Dynamical complexity of spatially embedded networks

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Living systems are embedded within physical space. While this embedding can be viewed as a restrictive constraint on structure, where the prohibitive costs of establishing or maintaining interactions over long distances mitigate against certain kinds of potential organisation, it can also be seen as an enabling factor, bringing about correlations, regularities and symmetries that can be exploited by evolution. Artificial life research on spatially embedded games, ecologies, networks, evolution, and agents has shown that projecting a well-mixed system into a low-dimensional medium and constraining interactions to be local can confer interesting properties (e.g., stability, honesty, robustness to parasites) that are otherwise absent or unstable. This paper explores the question: what is the contribution of spatial embedding to the dynamical complexity of networks.

In previous work some of us have developed a general framework for characterising the impact of spatial constraints on network topology (Barnett et al., Phys. Rev. E 76, 056115, 2007), and some of us have explored the dynamical complexity of spatially embedded artificial neural networks (Buckley & Bullock, ECAL 2007). Here we combine these two threads to discover what graph theoretic properties of networks confer high dynamical complexity, and to explore the extent to which spatial embedding tends to encourage exactly these topological properties in networks that are random in other respects.

We first return to the original formulation of the dynamical complexity measure due to Tononi, Sporns and Edelman (PNAS 91, 5033, 1994) and correct an error in a widely used approximation of this measure. This correction impacts on intuitions about the structural and functional roots of dynamical complexity. However, we are able to rescue these intuitions by re-deriving the approximation for a continuous-time dynamical system rather than the discrete dynamical system used in the original formulism. This process emphasises some key differences between the dynamics of continuous and discrete dynamical systems.

We then go on to derive and extend a graph theoretical interpretation of dynamical complexity for the corrected discrete measure and the new continuous measure. This allows us to strengthen our understanding of the relationship between properties of spatially embedded structures and high complexity. In particular, we are able concretise the notion that the structural contribution of spatial embedding to high dynamical complexity results from the introduction of cycles of connectivity at many structural scales. Furthermore, we are to able to address a misconceived equivalence between the small world property and systems of high "dynamical complexity". Specifically, we find that while systems of high dynamical complexity may possess the small world property, neither property is either necessary or sufficient for the other.