A COOPERATIVE ALGORITHM FOR DETERMINING SURFACE ORIENTATION FROM A SINGLE VIEW

Robert J. Woodham
Artificial Intelligence Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts, 02139

INTRODUCTION

This paper addresses the problem of determining local surface orientation from the intensity information contained in a single monocular view. It represents another look at the problem of obtaining shape from shading information. This problem was first formulated by <Horn 1970> as the solution of first-order non-linear partial differential equations. It was subsequently reformulated by <Horn 1975> to take advantage of the geometric insight provided by the gradient space approach popularized by <Huffman l97l,1975> and <Mackworth 1973>.

The goal of this work is to understand how the observed intensity variation across surfaces of objects forces conclusions about the local topography of those surfaces. The problem is formulated as a problem in image analysis. This paper adopts the position that it is important to examine ways of squeezing out the last ounce of information from the intensity values recorded in an image before taking recourse to high-level knowledge.

In order to exploit intensity information, it is vital to understand how intensity values arise in an image. The paper begins by introducing gradient space and reflectance map techniques. Having developed these tools, the physical constraints imposed by the light source, object surface and viewer geometry are re-examined to recast the <Horn l975> formulation as a cooperative computation which determines local surface orientation by matching points in an image to their corresponding points in gradient space. Finally, the method is illustrated using the simple example of a Lambertian sphere illuminated by a single distant light source.

GRADIENT SPACE AND THE REFLECTANCE MAP

In order to understand the correspondence between intensity data and local surface orientation, it is necessary to relate the geometry of the image forming process to the photometry of the object being imaged. The problem of determining local surface orientation from intensity can be characterized as a mapping:

$$T : I(u,v) \rightarrow LSO(u,v)$$

which assigns to each image intensity point I(u,v) a local surface orientation LSO(u,v). In a visual world consisting of opaque smooth objects immersed in a transparent medium, the mapping T is well defined since each image intensity point I(u,v) arises from a unique object point (which, in turn, defines a unique surface orientation). What is not obvious, however, is whether T can be determined from intensity data alone. The difficulty, of course, is that T is not a local operator. In

formulating an algorithm for determining local surface orientation from image intensity. It will be important to keep track of how additional physical constraints are used in the computation.

Now, LSO(u.v) is a two-valued function of u and v since two parameters are required to specify an arbitrary orientation in space. One might expect surface orientation to be difficult to represent explicitly. Gradient space is introduced as the appropriate formalism for reasoning about surface orientation.

Gradient space can be derived in several ways. For present purposes, it will be related directly to surface orientation. If the equation of a smooth surface is given by:

$$z = f(x,y)$$

then $(\partial f/\partial x, \partial f/\partial y, -1)$ defines a local (outward) surface normal. (Throughout this paper, smooth will interpreted to mean $z = f(x,y)$ has continuous first and second partial derivatives.) Now, let:

$$p = \partial f/\partial x$$
$$q = \partial f/\partial y$$

so that the local (outward) surface normal becomes (p,q,-l). The quantity (p,q) will be called the *gradient* and *gradient space* is defined to be the two-dimensional space of alt such points (p,q).

The fraction of light reflected by a surface in a given direction depends upon the optical properties of the object material, the surface micro-structure and the spatial and spectral distribution of the incident light. The important photometric observation underlying this work is the following: no matter how complex the distribution of incident illumination, for most surfaces, the fraction of the incident light reflected in a particular direction depends only on the local light source, object surface and viewer geometry. To make this observation more concrete, one can standardize the local representation of the light source, object surface and viewer geometry and tie this representation down to gradient coordinates p and q.

A surface photometric function ø(i,e,g) is typically defined in terms of the three angles i, e and g illustrated in figure l. These angles are called, respectively, the incident, emergent and phase angle. Here, the emergent angle e will be referred to as the *view angle*.
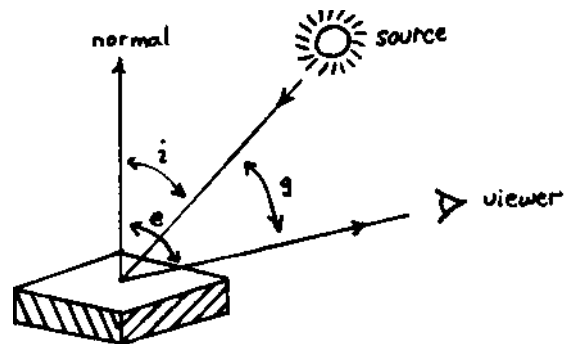

Figure 1

If both the viewing direction and the direction of incident illumination are known, then expressions for cos(i), cos(e) and cos(g) can be derived in terms of gradient space coordinates p and q. To simplify matters, consider an image forming system

that performs an orthographic projection. The Important simplification inherent in the assumption of an orthographic projection is that the viewing direction, and hence the phase angle g, is constant for all image points (u,v). To further simplify the mathematics, one can align the viewing direction with the negative Z-axis and assume a scaling in U and V that takes object point (x.y.z) to Image point (u,v) where u - x and v - y (see figure 2). Under this projection, the use of separate image coordinates (u,v) is redundant. Henceforth, image coordinates (x,y) and object coordinates (x.y) will be referred to interchangeably. With this geometry, object space vector (0,0,-1) points in the direction of the viewer. That is, the viewer is located at gradient space point (0,0).
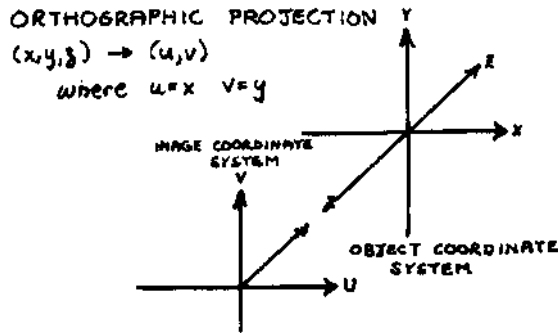

Figure 2

Now, for a given distribution of incident illumination, a given surf ace-viewer geometry and a given object material, the image intensity corresponding to a surface point with gradient (p,q) is unique. The intensities recorded at each (p,q) is called the *reflectance map* R(p,q). Reflectance maps can be determined empirically, derived from phenomenotological models of surface reflectivity or derived from analytic models of surface micro-structure. Once determined, however, the reflectance map Is independent of the shape of the objects being viewed. It represents explicit knowledge of intensities that can be recorded from objects made of a given material and viewed under a particular light source and viewer geometry. The reflectance map accounts for the physical intensities that can be recorded under varying components of diffuse and specular reflection. It does not, of course, account for the human perception of those intensities, (eg. A local specular component can make an otherwise "dull" surface appear "glossy")

Consider an image whose Intensity values I(x,y) have been normalized to equal the reflectance map values at the corresponding gradients (p,q). Then, the following equation describes the image forming process:

$$I(x.y) - R(p.q)$$

This is the basic equation relating image intensity to the geometry of the image forming process. It is one equation in the two unknowns p and q. Thus, the problem of determining local surface orientation from intensity becomes the problem of finding the point in gradient space (p,q) corresponding to the image intensity point I(x,y).

Now, the simplest case for Incident illumination Is that of a single distant point source. Choose such a source and place it so that object space vector $(p_s,q_s,-1)$ points in the direction of the source. That is, the source is located at gradient space point $(p_s,q_s,)$. Specifying a single distant point source is not a fundamental restriction on the development. Non-point sources can be modeled as the superposition of single point sources. The development does, however, assume equal illumination at all surface points. For non-convex surfaces, the reflectance map does not account for the fact that certain surface points can be shadowed with respect to one or more of the sources nor for the fact that certain surface points can receive additional illumination due to light reflected from other sections of surface (mutual illumination).

Recall that, object space vector (p.q.-l) is an (outward) normal to the surface point (x.y,z). That is, (p,q) is the gradient point corresponding to the surface point (x,y,i). Then, using standard vector algebra, the expressions for cos(i), cos(e) and cos(g) become:

$$\cos(i) = \frac{1 + pp_s + qq_s}{\sqrt{1 + p_s^2 + q_s^2}\sqrt{1 + p^2 + q^2}}$$

$$\cos(e) = \frac{1}{\sqrt{1 + p^2 + q^2}}$$

$$\cos(g) = \frac{1}{\sqrt{1 + p_s^2 + q_s^2}}$$

Using the above expressions, it is clear that one can transform an arbitrary surface photometric function 4(i,e,g) into a reflectance map R(p,q). By now, it should be clear how points in gradient space correspond to orientations in object space. However, it is possible to be more explicit about the correspondence between movement in gradient space and changes in local surface orientation.

Using elementary trigonometry, the expression for cos(e) can be rewritten as:

$$\tan(e) = \sqrt{p^2 + q^2}$$

This gives a first useful interpretation that the inclination of the surface with respect to the viewing direction varies monotonicalty with the distance from the origin in gradient space. Specifically, the distance from the origin is the tangent of the angle between the surface normal and the view vector. As the gradient (p,q) moves away from the origin, the inclination of the surface with respect to the viewer increases. As the gradient (p,q) moves toward the origin, the inclination of the surface with respect to the viewer decreases.

Now, the view angle e characterizes one of the two degrees of freedom associated with an arbitrary orientation in space. Note that the locus of points in object space having a constant view angle e defines a right circular cone oriented along the viewing direction. The angular position of each gradient (p,q) on the circle $p^2 + q^2 - \tan^2(e)$ defines the direction of steepest descent in image space along this cone. This gives a second useful interpretation that the angular position of a point (p.q) in gradient space corresponds to the direction of steepest descent in Image space along the original surface. Rotating object space about the view vector induces an equal rotation in gradient space.

RE-EXAMINING PHYSICAL CONSTRAINTS

One can formulate the problem of determining the point in gradient space (p,q) corresponding to the image intensity point I(x,y) analytically as has been done in <Horn 1970,1975>. This work will not be reviewed here. Instead, the formal nature of the computation will be put aside in order to first re-examine its basis in the physical world.

Two constraints of importance can be identified:
1. A given point on a physical surface has a unique orientation in space.
2. Matter is cohesive. It is separated into objects. The surfaces of objects are generally smooth compared with their distance from the viewer.

The reader will note that these are essentially the same two constraints <Marr and Poggio 1976> use as a basis for their computation of stereo disparity. As in their paper, the goal is to translate the above two physical constraints into rules for how points in an image can be matched to points in gradient space.

In their most general form, these rules can be expressed as.
1. UNIQUENESS: Each image point may be assigned to at most one location in gradient space
2. CONTINUITY: Surfaces vary smoothly almost everywhere Only a small fraction of the area of an image is composed of boundaries that correspond to discontinuities of surface.

The task ahead is to demonstrate that these rules can be explicitly embedded in a computation. The result is an algorithm which attempts to achieve a global correspondence between image points and points in gradient space via local, interactive constraints.

It has become fashionable to call such algorithms "cooperative" after similar phemomena in physics. Perhaps the reader will be more comfortable if the algorithm is presented as a "relaxation scheme". Nonetheless, what is important is to get the flavor of how local constraints can propagate back and forth to globally constrain possible matches between image points and points in gradient space.

SPECIFYING LOCAL CONSTRAINT

The basic equation
$$I(x,y) = R(p,q)$$
is one equation in the two unknowns p and q. With this equation alone, the gradient corresponding to a particular image point is constrained to lie on a one parameter (family of) contour(s) in gradient space. The goal is to apply further constraint in order to assign a unique location in gradient space to each image point.

The essential physical constraint to be exploited is the assumption that, compared to the viewing distance, surfaces vary smoothly almost everywhere. This surface smoothness assumption is translated into monotonicity rules on changes to view angle and changes to direction of steepest descent permitted between (closely spaced) image points.

One can illustrate how physical constraint adds additional constraint to the possible gradient space solutions to the basic equation $1(x,y) - R(p,q)$. Suppose, two (closely spaced) image points $(x,,y,)$ and (x_2,y_2) are hypothesized to correspond to object points on the same section of smooth surface. Further, suppose that the view angle increases in going from $(x,,y,)$ to (x_2,y_2) and that the direction of steepest descent decreases in going from $(x,.y,)$ to (x_2,y_2). Let $(p_1.q_1)$ and (p_2,q_2) be the gradient locations corresponding to $(x,,y,>$ and (x_2,y_2).

Suppose, further, that the basic equation $I(x,y) - R(p,q)$ constrains (p t .q i) and (p_2,q_2) to lie on the contours C, and C_2 respectively as shown In figure 3.
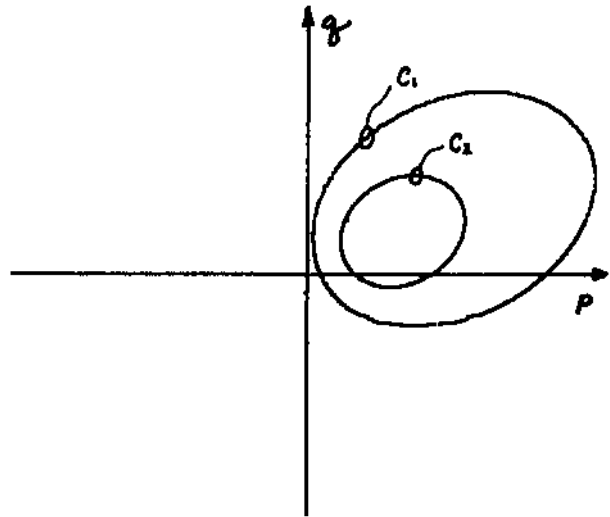

Figure 3

Figure 4 shows, superimposed on the two contours of figure 3, the gradient space circle corresponding to the maximum view angle interpretation of (p_2,q_2) and the gradient space line corresponding to the minimum direction of steepest descent interpretation of (p_2,q_2). Since the view angle increases in going from $(x,,y,)$ to (x_2,y_2), the contour of permissable (p,q,) can be restricted to include only those gradient points on C, lying on or within the circle of the maximum view angle interpretation of (p_2,q_2). Similarly, since direction of steepest descent decreases in going from $(x,,y,)$ to (x_2,y_2), the contour of permissable $(p_t,q_t>$ can be restricted to include only those gradient points on or above the line of the minimum direction of steepest descent interpretation of (p_2,q_2). Thus, without any additional constraint on (p_2,q_2), the assumed monotonicity relations between $(x,,y,)$ and (x_2,y_2) have been applied to the topography of the reflectance map to constrain the possible interpretation of (p,,q,) to include only those points of C, indicated by the solid line of figure 4.

Let (r J) denote the polar representation of the gradient space point (p,q). That is:
$$r \sqrt{p^2 + q^2}$$
$$\theta - atan(q/p)$$

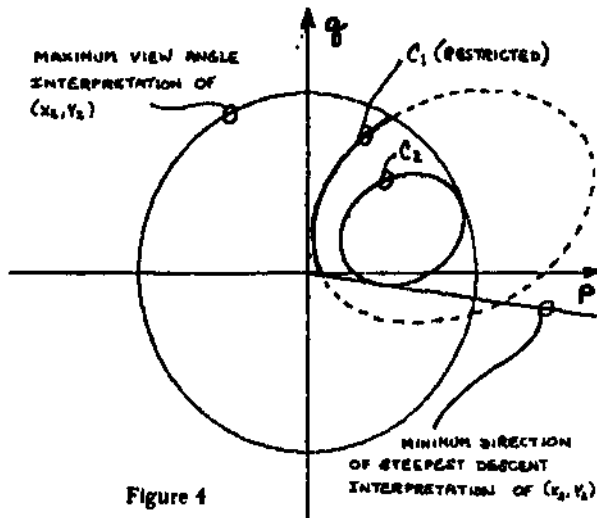Generalizing from the above illustration* the following two rules are stated:

Figure 4

MAXIMUM VIEW ANGLE INTERPRETATION OF (x_1, y_1)

C_1 (RESTRICTED)

C_2

MINIMUM DIRECTION OF STEEPEST DESCENT INTERPRETATION OF (x_1, y_1)

RULE I: CHANGES IN VIEW ANGLE

Let $I_1, I_2 \ldots I_n$ be a set of (closely spaced) image points hypothesized to correspond to object points on the same section of smooth surface that are monotonically non-decreasing in view angle. Let $C_1, C_2, . C_n$ be the corresponding set of gradient contours determined from the reflectance map. Then, each contour Cj can be further constrained such that, for each $(r, \theta) ei\ C_{it}\ i - 2,3 \ldots n-l$

min $\{r \mid (r, \$) \ c\ C_{i-1}\} < r < $ max $\{r \mid (rj)\ f\ C_{i+1}\}$

Similarly, if $1_1 1_2 - I_n$ *$ hypothesized to correspond to a set of object points on the same section of smooth surface that are monotonically non-increasing in view angle, then each contour Cj can be further constrained such that, for each $(r, 0)\ 1\ C_n$ $i - 2.3 \ldots n-l$

min $\{r \mid (rj) \ll C^\wedge\}\ s\ r\ s$ max $\{r \mid (rj)\ 1\ C_M\}$

RULE II: CHANGES IN DIRECTION OF STEEPEST DESCENT

Let $1_1, 1_2 \ldots \wedge_n$ be a set of (closely spaced) image points hypothesized to correspond to object points on the same section of smooth surface that are monotonically non-decreasing in direction of steepest descent. As above, let $C_1, C_2, . C_{ft}$ be the corresponding set of gradient contours. Then, each contour C, can be further constrained such that, for each $(r, f)\ 1\ C_n$ $i - 2,3 \ldots n-l$

min $\{\$ \mid (r, f)\ t\ C_{M}J\ s\ \$\ s$ max $\{\# \mid (rj) \in C_{Hl}]$

Similarly, if $I_i, I_2 \ldots I_n$ is hypothesized to correspond to a set of object points on the same section of smooth surface that are monotonically non-increasing in direction of steepest descent, then each contour Cj can be further constrained such that, for each $(r\ J)\ e\ Cj,\ i - 2.3 \ldots n-l$

min $\{\$ \mid (ri)\ 1\ C_M)\ s\ \$\ s$ max $(I \mid (rj)\ 1\ Cj.,\}$

HYPOTHESIZING MONOTONICITY RELATIONS

It is now time to turn to the question of how to hypothesize monotonicity relations between selected image points. To begin with, consider the worst possible approach. For some small value of n, one might explore all possible orderings, with respect to both view angle and direction of steepest descent, of selected (closely spaced) image points $I_1, I_2 \ldots, I_n$. The hope would be that only a small fraction of those orderings would have

admissable interpretations (ie. interpretations that included at least one gradient point for each image point). The constraints imposed by each local interpretation would propagate to neighboring sets of selected image points to provide further mutual constraint. Again, the hope would be that propagation of local constraint would converge to a correct global interpretation while "incorrect" propagations would (quickly) die out.

Consider a second possible approach. Suppose a particular surface interpretation is forced onto the data. Such an interpretation would provide a framework to (partially) order selected (closely spaced) image points with respect to changes in both view angle and direction of steepest descent. Instead of allowing all possible orderings to compete, this second approach pursues a particular interpretation. Again, the hope would be that propagation of local constraint would converge to a single global interpretation that represents a simple distortion of the particular interpretation being forced. (Here, simple distortion implies any surface that preserves the assumed monotonicity relations concerning changes to view angle and changes to direction of steepest descent.)

The method actually implemented corresponds to this second approach. The program has a small set of interpretations it is willing to pursue. Some are quite rigid, others are quite flexible. In the next section, a specific example is presented. For now, a brief analysis is given to show of how convexity can be used to hypothesize monotonicity relations between (closely spaced) image points.

By partially differentiating the basic equation $I(x,y) - R(p,q)$ with respect to X and Y we obtain two equations which can be written as the single matrix equation:

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} = \begin{bmatrix} p_x & q_x \\ p_y & q_y \end{bmatrix} \begin{bmatrix} R_p \\ R_q \end{bmatrix}$$

(subscripts denote partial differentiation). Similarly, the two first-order equations $dp - p_x dx + p_y dy$ and $dq - q_K dx + q_y dy$ can be written as the single matrix equation:

$$\begin{bmatrix} dp \\ dq \end{bmatrix} = \begin{bmatrix} p_x & p_y \\ q_x & q_y \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Recalling the original definitions of $p = \partial f / \partial x$ and $q = \partial f / \partial y$, observe that $p_y = q_x$ (for smooth surfaces, the order of differentiation can be interchanged). Thus, one can define a matrix H by:

$$H = \begin{bmatrix} p_x & q_x \\ p_y & q_y \end{bmatrix} = \begin{bmatrix} p_x & p_y \\ q_x & q_y \end{bmatrix}$$

H is the standard Hessian matrix (which captures the notion of surface curvature). The first observation to be made is that smoothness guarantees that H is symmetric. Thus, it is reasonable to ask under what conditions is H positive semidefinite (A real symmetric n x n matrix A is positive semidefinite if and only if $x'Ax > 0$. for all vectors x in R^n.) Now, H is positive semidefinite if and only if $z \cdot f(x,y)$ is convex. This result is used to show how convexity adds constraint. Suppose $z - f(x,y)$ is convex. Thus, H is positive semidefinite. Multiplying the two matrix equations on the left by (non-zero) $[R_p\ R_q]$ and (non-zero) $[dx\ dy]$ respectively, gives the two inequalities:

$$I_x R_p + I_y R_q > 0$$
$$dp\ dx + dq\ dy > 0$$

This first inequality can be viewed as an additional *a priori* constraint on the contour in gradient space of possible solutions to the basic equation $I(x,y) - R(p,q)$. The tangent vector $[R_p.R_q]$ to the contour of constant reflectance at any point (p,q) hypothesized to be a solution to the basic equation $I(x,y) = R(p,q)$ must have a non-negative component in the direction of the tangent vector $[I_x,I_y]$ to the contour of constant intensity at (x,y).

The second inequality can be viewed as an additional constraint on the possible movement $[dp,dq]$ in gradient space corresponding to a movement $[dx,dy]$ in the image. As above, the vector $[dp,dq]$ must have a non-negative component in the direction $[dx,dy]$ Thus, by choosing $[dx.dy]$ appropriately, it is possible to guarantee either the sign of the change to the view angle or the sign of the change to the direction of steepest descent.

ACHIEVING GLOBAL CONSTRAINT

Regardless of what mechanism is used to hypothesize (local) monotonicity relations between points in image space, it is stilt necessary to embed that mechanism in a computation to achieve global constraint. The implementation approach taken here is admittedly ad hoc. The program selects nine points in a simple 3×3 square pattern as its basic set of (closely spaced) image points. This pattern serves as the set $I_1, I_2....I_n$ for applying the local constraint criteria (Rules I and II). First, however, the set $I_1.I_2..,I_n$ is passed to the chosen hypothesizing routines to be (partially) ordered with respect to view angle and direction of steepest descent. The reflectance map $R(p,q)$ is then used to determine the initial contour of possible gradient space locations for each point I_i. Rules I and II are interatively applied to these contours until no further mutual constraint is provided.

The above describes the basic application of local constraint to each 3×3 template. The selection of successive 3×3 square patterns is allowed to overlap. Thus, each image point I, will eventually belong to nine templates. Each time a particular image point I, is further constrained by the application of local constraint to a template of which it is a member, each of its eight other templates is marked for reconsideration. Before moving on to a previously unconsidered template local constraint is applied recursively to each marked template, with additional marking added as required, until no marked templates remain to be reconsidered. Each time an image point Ii is considered, any additional constraint on the gradient space contour of possible solutions to the basic equation $I(x,y) - R(p,q)$ propagates through this local filtering mechanism to all other image points under consideration.

The next issue to arise is the question of how to terminate the growth of templates. Currently, the program terminates only when some image point under consideration has no admissable gradient space interpretation, in which case the "forced" interpretation is deemed to have failed, or when all boundary points to existing templates have view angle greater than some preassigned value, in which case the surface Is deemed to be too oblique for further expansion. Clearly, these termination conditions are not adequate for a general surface analysis routine. The question of termination also raises deeper issues. How important are boundaries to surface interpretation? How can a complex surface be segmented into simpler (convex/concave) subsections? How does one handle the inherent indentation/protrusion ambiguity in trying to piece together surfaces? These issues must be dealt with. But, for now, let us consider a specific example in order to illustrate the method in operation.

AN EXAMPLE

Consider the simple example of a "Lambertian" sphere illuminated by a single distant light source. The intensity space to gradient space correspondence will be derived analytically and then it will be used as a basis for Judging the performance of the computation.

The first task is to generate a reflectance map. To do this, a reflectivity function must be specified. Now, the term Lambertian refers to a phenomenological model of a perfect diffuse reflector such that the surface appears equally bright from all viewing directions. For such a surface, the reflectivity depends only on the forshortening effect of the varying angle of incidence. In particular, the reflectance function $ø(i.e.g)$ is given by:

$$ø(i,e,g) = \cos(i)$$

This reflectivity function is transformed Into a reflectance map by simply recalling the expression derived earlier for $\cos(i)$. The reflectance map is thus given by:

$$R(p,q) = \frac{1 + pp_0 + qq_0}{\sqrt{1 + p_0^2 + q_0^2}\ \sqrt{1 + p^2 + q^2}}$$

The second task is to determine the local surface orientation of each image point. In the example, this result is particularly easy to obtain. Let the object sphere be centered at the origin and have radius r. Thus, the equation of the sphere is:

$$x^2 + y^2 + z^2 + r^2$$

Elementary calculus will verify that the vector (x,y,z) defines an (outward) normal at each point (x,y,z) on the surface. The appropriate gradient is obtained by rewriting this normal as $(-x/z,-y/z,-I)$. Now, for each (x,y), there are actually two possible z values to be considered. Note, however, that in this example the hemi-sphere actually in view corresponds to negative values of z (recall figure 2).

The parameters of the image forming system can be factored out by assuming that the image intensity has been normalized to correspond directly to reflectivity. In this case, the intensity recorded at each image point (x,y) is equal to the value of the reflectance map at the corresponding (p,q). Thus, the equation for image intensity is:

$$I(x,y) = R(-x/z,-y/z)\ \text{where}\ z = -\sqrt{r^2 - x^2 - y^2}$$

Finally, the mechanism by which monotonicity relations were hypothesized for this example must be specified. One additional assumption was used. The algorithm assumes that it knows at least one image point corresponding to an object point oriented directly facing the viewer. Now, let such a point define a *pseudo-origin* in image space. Since the view angle is

assumed to be zero at the pseudo-origin, the only possible interpretation is that, in a particular direction, the view angle is locally non-decreasing with increasing Image distance from the pseudo-origin. In general, one can not hope to assert any local monotoniclty relation on direction of steepest descent based on angular position about the pseudo-origin. If, however, the surface is known to be convex, the direction of steepest descent is (locally) non-decreasing with increasing angular position about the pseudo-origin. For the example, the set of image points $l_1l_2,...,l_n$ was ordered in view angle according to their distance in image space from the pseudo-origin and ordered in direction of steepest descent according to their angular position about the pseudo-origin. Applied together, these hypothesis rules are equivalent to the strong assumption that the surface in question is a convex solid of revolution.

For the example, $p_s = 0.7$, q_s - 0.3 and R - 60. A 128 x 128 test image was considered. The algorithm was applied to this image using 3 x 3 square templates sampled at an image spacing of 5 points (in both X and Y). The pseudo-origin was defined as x - 0 and y = 0. The results are first presented as a pair of figures. Figure 5 shows the reflectance map R(p,q) - cos(i) drawn as a series of contours (spaced 0.1 units apart). Superimposed are crosses marking the known gradient points (p,q) corresponding to the image points (x,y) sampled from the image. All gradient points to the left of the contour R(p,q) - 0.0 correspond to surface points oriented more than 90° away from the direction of incident illumination and hence to the self-shadowed region of the sphere. Figure 6 shows the restricted subsection of contour determined for each sampled image point. Again, crosses are superimposed to illustrate how well the algorithm has performed. The crosses mark the correct gradient points, determined analytically, while the corresponding subsection of contour marks how well the algorithm has isolated those points.
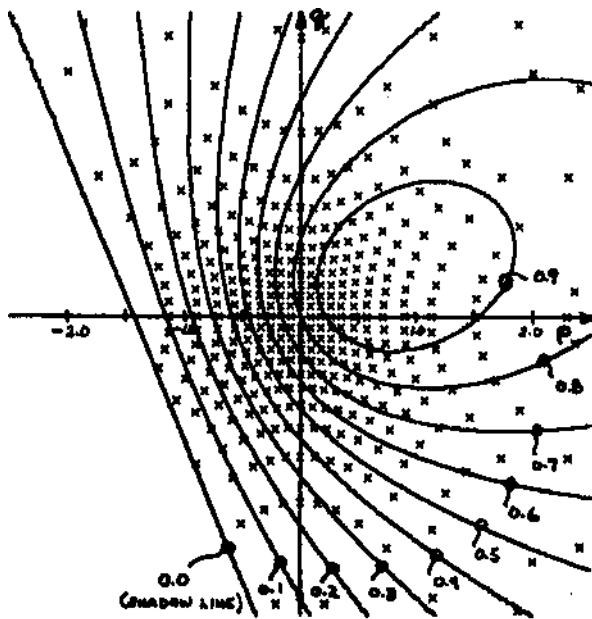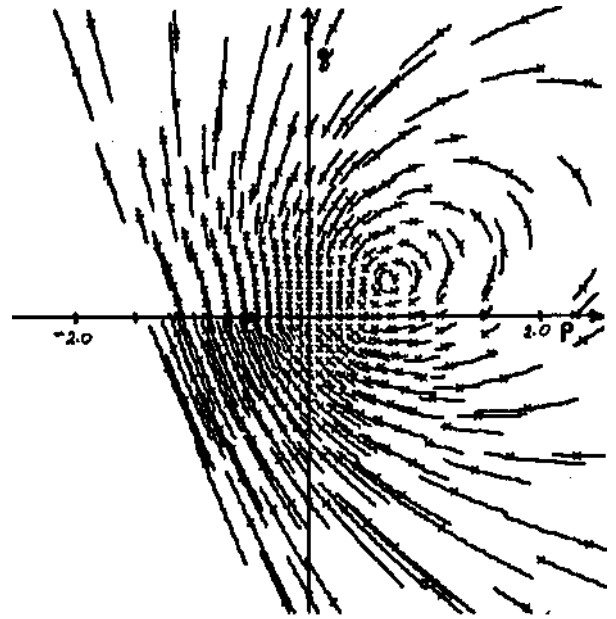

Figure 6

DISCUSSION

The ultimate criteria for judging the performance of this method is, of course, to discover whether it can be used to solve interesting problems. For now, however, the discussion is restricted to some simple statistics and some qualitative observations.

How well has the algorithm performed on this example? To answer this question, the freedom remaining at each image point l_i can be examined. Since each point in gradient space defines an orientation, this freedom can be characterized by the angular spread remaining in the corresponding gradient contour C_r This freedom has been measured in two ways. First, define the angular spread in local surface orientation at image point l_i by:

$$\max \{\theta \mid \cos(\theta) - \frac{(p_1,q_1,-1)(p_2,q_2,-1)}{|(p_1,q_1,-1)||(p_2,q_2,-1)|} \quad (p_1,q_1),(p_2,q_2) \in C_i\}$$

Second, define the angular spread in view angle e at image point l_i:

$$\max \{(\theta_1 - \theta_2) \mid \text{where } j - 1,2; (p_j,q_j) \in C_i; \tan(\theta_j) - \sqrt{p_j^2 + q_j^2}\}$$

(View angle is useful because it determines the degree of surface forshortening at each image point. Knowing the view angle, one can calculate the area of surface equivalent to a given area of image.) To tabulate the results, the sample points were split into two classes. First, consider all sample points within 45° view angle (ie. lying within gradient space circle $p^2 + q^2 = l$). Second, consider all sample points within 60° view angle (ie. lying within gradient space circle $p^2 + q^2 - 3$). Table 1 summarizes the results:

Figure 5

	Points Within 45° View Angle	Points Within 60° View Angle
Local Surface Orientation:		
Mean Angular Spread	8.9°	9.9°
Standard Deviation	6.1°	7.0°
Best Case	0.5°	0.5°
Worst Case	56.3°	41.7°
View Angle:		
Mean Angular Spread	4.7°	5.0°
Standard Deviation	2.4°	2.5°
Best Case	0.5°	0.5°
Worst Case	10.9°	11.9°

Table I

Note that these measures refer to total angular freedom. If a choice algorithm is adopted which selects the "correct" answer to be at the midpoint of the angular spread, this choice is guaranteed to be no more than half the angular spread in error. Thus, the upper right portion of table 1 can be interpreted as follows:

On the average, by sampling at an image spacing of 5 points in X and Y, the algorithm was able to position image points, corresponding to surface points less than 60° in view angle, to within 5° of their true orientation in space. The standard deviation of this measure over all such points was 3.5° while the worst case point was located to within 2½° of its true orientation in space.

The performance of the algorithm depends critically on two factors: the ability to hypothesize monotonicity relations between selected image points and the topography of the reflectance map. The discussion of convexity gives some indication of how one can use *a priori* assumptions about surface geometry to specify monotonicity relations. In any event, given a particular hypothesis mechanism, the algorithm generates a constrained interpretation consistent with that hypothesis mechanism (or demonstrates that no such consistent interpretation is possible).

The topography of the reflectance map is determined by two factors: the local photometry of the surface being viewed and the light source, object surface and viewer geometry. In general, one would not expect to have much control over the local photometry of the surfaces being viewed. In principle, however, the user is free to vary the light source and viewer geometry to achieve optimal results with the algorithm. This approach has proven particularly useful in extending these techniques to the problem of inspecting for surface defects in metal castings (where it is feasible to construct inspection stations with independently controllable light sources <Woodham 1977>).

CONCLUSION

The notions of local surface orientation, gradient space and reflectance maps represent important conceptual tools for image analysis. The beauty of the gradient space formulation is that it allows one to formulate physical constraints on the object surface as simple geometric constraints on the gradient space contour of possible solutions to the basic image forming equation $I(x,y) = R(p,q)$. The beauty of the "cooperative" algorithm is that it provides a simple mechanism for propagating these geometric constraints.

REFERENCES

Horn, Berthold K. P. (1970), "Shape From Shading: A Method for Obtaining the Shape of a Smooth Opaque Object from One View", MAC TR-79, M.IT. Project MAC, November 1970.
(See also: Horn, Berthold K. P., "Obtaining Shape from Shading Information", in *The Psychology of Computer Vision*, P. H. Winston (editor), McGraw-Hill, 1975, pp 115-155.)

Horn, Berthold K. P. (1975). "Image Intensity Understanding", AI-MEMO 335, AI Laboratory, MIT, August 1975.
(See also: Horn, Berthold K. P., "Understanding Image Intensities", in Artificial Intelligence. Vol 8, 1977 <in publication>)

Huffman, David A. (1971), "Impossible Objects as Nonsense Sentences", in *Machine Intelligence* 6, R. Meltzer and D Michie (editors), Edinburgh University Press, pp 295-323.

Huffman, David A. (1975), "Curvature and Creases: A Primer on Paper", Proceedings of Conference on Computer Graphics, Pattern Recognition and Data Structures. May 1975, pp 360-370.

Mackworth. Alan K. (1973), "Interpreting Pictures of Polyhedral Scenes", in Artificial Intelligence. Vol 4,1973 pp 121-137.

Marr, David *8c* Poggio, Tomaso (1976), "Cooperative Computation of Stereo Disparity", AI-MEMO 364, AI Laboratory, MIT, June 1976.

Wood ham, Robert J. (1977), "Reflectance Map Techniques for Analyzing Surface Defects in Metal Castings", Ph.D. Thesis, AI Laboratory, M.I.T., <in preparation>