mathcal{B} .

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \rightarrow (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

Theorem A.0.2. For any regions \mathcal{X}_0 , \mathcal{X}_1 , \mathcal{X}_2 , if $\mathcal{X}_0 <_s \mathcal{X}_1$ and $\mathcal{X}_1 <_s \mathcal{X}_2$, then $\mathcal{X}_0 <_s \mathcal{X}_2$.

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \wedge \mathcal{X}_1 <_s \mathcal{X}_2 \rightarrow \mathcal{X}_0 <_s \mathcal{X}_2$$

Theorem A.0.3. For any region \mathcal{X} , \mathcal{X} is of the same size as itself.

$$\forall \mathcal{X} \bullet \mathcal{X} =_s \mathcal{X}$$

Theorem A.0.4. *For all regions \mathcal{X}_0 and \mathcal{X}_1 , it holds that either \mathcal{X}_0 is smaller than \mathcal{X}_1 , or \mathcal{X}_0 equals to \mathcal{X}_1 , or \mathcal{X}_1 is smaller than \mathcal{X}_0 .*

$$\forall \mathcal{X}_0, \mathcal{X}_1 \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \vee \mathcal{X}_0 =_s \mathcal{X}_1 \vee \mathcal{X}_1 <_s \mathcal{X}_0$$

Proofs of the above theorems are listed in Appendix B.

Appendix B

Theorem proof sketches

Theorem 5.2.1 Given \mathcal{A} an object region, and \mathbf{X} be a category of an object region. There is \mathcal{X} satisfying that \mathcal{X} is a member of \mathbf{X} such that if \mathcal{X} is connected with \mathcal{A} , then there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{Y} is connected with \mathcal{W} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is a member of \mathbf{X} and connected with \mathcal{A} such that \mathcal{V} is connected with \mathcal{W} .

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

Proof sketch:

(1)[*definition of \equiv*]

$$\forall \mathcal{W} \bullet \mathbf{C}(\mathcal{W}, \mathcal{X}) \equiv \mathbf{C}(\mathcal{W}, \mathcal{X})$$

(2)[**Axiom 5.2.3**]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \mathbf{C}(\mathcal{A}, \mathcal{X})$$

(3)[(1), (2)]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{x} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{X}) \equiv \mathbf{C}(\mathcal{A}, \mathcal{X}) \wedge \mathbf{C}(\mathcal{W}, \mathcal{X}))$$

(4)[\exists – *introduce*]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{x} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} \bullet \mathcal{V} \in \mathbf{x} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \wedge \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

(5)[$|$ – *introduce*]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{x} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{x} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

The existence of $\mathcal{A}^{\mathbf{x}}$ **Proof sketch:**(1)[**Theorem 5.2.1**]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{x} \bullet \mathbf{C}(\mathcal{X}, \mathcal{A}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{x} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

(2)[**Axiom 5.2.1**]

$$\forall \mathcal{A} \bullet \mathbf{C}(\mathcal{A}, \mathcal{A}) \rightarrow \forall \mathcal{Z} \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{z} \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{A}, \mathcal{Z})$$

(3)[(2), **Axiom 5.2.2**]

$$\forall \mathcal{Z} \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{z} \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{A}, \mathcal{Z})$$

(4)[(3), \forall – *elimination*, $p \wedge p \equiv p$]

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{x} \bullet \mathbf{C}(\mathcal{A}, \mathcal{X})$$

(5)[(1), (4), **Axiom 5.2.3**]

$$\exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{x} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

The uniqueness of \mathcal{A}^x

Proof sketch:

(1)[**Definition 5.2.2, Theorem 5.2.1**]

$$\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}_1) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

(2)[**Definition 5.2.2, Theorem 5.2.1**]

$$\forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}_2) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

(3)[(1), (2), *definition of \equiv*]

$$\forall \mathcal{W} \bullet \mathbf{C}(\mathcal{W}, \mathcal{Y}_1) \equiv \mathbf{C}(\mathcal{W}, \mathcal{Y}_2)$$

(4)[(3), **Definition 5.2.1**]

$$\mathbf{P}(\mathcal{Y}_1, \mathcal{Y}_2) \wedge \mathbf{P}(\mathcal{Y}_2, \mathcal{Y}_1)$$

(5)[(4), **Axiom 5.2.4**]

$$\mathcal{Y}_1 = \mathcal{Y}_2$$

Theorem 5.2.2 Given object regions \mathcal{A} and \mathcal{X} , \mathcal{A} is a part of the *near* extension of \mathcal{A} by \mathcal{X} .

$$\forall \mathcal{A}, \mathcal{X} \bullet \mathbf{P}(\mathcal{A}, \mathcal{A}^x)$$

Proof sketch:

(1)[**Axiom 5.1**]

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}, \mathcal{B}) \rightarrow \forall \mathcal{Z} \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{Z} \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{Z})$$

(2)[(1), \forall – *elimination*]

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}, \mathcal{B}) \rightarrow \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{Z})$$

(3)[**Definition 5.2.2** of $\mathcal{A}^{\mathcal{X}}$]

$$\forall \mathcal{W} \bullet \mathbf{C}(\mathcal{W}, \mathcal{A}^{\mathcal{X}}) \equiv \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{A}, \mathcal{Z}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{Z})$$

(4)[(3), \forall - *elimination*, $\exists x | p \bullet q \equiv \exists x \bullet p \wedge q$]

$$\mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}}) \equiv \exists \mathcal{Z} | \mathcal{Z} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{A}, \mathcal{Z}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{Z})$$

(5)[(2), (4)]

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}})$$

(6)[(5), **Axiom 5.2.3**]

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{B}, \mathcal{A}) \rightarrow \mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}})$$

(7)[**Definition of P** in **Definition 5.2.1**]

$$\forall \mathcal{A}, \mathcal{X} \bullet \mathbf{P}(\mathcal{A}, \mathcal{A}^{\mathcal{X}})$$

Theorem 5.2.3 All object regions \mathcal{A} , \mathcal{B} and \mathcal{X} , it holds that the *near* extension of \mathcal{A} by \mathcal{X} is connected with \mathcal{B} , if and only if \mathcal{A} is connected with the *near* extension of \mathcal{B} by \mathcal{X} .

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \equiv \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}})$$

Proof sketch:

(1)[**Axiom 5.2.3**]

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}})$$

(2)[**Axiom 5.2.3**]

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B})$$

(3)[(1), (2), **Definition of \equiv**]

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \equiv \mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}})$$

(4)[**Definition 5.2.2 of $\mathcal{A}^{\mathcal{X}}$** , $\exists x|p \bullet q \equiv \exists x \bullet p \wedge q$]

$$\mathbf{C}(\mathcal{B}, \mathcal{A}^{\mathcal{X}}) \stackrel{\text{def}}{=} \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{A}, \mathcal{V}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{V})$$

(5)[$p \wedge q \equiv q \wedge p$]

$$\exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{A}, \mathcal{V}) \wedge \mathbf{C}(\mathcal{B}, \mathcal{V}) \equiv \mathbf{C}(\mathcal{B}, \mathcal{V}) \wedge \mathbf{C}(\mathcal{A}, \mathcal{V})$$

(6)[**Definition 5.2.2 of $\mathcal{B}^{\mathcal{X}}$** , $\exists x|p \bullet q \equiv \exists x \bullet p \wedge q$]

$$\mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}}) \stackrel{\text{def}}{=} \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{B}, \mathcal{V}) \wedge \mathbf{C}(\mathcal{A}, \mathcal{V})$$

(7)[(3), (4), (5), (6)]

$$\forall \mathcal{A}, \mathcal{B}, \mathcal{X} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \equiv \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}})$$

Theorem 5.2.4 Let \mathcal{C} be a constructed region, and \mathbf{X} be a category of an object region. There is \mathcal{X} satisfying that \mathcal{X} is a member of \mathbf{X} such that if \mathcal{X} is connected with \mathcal{C} , there is \mathcal{Y} such that all \mathcal{W} , it holds that \mathcal{Y} is connected with \mathcal{W} , if and only if there is \mathcal{V} satisfying that \mathcal{V} is a member of \mathbf{X} and connected with \mathcal{C} such that \mathcal{V} is connected with \mathcal{W} .

$$\exists \mathcal{X} | \mathcal{X} \in \mathbf{X} \bullet \mathbf{C}(\mathcal{X}, \mathcal{C}) \rightarrow \exists \mathcal{Y} \forall \mathcal{W} \bullet (\mathbf{C}(\mathcal{W}, \mathcal{Y}) \equiv \exists \mathcal{V} | \mathcal{V} \in \mathbf{X} \wedge \mathbf{C}(\mathcal{C}, \mathcal{V}) \bullet \mathbf{C}(\mathcal{W}, \mathcal{V}))$$

Proof sketch: similar with the proof sketch of **Theorem 5.2.1**.

The proofs of the existence and the uniqueness of $\mathcal{C}^{\mathcal{X}}$ are similar to the proofs of those of $\mathcal{A}^{\mathcal{X}}$.

Theorem A.0.1 Given two regions, \mathcal{X}_0 (of category \mathbf{X}_0) and \mathcal{X}_1 (of category \mathbf{X}_1), all regions \mathcal{A} and \mathcal{B} , it holds that if $\mathcal{X}_0 <_s \mathcal{X}_1$, then if $\mathcal{A}^{\mathcal{X}_0}$ is connected with \mathcal{B} , then $\mathcal{A}^{\mathcal{X}_1}$

is also connected with \mathcal{B} .

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \rightarrow (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

Proof sketch:

$$(1) [p \vee \neg p \equiv \text{true}]$$

$$\forall \mathcal{A}, \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \vee \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B})$$

$$(2) [(1), p \vee q \equiv q \vee p]$$

$$\forall \mathcal{A}, \mathcal{B} \bullet (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B})) \vee (\neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \vee \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

$$(3) [\neg p \vee q \equiv \neg(p \wedge \neg q), \neg p \vee q \equiv p \rightarrow q]$$

$$\forall \mathcal{A}, \mathcal{B} \bullet (\neg(\neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))) \vee (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

$$(4) [\text{Definition A.0.1 of } <_s]$$

$$\forall \mathcal{A}, \mathcal{B} \bullet \neg(\mathcal{X}_0 <_s \mathcal{X}_1) \vee (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

$$(5) [(4), \neg p \vee q \equiv p \rightarrow q]$$

$$\forall \mathcal{A}, \mathcal{B} \bullet (\mathcal{X}_0 <_s \mathcal{X}_1 \rightarrow (\mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \rightarrow \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B})))$$

Theorem A.0.2 For any regions $\mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2$, if $\mathcal{X}_0 <_s \mathcal{X}_1$ and $\mathcal{X}_1 <_s \mathcal{X}_2$, then $\mathcal{X}_0 <_s \mathcal{X}_2$.

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\mathcal{X}_0 <_s \mathcal{X}_1 \wedge \mathcal{X}_1 <_s \mathcal{X}_2 \rightarrow \mathcal{X}_0 <_s \mathcal{X}_2)$$

Proof sketch:

(1)[**Definition A.0.1** of $<_s$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\mathcal{X}_0 <_s \mathcal{X}_1 \wedge \mathcal{X}_1 <_s \mathcal{X}_2)$$

$$\stackrel{\text{def}}{=} \forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet ((\exists \mathcal{A} \exists \mathcal{B} \bullet \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B})) \wedge \mathcal{X}_1 <_s \mathcal{X}_2)$$

(2) [(1), $((\exists x \bullet p(x)) \wedge p) \rightarrow (\exists x \bullet (p(x) \wedge p))$)]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\exists \mathcal{A} \exists \mathcal{B} \bullet \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \wedge \mathcal{X}_1 <_s \mathcal{X}_2)$$

(3) [(2), \exists - elimination]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\neg \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_0}, \mathcal{B}_0) \wedge \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_1}, \mathcal{B}_0) \wedge \mathcal{X}_1 <_s \mathcal{X}_2)$$

(4) [(3), $p \wedge q \rightarrow q$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\mathbf{C}(\mathcal{A}_0^{\mathcal{X}_1}, \mathcal{B}_0) \wedge \mathcal{X}_1 <_s \mathcal{X}_2)$$

(5) [(4), $p \wedge q \rightarrow p$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_1}, \mathcal{B}_0)$$

(6) [(4), $p \wedge q \rightarrow q$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet \mathcal{X}_1 <_s \mathcal{X}_2$$

(7) [(5), (6), **Theorem A.0.1**]

$$\mathbf{C}(\mathcal{A}_0^{\mathcal{X}_2}, \mathcal{B}_0)$$

(8) [(3), $p \wedge q \rightarrow p$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet \neg \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_0}, \mathcal{B}_0)$$

(9) [(3), (7), (8)]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\exists \mathcal{A}_0 \exists \mathcal{B}_0 \bullet \neg \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_0}, \mathcal{B}_0) \wedge \mathbf{C}(\mathcal{A}_0^{\mathcal{X}_2}, \mathcal{B}_0))$$

(10) [(9), **Definition A.0.1** of $<_s$]

$$\forall \mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2 \bullet (\mathcal{X}_0 <_s \mathcal{X}_2)$$

Theorem A.0.3 For any region \mathcal{X} , \mathcal{X} is of the same size as itself.

$$\forall \mathcal{X} \bullet \mathcal{X} =_s \mathcal{X}$$

Proof sketch:

(1) [Definition A.0.2 of $=_s$]

$$\forall \mathcal{X} \bullet \mathcal{X} =_s \mathcal{X}$$

$$\stackrel{\text{def}}{=} \forall \mathcal{A}, \mathcal{B} \bullet ((\mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B})) \vee (\neg \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B})))$$

(2) [(1), $p \wedge p \equiv p$]

$$\forall \mathcal{A}, \mathcal{B} \bullet (\mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}}, \mathcal{B}))$$

(3) [(2), $p \vee \neg p \equiv \text{true}$]

true

Theorem A.0.4 For all regions \mathcal{X}_0 and \mathcal{X}_1 , it holds that either \mathcal{X}_0 is smaller than \mathcal{X}_1 , or \mathcal{X}_0 equals to \mathcal{X}_1 , or \mathcal{X}_1 is smaller than \mathcal{X}_0 .

$$\forall \mathcal{X}_0, \mathcal{X}_1 \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \vee \mathcal{X}_0 =_s \mathcal{X}_1 \vee \mathcal{X}_1 <_s \mathcal{X}_0$$

Proof sketch:

(1) [Suppose $\neg(\mathcal{X}_0 <_s \mathcal{X}_1)$]

$$\neg(\mathcal{X}_0 <_s \mathcal{X}_1)$$

(2) [Definition A.0.1 of $<_s$]

$$\neg(\exists \mathcal{A} \exists \mathcal{B} \bullet \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}))$$

$$(3) [(2), (\neg \exists x \exists y \bullet p \rightarrow \forall x \forall y \bullet \neg p)]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet (\mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}))$$

$$(4) [(3), p \equiv p \wedge (q \vee \neg q)]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet ((\mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B})) \wedge (\mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})))$$

$$(5) [(4), (p \vee q) \wedge (s \vee t) \equiv (p \wedge s) \vee (p \wedge t) \vee (q \wedge s) \vee (q \wedge t)]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$\vee \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$(6) [(5), (p \wedge \neg p) \equiv \text{false}; p \wedge p \equiv p]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$\vee \text{false} \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$(7) [(6), p \vee \text{false} \equiv p]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$(8) [(7), p \equiv p \wedge (q \vee \neg q)]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet (\mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}))$$

$$\wedge (\mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}))$$

$$(9) [(8), (p \vee q) \wedge (s \vee t) \equiv (p \wedge s) \vee (p \wedge t) \vee (q \wedge s) \vee (q \wedge t)]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B})$$

$$\vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B})$$

$$\vee \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B})$$

$$\vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B})$$

$$(10) [(9), (p \wedge \neg p) \equiv \text{false}; p \wedge p \equiv p]$$

$$\forall \mathcal{A} \forall \mathcal{B} \bullet \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \vee \text{false} \vee \text{false}$$

$$\vee \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$\vee \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_0}, \mathcal{B})$$

$$(11) \quad [(10), p \vee \mathbf{false} \equiv p; p \vee p \equiv p]$$

$$\begin{aligned} \forall \mathcal{A} \forall \mathcal{B} \bullet & \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \vee \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \\ & \vee \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_0}, \mathcal{B}) \end{aligned}$$

$$(12) \quad [(11), \mathbf{Theorem\ 5.2.3}]$$

$$\begin{aligned} \forall \mathcal{A} \forall \mathcal{B} \bullet & \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_0}) \wedge \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_1}) \vee \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_0}) \wedge \neg \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_1}) \\ & \vee \neg \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_1}) \wedge \neg \mathbf{C}(\mathcal{A}, \mathcal{B}^{\mathcal{X}_0}) \end{aligned}$$

$$(13) \quad [(12), \mathbf{Axiom\ 5.2.3}]$$

$$\begin{aligned} \forall \mathcal{A} \forall \mathcal{B} \bullet & \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \wedge \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \vee \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \wedge \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \\ & \vee \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \wedge \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \end{aligned}$$

$$(14) \quad [(13), (\forall x \forall y \bullet p) \rightarrow (\forall y \forall x \bullet p)]$$

$$\begin{aligned} \forall \mathcal{B} \forall \mathcal{A} \bullet & \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \wedge \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \vee \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \wedge \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \\ & \vee \neg \mathbf{C}(\mathcal{B}^{\mathcal{X}_1}, \mathcal{A}) \wedge \mathbf{C}(\mathcal{B}^{\mathcal{X}_0}, \mathcal{A}) \end{aligned}$$

$$(15) \quad [(14), \mathbf{Definition\ A.0.1, A.0.2\ of\ } <_s \text{ and } =_s]$$

$$\mathcal{X}_0 =_s \mathcal{X}_1 \vee \mathcal{X}_1 <_s \mathcal{X}_0$$

$$(16) \quad [(1), (15)]$$

$$\forall \mathcal{X}_0, \mathcal{X}_1 \bullet \neg(\mathcal{X}_0 <_s \mathcal{X}_1) \rightarrow (\mathcal{X}_0 =_s \mathcal{X}_1 \vee \mathcal{X}_1 <_s \mathcal{X}_0)$$

$$(17) \quad [(16), (p \rightarrow q) \equiv \neg p \vee q]$$

$$\forall \mathcal{X}_0, \mathcal{X}_1 \bullet \mathcal{X}_0 <_s \mathcal{X}_1 \vee \mathcal{X}_0 =_s \mathcal{X}_1 \vee \mathcal{X}_1 <_s \mathcal{X}_0$$

Theorem 5.2.5 Let \mathcal{A} and \mathcal{B} be two object regions, $\mathbf{Front}(\mathcal{A}, \mathcal{B})$, $\mathbf{Left}(\mathcal{A}, \mathcal{B})$, $\mathbf{Right}(\mathcal{A}, \mathcal{B})$ and $\mathbf{Behind}(\mathcal{A}, \mathcal{B})$ are pairwise disjoint.

$$\mathbf{Front}(\mathcal{A}, \mathcal{B}) \wedge \mathbf{Left}(\mathcal{A}, \mathcal{B}) \equiv \mathbf{false}$$

Proof sketch:

(1) [Definitions 5.2.6, 5.2.7 of Front and Left]

 $\text{Front}(\mathcal{A}, \mathcal{B}) \wedge \text{Left}(\mathcal{A}, \mathcal{B})$ $\stackrel{\text{def}}{=} (\forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\})$ $\bullet p \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, p)$ $\wedge (\forall p | p \in \{\mathcal{B}.\text{left}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right}, \mathcal{B}.\text{back}\})$ $\bullet p \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, p)$ (2) [(1), $\forall x \bullet p(x) \equiv p(x_0) \wedge p(x_1) \wedge \dots$] $\mathcal{B}.\text{left} \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{left})$ $\wedge \mathcal{B}.\text{front} \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{front})$ $\wedge \mathcal{B}.\text{right} \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right})$ $\wedge \mathcal{B}.\text{back} \neq \mathcal{B}.\text{front} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{back})$ $\wedge \mathcal{B}.\text{left} \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{left})$ $\wedge \mathcal{B}.\text{front} \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{front})$ $\wedge \mathcal{B}.\text{right} \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{right})$ $\wedge \mathcal{B}.\text{back} \neq \mathcal{B}.\text{left} \rightarrow \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{back})$ (3) [(2), $\text{true} \rightarrow p \equiv p$; $\text{false} \rightarrow p \equiv \text{true}$] $\text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{left}) \wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{right})$ $\wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{back}) \wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{front})$ $\wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{right}) \wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{back})$ (4) [(3), $p \wedge q \rightarrow p$] $\text{nearer}(\mathcal{A}, \mathcal{B}.\text{front}, \mathcal{B}.\text{left}) \wedge \text{nearer}(\mathcal{A}, \mathcal{B}.\text{left}, \mathcal{B}.\text{front})$

(5) [(4), **Definition 5.2.4** of *nearer*]

$$\begin{aligned} & \exists \mathcal{X}_1 \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}.\text{front}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_1}, \mathcal{B}.\text{left}) \\ & \wedge \exists \mathcal{X}_2 \bullet \mathbf{C}(\mathcal{A}^{\mathcal{X}_2}, \mathcal{B}.\text{left}) \wedge \neg \mathbf{C}(\mathcal{A}^{\mathcal{X}_2}, \mathcal{B}.\text{front}) \end{aligned}$$

(6) [(5), *Let x_1 be \mathcal{X}_1 and x_2 be \mathcal{X}_2*]

$$\begin{aligned} & \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}.\text{front}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}.\text{left}) \\ & \wedge \mathbf{C}(\mathcal{A}^{x_2}, \mathcal{B}.\text{left}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_2}, \mathcal{B}.\text{front}) \end{aligned}$$

(7) [(6) $p \wedge q \equiv q \wedge p$]

$$\begin{aligned} & \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}.\text{front}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_2}, \mathcal{B}.\text{front}) \\ & \wedge \mathbf{C}(\mathcal{A}^{x_2}, \mathcal{B}.\text{left}) \wedge \neg \mathbf{C}(\mathcal{A}^{x_1}, \mathcal{B}.\text{left}) \end{aligned}$$

(8) [(7), **Definition A.0.1** of $<_s$]

$$x_1 <_s x_2 \wedge x_2 <_s x_1$$

(9) [(8), **Theorem A.0.2**]

$$x_1 <_s x_1$$

(10) [(9), **Theorem A.0.3**]

false

The proofs of the existence and the uniqueness of *fiat* containers are similar to the proofs of those of $\mathcal{A}^{\mathcal{X}}$.

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