# An Inconsistency Tolerant Model for Belief Representation and Belief Revision

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## Abstract

We propose a new model for representing and revising belief structures, which relies on a notion of partial language splitting and tolerates some amount of inconsistency while retaining classical logic. The model preserves an agent's ability to answer queries in a coherent way using Belnap's four-valued logic. Axioms analogous to the AGM axioms hold for this new model. The distinction between implicit and explicit beliefs is represented and psychologically plausible, computationally tractable procedures for query answering and belief base revision are obtained.

## 1 Introduction

This paper is motivated by several concerns in the area of belief representation and belief revision.

Minimal Change: When a belief structure is revised in the face of new information, we would like the new belief structure to be as similar to the old one as possible. This requirement goes back to Quine and has been repeatedly emphasized by Gardenfors as the *preservation criterion* (cf. [Gardenfors, 1988].)

Computational Tractability: Since we are seeking to represent the beliefs of real people, and since the satisfiability problem is NP-complete, even at the propositional level we need to be aware of the computational limitations of real people or even real computers and try, if possible, to take them into account in our belief representation models.

Explicit vs Implicit Beliefs: There are beliefs which are explicit - they may be actually asserted or at least agreed to. Other beliefs are implicit in the sense that an agent can be made to agree to them after a discussion. If the set of explicit beliefs is consistent, then we can define the set of implicit beliefs as just their logical consequences. However, this becomes implausible if - unknown to the agent - the explicit beliefs are inconsistent.

Inconsistency Tolerance: The beliefs of real individuals are usually inconsistent from a global point of view. This is also likely to be the case with large data bases. So we need to model how an agent deals with possible inconsistencies in its belief base.

In this paper we propose a model which comes to grips with all four issues. Our work is related to the *splitting languages* framework of [Parikh, 1996]. That paper showed how an agent's beliefs can be (uniquely) subdivided into sub-areas and how this division can be used in belief revision. However, it did not address the issue of explicit vs implicit beliefs, not did it tackle the question of inconsistent explicit beliefs. In the current paper we do both.

We take the celebrated AGM axioms [Alchourron *et. al*, 1985] as our starting point, but modify them somewhat and represent an agent's beliefs, not by a *theory* as is usual, but by what we call a B-structure, a notion which generalizes the notion of a theory. We begin by giving some notation and re-stating the AGM axioms.

Notation: In the following, $L$ is a finite propositional language. (However, most results continue to hold for a countably infinite first order language without equality.) The constants *true*, *false* are in $L$. The letter $L$ stands both for a set of propositional symbols and for the formulae generated by that set. It will be clear from the context which is meant. $A \Leftrightarrow B$ means that $A \leftrightarrow B$ is a tautology, i.e. true under all truth assignments. Similarly, $A \Rightarrow B$ means that $A$ -4 $B$ is a tautology. If $X$ is a set of formulae then $Cn(X)$ is the logical closure of $X$. Thus $X$ is a theory iff $X = Cn(X)$. Letters $T, T'$ denote theories. $T * A$ *is* the revision of $T$ by $A$, and finally, $T \dot{+} A$ *is* $Cn(T \cup \{A\})$, .e. the result of a brute addition of $A$ to $T$ (followed by logical closure) without considering the need for consistency. Slightly departing from the AGM usage, we always assume that both $T$ and $A$ are *individually* consistent, only their union might not be.

AGM have proposed the following widely accepted axioms for the revision operator *. We omit their axioms 7,8 having to do with conjunction. See [Parikh 1996] for a discussion of those two axioms and why they are less plausible than the other six.

1. $T * A$ is a theory.
2. $A \in T*A$

3. If $A \Leftrightarrow B$, then $T * A = T * B$.

4. $T * A \subseteq T \dot{+} A$

5. If $A$ is consistent with $T$, i.e. it is not the case that $\neg A \in T$, then $T * A = T \dot{+} A$.

6. $T * A$ is consistent if $A$ is.

However, the AGM axioms are consistent with the *trivial update,* which is defined by:

*If A is consistent with* T, *then* $T * A = T \dot{+} A$, *otherwise* T * A $= Cn(A)$.
Thus in case *A* is inconsistent with T, under this update, all information in T is simply discarded, which is not in the spirit of the minimal change requirement. The work in [Parikh, 1996], discussed below, was motivated in part by a desire to block the trivial update.

We now give some details of the LS model on which our new notion of a B-structure is based.

## 2 The LS Model

[Parikh, 1996] introduced a language splitting model which addressed *two* of the four issues we mentioned at the beginning, namely minimal change and computational tractability. We review his results below before going on to describe our new i?-structures model which addresses inconsistency tolerance as well.

The intuition behind the LS model is that our beliefs are subdivided into *disjoint* areas which do not affect each other. This separation allows us to revise our beliefs *locally* and to minimize the amount of computation we have to do when we receive a new piece of information, both in checking whether it is consistent with the old set of beliefs and in revising our beliefs in view of the new information.

Definition LSI: 1) Suppose T is a theory in the language *L* and let $L_1, L_2, .., L_n$ be a partition of *L.* Then $L_1, ..., L_n$ *split* the theory T if there exist formulae $A_i \in L_i$ such that $T = Cn(A_1, ..., A_n)$. We may also say that $\{L_1, ..., L_n\}$ is a T-splitting.

2) If $L_1 \subset L$ then we say that T is *confined* to *L* if $T = Cn(T \cap L_1)$.

In part 1 of the definition, we can think of T as being generated by the various Ti in languages Li. Then the condition implies that T contains no "cross-talk" between *Li* and *Li* for distinct $i, j$. Part 2 of the definition says that T knows nothing about the part $L - L_1$ of $L$.

Example: Let $T$ be $Cn(R \vee C, B \rightarrow S)$. Then $L(T) = \{R, C, B, S\}$ can be split into $\{R, C\}$ and $\{B, S\}$ with corresponding theories $Cn(R \vee C)$ and $Cn(B \rightarrow S)$. Further splitting is now impossible. Suppose I believe that it will be either raining or clear tomorrow and that if I eat bananas then I will get a stomach ache. These beliefs can then be separated.

Lemma LSI: [Parikh, 1996] Given a theory T in the language L, there is a unique *finest* T-splitting of L, i.e. one which refines *every other* T-splitting.

Lemma 1 says that there is a unique way to think of T as being composed of disjoint information about certain subject matters.

Lemma LS2: Given a formula A, there is a *smallest* language $L'$ in which *A* can be expressed, i.e., there is $L' \subseteq L$ and a formula $B \in L'$ with *A* ⇔B, and for all $L''$ and *B''* such that $B'' \in L''$ and *A* ⇔ B'', $L' \subseteq L''$.

Although *A* is equivalent to many different formulas in different languages, lemma 2 tells us that nonetheless, the question, "What is *A* actually *aboutV* can be uniquely answered by providing a smallest language in which (a formula equivalent to) *A* can be stated. For example if *A* is the formula $P \wedge (Q \vee \neg Q)$, then *A* ⇔ *P* and hence $L(A) = \{P\}$. The language *V* will be referred to as $L(A)$. If $A \Leftrightarrow B$, then $L(A) = L(B)$.

The general rationale for axiom P, below, is that if we have information about two or more subject matters which, as far as we know, are unrelated (are split) then when we receive information about *one* of them, we should only update our information in that subject and leave the rest of our beliefs unchanged.

Axiom P: If $T = Cn(A, B)$ where *A, B* are in $L_1, L_2$ respectively and C is in *Li*, then $T * C = Cn(A) *' C \dot{+} B$, where $*'$ is a (local) update operator for *Li.*

It follows from axiom P that if T is split between $L_1$ and L2, A, *B* are in $L_1$ and $L_2$ respectively, then T * *A* * $B = T * B * A$. While axiom P looks binary, it implies the n-ary cause. Axiom P is independent of the AGM axioms. We now get

Theorem: [Parikh, 1996] There is an update procedure which satisfies the six AGM axioms and axiom P. Moreover, the trivial update procedure does not satisfy axiom P.

### 2.1 Comments on the LS Model

The LS model insists that the division of our beliefs uses *disjoint* sub-languages. But then global inconsistencies cannot be represented. Suppose an agent believes theories *Ti* in languages *Li* and the $L_i$ are mutually disjoint. Then if the *Ti* are individually consistent, they are also jointly consistent. Thus the LS model cannot explain how an agent can be locally logically omniscient i.e. derive logical consequences *within* each *Li* but still fail to be globally consistent.

We would like to model a more realistic agent, i.e. one who is locally logically omniscient, who does not believe any outright inconsistencies, but whose global belief structure might well be inconsistent without his being aware of it. This means that the languages *Li* must be allowed to overlap on occasion. The B-structure model which we introduce below does just that and shows how the logic is handled using Belnap's four truth values, $\{\perp, true, false, \top\}$ [Belnap, 1977].

We feel that this relaxation is psychologically realistic. E.g. we may imagine that Clinton's problems with Lewinsky might be connected with the bombing of Iraq, which may affect the price of oil and which in turn may affect our airfare to a conference. But it is unrealistic to think that we might reason with all these beliefs together at one time. Rather they will be separated, though only partially; the connections may be noticed on occasion, if there is an article in the newspaper connecting a price hike in airfares with the bombing of Iraq.

The sort of model we are considering will also apply to group reasoning. Each agent will have its own language and be individually consistent in it. But the languages of different agents may overlap and a particular question *A* may be resolved by consulting those agents whose languages overlap with the language of *A*. Options A and B discussed below will show how this is handled at a technical level. Even a single agent may be thought of as consisting of a collective entity, see [Minsky, 1986].

## 3 The B-structure model

Definition 1: a belief structure *B* on *L* is a set $\{(L_1, T_1), ..., (L_n, T_n)\}$ such that $L = \bigcup L_i : i \leq n$, and each $T_i$ is a theory in $L_i$. (We can imagine that $T_i = Cn(\Gamma_i)$ where the *Ti* are the explicit beliefs of some agent in language $L_i$. If $n = 1$, then *B* will be just a theory.)

As in the case of belief bases, the object of revision is a set of beliefs that is not necessarily logically closed.

While we axe not requiring the Li to be disjoint, we do want to retain *some* amount of disjointness. The following notion does that.

Definition 2: Given a finite propositional language *L* and languages $\{L_1 \ldots L_n\}$ such that $L = L_1 \cup \ldots \cup L_n$, $\{L_1 \ldots L_n\}$ is a *k-partition* of *L* if any propositional symbol *q* occurs in at most *k* of the *Li*.

The presence of such an overlap of symbols may be thought of as "cross-talk" amongst the Ti; beliefs in some of the *Ti* may be relevant or related to beliefs in others. A k-partition with a smaller *k* will be *more* disjoint than one with a larger *k* and indicates a finer partitioning of her beliefs by the agent. In particular, a normal partition is just a 1-partition. A k-partition is *a fortiori* a k + 1 - partition and hence, for instance, a maximally fine 2-partition will usually be finer than the finest 1-partition whose existence is guaranteed by lemma L S I. Tolerating some overlap makes it easier to organize one's beliefs in smaller chunks. For example, if *T* is axiomatized by the formula $(P \vee Q) \wedge (\neg Q \vee R)$ then $L(T) = \{P, Q, R\}$ has only a trivial 1-partition. However, there is a 2-partition into sets $\{P, Q\}$ and $\{Q, R\}$ with the two theories being generated b y $P \vee Q$ a $\neg Q \vee R$ respectively.

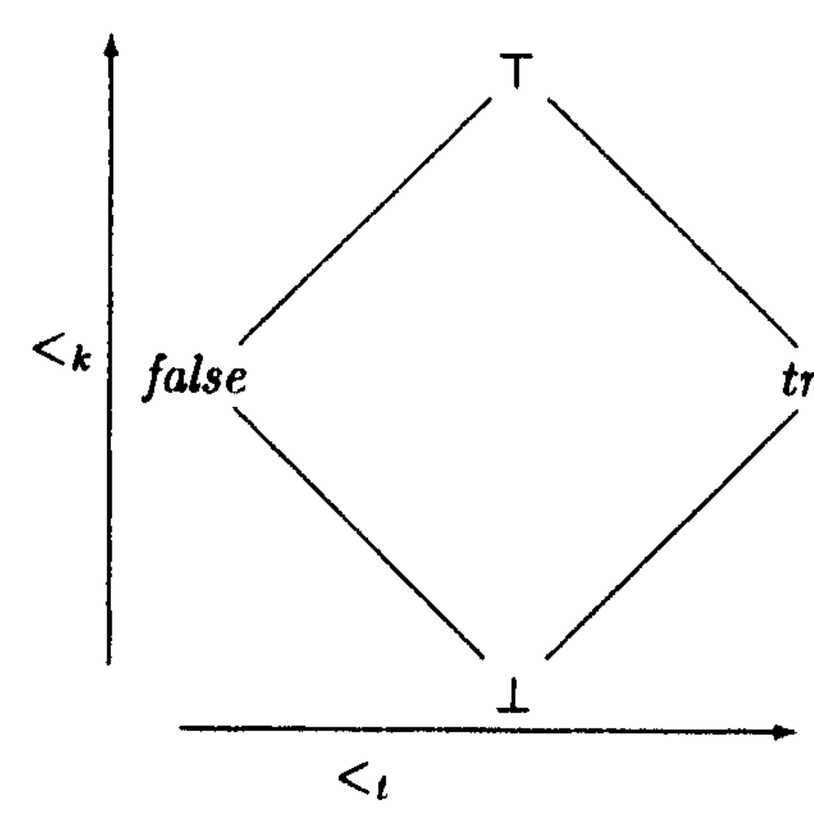
Definition 3: *B* as in definition 1 is m-consistent if any m of the *Ti* are jointly consistent.

We do not require that the whole collection be consistent, but the *amount* of inconsistency is limited by this requirement. The larger the m, the stronger the requirement imposed by m-consistency. Thus, for instance, *B* is 1-consistent iff the *Ti* are *individually* consistent, but $\mathcal{B} = \{(L_1, T_1), ..., (L_n, T_n)\}$ is n-consistent iff $\bigcup T_i$ is consistent.

A very natural example of such a collection with the *bald person paradox.* Suppose that a person with 10,000 hairs is not bald but that a person with 0 hairs *is* bald. Also, suppose that a non-bald person cannot become bald through the removal of just one hair. This gives us the axioms $B(0)$, *B(n)* -> $B(n+1)$, for n = 0 to $n =$ 9,999, and $\neg B$(10,000). These 10,002 axioms are inconsistent, but any 10,001 of them are consistent. This gives us a 10,001-consistent, inconsistent collection of axioms, all of which are accepted, and used.

For a more practical example, consider an airline which has 100 seats on a flight, and accepts 120 reservations. If $C_i$ says that customer number *i* has a seat, then the airline is asserting Ci for $1 \leq i \leq$ 120, and also that at most 100 of the $C_i$ hold. This gives a B-structure *B* with 121 languages: $\{C_1\}, ...\{C_{120}\}, \{C_1, ..., C_{120}\}$ and theories $T_1, ..., T_{120}, T_{121}$. Here, f o $i \leq$ 2 0 , $T_i$ is generated by $C_i$ and T121 is generated by a gigantic formula which says that at most 100 of the $C_i$ are true. This B-structure is 100-consistent, but inconsistent. But its inconsistency will not matter if fewer than 100 of the customers claim their seats.

To represent how an agent *uses* his B-structure and revises it, we make use of the four valued semantics [Belnap, 1977], [Fitting, 1989], where the four truth values $\perp$, *true, false* $\top$ stand for, respectively, *no information, true, false,* and *over-defined* (or inconsistent). The first three truth values are intuitively obvious. The last, *over-defined,* can arise when we are given contradictory information by two people both of whom are trustworthy. In this four valued bi-lattice there are two orderings, the *knowledge* ordering under which $\perp <_k$ *true, false* $<_k \top$ and the *truth* ordering under which *false* $<_t \perp, \top <_k$ *true, true* is truer than $\top$ which is truer than *false.* But $\perp, \top$ are incomparable under the truth ordering. Similarly, *true, false* are incomparable under the knowledge ordering. The truth values with their orderings are displayed in the double Hasse diagram below.




Now we consider how a B-structure responds to queries and how it is revised.

## 4 Answering queries

Given a formula *A* and B-structure *B* the query *"A?"* is answered by giving the value $v_B(A)$ - abbreviated as

$v(A)$ - which is defined as follows: Let $\Gamma_A = \bigcup T_i : L_i \cap L(A) \neq \emptyset$, where $L(A)$ is the *smallest language* in which $A$ can be expressed (cf. Lemma LS2).

If $\Gamma_A$ is consistent, then

if $\Gamma_A \vdash A$, then $v(A) = true$

if $\Gamma_A \vdash \neg A$, then $v(A) = false$, and

$v(A) = \bot$ otherwise.

If $\Gamma_A$ is inconsistent, then $v(A) = \top$.

Intuitively we see *which* of the theories $T_i$ *could be* relevant to $A$ and put them together to get $\Gamma_A$. $\Gamma_A$ is used to answer questions about $A$ and the rest of the theories $T_i$ are *not* brought into play.

Controlling Inconsistency: Our approach, while seeking to handle this problem, is distinct from the usual paraconsistent approach in that we do allow full use of classical logic, albeit locally.

Limited consistency: Suppose that $A$ has at most $l$ distinct symbols, the $L_i$ are a k-partition, and $m \geq k \times l$. Then $\Gamma_A$ will be a union of at most m of the theories $T_i$. If the collection $\{T_i : i \leq n\}$ is m-consistent, then $\Gamma_A$ will be consistent and *exactly one* of the first three values will be given. In other words, an agent whose B-structure is *fairly consistent,* and who is responding to a *short* query, will *always* give a consistent response.

We say that the person with belief base $B$ *implicitly believes* $A$ if $v(A) = true$. If so, it is true that $A$ follows from her explicit beliefs but the converse does not hold in general. If the explicit beliefs are jointly inconsistent, then their consequences will include all formulae, but the agent may still have implicit beliefs which are a reasonable set. The implicit beliefs will always include the explicit ones and will be *locally* closed under logical rules.

What about adjunction? If an agent implicitly believes $A$ and B, i.e. if the answers to $A$ and B are both *true* and $L(A \wedge B)$ has no more than $l$ symbols, then the answer to $A \wedge B$ will also be *true.* However, cases can arise where the longer formula $A \wedge B$ forces the agent to simultaneously consider several of his beliefs and the underlying inconsistency is d e t Let $L_1 = \{P, R\}$, $L_2 = \{Q, R\}$, $T_1 = Cn(P,R)$ and $T_2 = Cn(Q, \neg R)$ Then the answers to the queries '$P$?' and '$Q$?' will both be *yes,* but to '$P \wedge Q$?' it will be $\top$.

## 5 Base Refinement

A person who has explicit beliefs $T$ in language $L$ may organize these beliefs in smaller or larger chunks. Clearly if r is inconsistent, then it is un-workable to organize them in a single chunk. Moreover, organizing in larger chunks can make the computational problems harder. On the other hand it does have the advantage that more implicit beliefs can be derived in the sense described below.

Definition 4: A B-structure $B$ *refines* another, $B'$ if (i) every language Li is a subset of another $L'_j$, (ii) every $L'_j = \bigcup L_i | L_i \subseteq L'_j$, and (iii) every $T'_j = Cn(\bigcup T_i | L_i \subseteq L'_j)$.

Example: Suppose that the B-structure $B$ has the three languages $\{P,Q\}, \{Q,R\}, \{R,S\}$ and theories $T_i : i \leq 3$ generated by the formulae $P \rightarrow Q$, $\neg Q \wedge R$ and $R \wedge$ 5respectively, the B-structure $B'$ has the two languages $\{P,Q,R\}, \{R,S\}$. and t h e es $T'_j : j \leq 2$ e r a t e d by the formulae $\neg P \wedge \neg Q \wedge R$ and $R \wedge S$ respectively. Clearly $B$ refines $B'$. Now to the query '$P$?', $B$ will give the answer $\bot$ and $B'$ will give the answer 'yes' or *true.*

Theorem 1: Let $B$ refine $B'$ and $A$ be a formula. Then $v(A) \leq_k v'(A)$ where $v(A), v'(A)$ respectively stand for $V_B(A)$ and $V_{B'}(A)$.

*Proof:* Given a formula $A$ and B-structure $B$ the query A?" is answered by giving the value $v_B(A) = v(A)$ which is defined using $\Gamma_A = \bigcup T_i : L_i \cap L(A) \neq \emptyset$, where $L(A)$ is the language of $A$. Similarly for $B'$

Now we note that if $L_i \subseteq L'_j$ and $L(A)$ intersects $L_i$ then it also intersects $L'_j$. Hence $\Gamma_A \subseteq \Gamma'_A$. This immediately yields $v(A) \leq_k v'(A)$. □

In other words, $B'$ always yields more information than $B$. The downside is that $B^1$ may give the inconsistent answer $\top$ to a query $A$ whereas $B$ may have given a *true* or *false* answer to it. The partitioning of information into separate languages may on occasion miss some answers which might have been obtained without such partitioning, but it is more likely to be consistent in particular queries.

## 6 B-Structure Revision

What will happen to a B-structure when an agent receives some information which overlaps two sub-languages? One possibility is that the agent accepts the consequences *within* each sub-language while still keeping them separate. If I learn that both Beijing and London had cold winters, I am not likely to merge all my other beliefs about the two cities. However, if I repeatedly receive information which overlaps two sub-languages, I may decide that the division is artificial and should be abandoned. Options A and B below correspond to these two different attitudes.

option A - the non-merging option: Assume that each of the languages $L_i$ has its own AGM style revision operator. Given a belief structure $B$ and a new input A, define, for each i, the *i-shadow* of $A$ to be the set $\{B | B \in L_i \wedge A \vdash B\}$. This will be a theory $T'_i$ in $L_i$. $T'_i$ is what $A$ has to say about the language $L_i$. Let $A_i \in L_i$ be such that $T'_i = Cn(A_i)$. Now define the theories $T''_i$ by:

$T''_i = T_i \dot{+} A_i$ if $A_i$ is consistent with $T_i$, and

$T''_i = T_i *_i A_i$ otherwise where $*_i$ is a local revision operator for $L_i$.

Then the revised B-structure $B * A$ will equal $\{(L_1, T''_1), ..., (L_n, T''_n)\}$.

In practice, if $L(A) \cap L_i$ is empty, we can just leave $T_i$ unchanged, saving computational time. We define $B \dot{+} A$ analogously, except that we use the operator $\dot{+}$ on the various $T_i$.

option B - the merging option: Given a formula

$A$ as input and a belief base B, assume without loss of generality that $A$ has been written in the *smallest language* in which it can be expressed [Parikh, 1996]. Let as before $\Gamma_A = \bigcup T_i : L_i \cap L(A) \neq \emptyset$, where $L(A)$ is the language of $A$. Let $T_A$ be $Cn(\Gamma_A)$ and $L_A = \bigcup L_i | L_i \cap L(A) \neq \emptyset$. Now replace all those languages $L_i$ such that $L_i \cap L(A) \neq \emptyset$ by the single language $\bigcup L_i$ $L_i \cap L(A) \neq \emptyset$, which is their union. At the same time, replace all the corresponding $T_i$ by the theory $T_A * A$. This new way will specialize to the procedure in [Parikh, 1996] where the languages were all assumed to be disjoint but the receipt of information resulted in joining those theories whose languages overlapped the language of the new information.

### 6.1 Properties of B-structure revision

Both options A and B are *computationally efficient.* Generally we assume that each $L_i$ has relatively small size, say under some fixed p, while the cardinality $n$ of the whole language $L = \bigcup L_i$ might be quite large. Then given a k-partition and a formula $A$ with at most $l$ distinct symbols, the query answering procedures run in time which is exponential in $l \times k \times p$, but *linear* in $n$. (For option A, the procedure is linear in n, $k$ and exponential only in $l + p$). Thus if $k \times l \times p$ - the number of atomic propositions *relevant* to $A$ - is small compared to n, as is usually the case, the computational cost will be much smaller than that of usual update procedures which are exponential in n.

The update procedures need not preserve refinements. If $B$ refines $B'$ and there is new information $A$, it may reveal an inconsistency in B', even though $B$ is unfazed. It may seem foolish not to notice one's own inconsistencies, but these are often unavoidable, as with the well known Preface and Lottery paradoxes [Kyburg, 1961], [Kyburg, 1997]. The ability to retain a measure of consistency and act on the basis of one's implicit beliefs may have much to say in its favour.

Option B results in two formally distinct subject areas merging as the result of some new information which straddles them. In real life this is likely to happen only occasionally. Suppose I have a B-structure which keeps my beliefs about Turkey, Iraq and Iran separate. If I now receive a great many pieces of information about the Kurds who are scattered over these three countries then I may simply create the new subject *Asia Minor* and give up my attempt to deal with the three countries separately.

### 6.2 Analogues of the AGM axioms

Let $B * A$ denote the revision of B-structure $B$ by the formula $A$ according to option B. The axioms B1-B5 below hold. The set theoretic notions $\in, \subseteq$ used in stating the AGM axioms are now replaced by more sophisticated generalizations which enter in with Belnap's four truth values. For option A, axiom B2 needs the caveat that $A \in L_i$ for some $i$ or, at least that $A \Leftrightarrow B_1 \wedge ... \wedge B_p$ where each $B_k$ is in some $L_i$ and $p$ is small. However *disjunctive* information which straddles two of the $L_i$ may be lost if we insist, as we do in option A, on keeping the $L_i$ separate.

Let $B$ be a belief structure, $A$ a new piece of information, * the revision operator. We then have the following axioms: (read $A \in B$ as '$A$ is an implicit belief according to $B$'.)

B1. $B * A$, the revision of $B$ by A, is a belief structure.

B2. $A \in B*A$

B3. If $A \Leftrightarrow$ B, then $B * A = B*B$

B4. $B * A \subseteq B \dot{+} A$. i.e. if $B * A$ and $B \dot{+} A$ give values $v, v'$ to $A$, then $v <_k v'$.

B5. If $A$ is consistent with B, i.e it is not the case that $v_B(A) \in \{false, \top\}$, then $B * A = B + A$

Other Issues: Issues commonly raised in belief revision literature include, for instance, the axiom of recovery, revision by conjunctions, by sets of propositions etc. Most of these properties *will* hold at the local level provided that the original local operations have them. At the non-local level when several of the Ti interact in a particular case of a belief revision, more complex patterns of behaviour will emerge. These require further investigation.

### 6.3 Review of Previous Work

We briefly discuss how other authors have addressed the issues mentioned at the outset.

Work on the minimal change model has concentrated on minimizing the number of beliefs given up during the *contraction* operation. To this end, operations such as partial meet contraction using selection functions were defined in [Alchourron *et* al, 1985]. A motivation for the choice of beliefs to be dropped is given by the notion of *epistemic entrenchment* introduced by [Gardenfors and Makinson, 1988] and refined for iterated belief change by [Nayak, 1994], [Darwiche and Pearl, 1994], [Lehmann, 1995]. [Williams, 1996] uses the concept of *maxi-adjustment* to achieve maximal inertia of information under iterated belief revision. [Georgatos, 1999] presents a generalization of entrenchment that serves as a representation of the AGM axioms. The notion of *epistemic relevance* is used for minimal contraction in [Hansson, 1992] and [Nebel, 1992].

The distinction between implicit and explicit beliefs, has been explored by the proponents of the *belief base* method such as [Fuhrmann, 1991], [Nebel, 1992], [Hansson 1991; 1992]. [Rott, 1992] combines some intuitions in showing how epistemic entrenchment orderings can be carried out for *safe contractions* for belief bases.

Belief revision for inconsistent belief bases has been studied in an alternative approach by [Brewka, 1991]. The possibility of paraconsistent belief revision is explored by [Tanaka, 1997] while [Restall and Slaney, 1995] have developed a paraconsistent semantical representation based on the revision of models approach suggested in [Grove, 1988]. The work of [Schotch and Jennings, 1980] predates the AGM approach to belief revision. They consider an approach based on giving up the adjunction rule: fro $\Gamma \vdash A$ and $\Gamma \vdash B$ conclude $\Gamma \vdash A \wedge B$. As we saw, our treatment retains this rule at the local level.

The investigation of complexity procedures in [Nebel, 1992], via a fine-grained set of complexity classes, has shown that the complexity of base revision procedures satisfying AGM postulates is that of ordinary propositional derivability. Nebel's comparison of different revision methods shows that model-based revision methods such as those of [Dalai, 1988] have a complexity which exceeds both NP and co-NP.

Conclusion: We started this paper by indicating four desiderata which a framework for answering queries and for belief revision should try to meet. The B-structures framework meets all four. In future work we intend to carry out a thorough study of this interesting new model for belief representation and revision and to implement the query answering and revision procedures.

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