An Artificial Chemistry-based Model of Economies

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Abstract

Economies can be modelled using Artificial Chemistry approaches. In this contribution we discuss the development of such a model starting from the well-known von Neumann's technology matrices. Skills and technologies that allow the transformation of raw materials into products are introduced in a form akin to chemical reactions. The dynamic flow of materials in such a system is simulated and connected through an agent-based market mechanism that assigns value to raw materials, labour, and products. Starting from a fixed set of raw materials, energy and labor, we observe the appearance of new products, the use of consumables and the general increase in complexity of such a system. Real evolutionary dynamics including waves of innovation can be demonstrated.

Introduction

Economies are notoriously difficult to understand. Multiple approaches have been tried, yet progress has been very slow and today we are still not able to understand or predict the dynamics of an economy. much less its structural evolution.

Part of the reason is that economists are notoriously conservative in their approaches to elucidate these problems. While many disciplines have whole-heartedly embraced the new non-linear paradigms of Science, self-organization, the emergence of new function, chaos and complexity, non-equilibrium systems dynamics and evolution, economists have been skeptical and mostly seem to be concerned with equilibria, exact mathematical solutions to differential equations and systems without surprises.

Another reason is simply that economies are complex, dynamic, non-linear, and innovative. New products and companies arise constantly, grow in dominance in the marketplace, get competition, weaken - perhaps gradually - and are finally replaced by others. This unabating renewal process is one of the most fascinating, yet poorly understood aspects of economies. Schumpeter called it the "gales of creative destruction" (Schumpeter, 1939).

While there is a field of "Evolutionary Economics" (Metcalfe, 1998; Witt, 2006), it can be argued that evolution really does not play a key role in that area, since the key aspect of innovation is not appropriately modeled in Evolutionary Economics to date.

What we are going to do here is to take the evolutionary aspect of an economy seriously. For that to happen, we need the ability of our system to generate new products, new technologies and new companies. Artificial Chemistry has for a long time been proposed as a means to study constructive (innovative) aspects of systems. An artificial chemistry in its broadest definition (Dittrich *et al.*, 2001) consists of a collection of objects, transformation rules and an algorithm that drives the dynamics of their transformation. Here we will make use of an artificial chemistry to model the production system of the economy to be simulated. Objects can be goods such as raw materials (or labour and consumables), products of production processes created through rules of transformation, and technologies (which can be considered catalysts in a chemical sense).

The next section will introduce the von Neumann technology matrices and how we can use this method as a starting point for a production system. We will then introduce a simple system that will allow us to simulate an economy. It will be based on natural numbers being symbols of raw materials (prime numbers), products (products of natural numbers) and skills/technologies (in the form of indices). The fourth section will introduce the production agents, i.e. the "companies", and the marketplace, key elements of a viable economy which bring a valuation to the various goods of the production system introduced previously. The fifth section will demonstrate a run of the system, and explain how new products or technologies can be created (and observed). The 6th section will discuss typical runs watching for waves of innovation, competition for market dominance, and in general, the evolutionary dynamics.

Von Neumann Technology Matrices

Von Neumann proved the existence of a general economic equilibrium in an economic system undergoing balanced growth (von Neumann, 1946). He used matrices to model the transformation of input to output in an economic system. Before we can make use of this notation, we need to explain it in some detail.

Suppose we have m activities and n commodities in an economy. The activities can be regarded as production processes, skills or technologies and are denoted by a vector $t_1, ..., t_m$, whereas the commodities can be regarded as labour, capital, raw materials, or products, and are denoted by a vector $c_1, ..., c_n$.

We can then formulate an input matrix I to our production process, and an output matrix O. Here is an example with $m = 4$ and $n = 5$.

Table 1: Input and output matrices representing technology and the products involved

commodities	*input matrix*				*output matrix*			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
c_1	1	1	2	0	0	0	0	6
c_2	0	1	0	0	1	0	0	0
c_3	0	0	1	0	0	4	0	0
c_4	0	0	1	0	0	0	2	0
c_5	0	0	0	1	0	0	$\frac{7}{3}$	0

The columns of Table 1 can be read the following way: $c_1 \overrightarrow{t_1} c_2,\ c_1 + c_2 \overrightarrow{t_2} 4 \times c_3,\ 2 \times c_1 + c_3 + c_4 \overrightarrow{t_3} 2 \times c_4 + \frac{7}{3} \times c_5,$ and $c_5 \overrightarrow{t_4} 6 \times c_1$, that is, each of the pairs of columns in Table 1 indicates a process that transforms a combination of commodities into a set of output commodities.

The original purpose of the von Neumann Technology matrices was to be able to formally show that an equilibrium can be achieved with certain production factors appropriately chosen. It later turned out that a "balanced growth" scenario could be supported by this formalism as well in which all production processes expand at the same rate. For example, the matrices in Table 1 allow a constant growth rate of 2 when the activity vector z that denotes the number of times each of the m processes is executed per time step equals $\{6, 3, 6, 7\}$. Indeed $(O - 2I).z = 0$.

However, the notation is not restricted to equilibrium situations at all. We can make use of the same model, and include innovation by the addition of new columns (an innovation in technology) or new rows (new products appearing). Most of the time, a mixture of both would be necessary. A good outline of technology matrices is provided by Blatt (1983).

A Simple Economy based on Natural Numbers

The particular production system we are going to use here is inspired by the $\mathbb{N}$ economy using natural numbers to represent commodities (Herriot and Sawhill, 2008). The idea is to have multisets of objects, in this case natural numbers, which can be transformed into others using the operation of multiplication.

This system is indeed an Artificial Chemistry, with the reactions being the multiplication, and the numbers the equivalent to chemical species. As such, the system has some similarity with the prime number reaction system introduced in Banâtre *et al*. (1988); Banzhaf *et al*. (1996).

Just to briefly recall, an Artificial Chemistry can be represented by a triple (S, R, A), where S is the set of all possible molecules, R is a set of collision or reaction rules, and A is an algorithm describing the domain and how the rules are applied to the molecules (Dittrich *et al*., 2001). This general notation for an artificial chemistry can be applied to develop a framework for an artificial economy. In a free-market economy, the incentives for production are provided by the market. The algorithm should therefore represent a market in order to provide the rules for what is produced, when, and where. In addition, S – the set of all products – and R – the set of all production processes – need to be defined.

In the $\mathbb{N}$ economy the set S of molecules is the set P of products (which consists of commodities such as raw materials, products, capital and labour), with each product being represented by a natural number. Suppose the product set P consists of the following goods:

$$P = \{2, 3, 5, 7, 11, 13, 130, 260, 104\}$$

Here we have chosen to represent commodities by integers: prime numbers for raw materials, and products of integers for composites. This allows us to use the structure offered by the natural numbers, and products literally can be decomposed into their prime factorisation to see what raw materials are involved in making them. In this model, labour is treated explicitly as a commodity (the number `2`). Furthermore, there is a special product, which serves as money, and this is the number `3`.

Suppose we have a small product set, as above, and five production processes per time interval, as displayed in Table 2.

Table 2: Overview of the production processes during one time interval

2× 2					→	2× 5
2× 2	+	2× 5	+	2× 13	→	3× 130
3× 2	+	3× 130			→	6× 260
6× 2	+	6× 260			→	6× 104
6× 104					→	13× 2

Note that there is no production of commodity `13`, yet it is required in one of the production processes. This product is assumed to be a free good, such as sunlight. It can easily be verified that the production in Table 2 is in balance; total labour (product `2`) used equals total labour generated, and the products required for the generation of this labour are also covered, so that the required input per time interval is exactly the same as the output at the end of the time interval. Based on these production activities per unit of time, it is possible to determine a price set that will allow the system to function. In order for this to be the case, a producer needs to be able to finance the inputs for the next time interval with the sale of the current production. In other words, the price of the output should be greater than or equal to the price of the input. Assuming that activity levels for the next time step will remain the same, and since there is no surplus in the system, this gives the following set of equalities. When p_i stands for the price of product i, then

$$
\begin{aligned}
2p_{5} &= 2p_{2} \\
3p_{130} &= 2p_{2} + 2p_{5} + 2p_{13} \\
6p_{260} &= 3p_{2} + 3p_{130} \\
6p_{104} &= 6p_{2} + 6p_{260} \\
13p_{2} &= 6p_{104}
\end{aligned}
$$

When the price of one product (the numeraire good) is taken as a price unit, for example the price of labour $p_2 = 1$ and the free good $p_{13} = 0$, then this system of equalities has a unique solution, namely $p_2 = 1, p_5 = 1, p_{13} = 0, p_{130} = 4/3, p_{260} = 7/6, p_{104} = 13/6$.

The von Neumann Technology matrix for the example above is shown in Table 3.

Table 3: Input and output matrices representing technology and the products involved

products	*input matrix*					*output matrix*				
2	1	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0
5	0	$\frac{2}{3}$	0	0	0	1	0	0	0	0
13	0	$\frac{2}{3}$	0	0	0	0	0	0	0	0
130	0	0	$\frac{1}{2}$	0	0	0	1	0	0	0
260	0	0	0	1	0	0	0	1	0	0
104	0	0	0	0	$\frac{6}{13}$	0	0	0	1	0

Now that we have described the production system in some detail, let us turn our attention to what happens with all these products.

Economic Agents and their Marketplace

A production system alone is not sufficient to simulate an economy. In addition to production processes and products/raw materials, active entities like companies, here called agents, are necessary to actually produce quantities of these objects. Further, a validation process for objects/products will be required that allows to close the circle by exchanging objects between agents. This is the price setting mechanism which in our economy is realized by an auctioneer who keeps an eye on stock levels and who reports prices to agents.

Space

At the cost of labour, raw materials are extracted from the land, which has been divided into cells of equal size. As far as raw materials are concerned, the space is homogeneous. If desired, it is possible to experiment with more interesting distributions of resources.

Each cell provides a free resource, 13, which can be thought of as energy from sunlight. The distribution of free energy can be varied, but currently it is such that there is always an abundance.

On top of the land is a connection network, which represents the presence of trade links between cells. Links go both ways, so if a cell a is connected to a cell b, then b is connected to a, and all agents on a can trade with all agents on b, and vice versa. Initially the network is empty, but as agents obtain skills/technologies and require inputs for these skills, they can establish connections with providers. A wide range of rules of how to expand or contract an agent's trade network is possible. For example, an agent can randomly select one of its current providers and subsequently do a local search around the selected trade partner in an attempt to find an additional provider. Or an agent may be allowed to connect to the nearest provider. In addition, it is possible that an agent looses one of the more distant connections and subsequently attempts to find a supplier nearer by. Such rules obviously attempt to keep the social network consisting of trading partners compact. The rules according to which agents behave are described in the next section.

Agents

The grid space is home to a number of economic agents. They all have a fixed location and an identity number, so that we can distinguish different agents on one location (cell). Furthermore, agents possess assets: resources, other products present in the economy and skills. All assets are stored in a list. Therefore, an agent can be represented by

$$\{\{x, y, z\}, \{c_1, \ldots, c_n\}, \{t_1, \ldots, t_m\}\}$$

where x, y is the location, z is the identity number for that agent, $c_1, \ldots, c_n$ lists the possession of n commodities, and $t_1, \ldots, t_m$ is a boolean list to specify which of the m technologies the agent possesses for some $m, n \in \mathbb{N}$. Different agents have different skills.

Having a technology should be regarded as having the skill or knowledge to perform a certain transformation when you have the resources to do so. For actions that require capital, in addition to having the skill, the agent must also possess the appropriate capital goods in order to execute that particular action. Capital goods are a subset of the product list P. In particular, they are products, analogous to catalysts, that are necessary for the production of other products, but which are not used up in the process – that is, they are recovered (less a fraction representing depreciation) at the end of the production process.

As mentioned before, some products are consumables, while others are capital or intermediate products. The consumables can be converted into labour, 2, according to certain columns in the technology matrices. The actions of these particular columns represent consumption, and consumption is restricted to a certain type of agent, namely the group of consumers. We thus distinguish two types of agents: consumers and producers. The latter can have any of the other technologies. Producers depend on consumers for the required labour, while consumers depend on producers for the consumables. Neither group can have skills that belong to the other group: an agent is either consumer, or producer, but never both. This is not crucial to the functioning of the model and could easily be relaxed later.

Agents in action

The production of goods, mentioned in the previous section, does not simply happen by itself. It occurs because somebody (something) somewhere actually does the job. Therefore, we need actual agents in space to serve as economic individuals to drive the economy. The whole environment of space, networks, agents, product set and technology is called the economy. All changes that take place in the economy occur because agents perform some or all of the following actions:

Win free resources: Free resources are distributed over the cells; some agents will use them.

Expand network: An agent asks around among its trading partners (all agents it is linked with) to inquire what they have in store. It compares this list with what is possibly required for executing its own skills. If there are any products it does not have access to, it will randomly choose a trading partner, then scan the surroundings of the trading partner for interesting new partners. A connection to the cell that contains the agent that offers most of what was not yet available will be added to the connection network.

Make plan: What an agent can do, firstly depends on its skills. Secondly, resources need to be available for the transformation of commodities into products. Resources can be bought, but the agent is limited by what its partners have to offer. A surplus of products can be sold to generate money to buy the required resources. However, the quantity of products being sold is dependent on the monetary resources of trading partners. In the end, the agent's initial possession, plus what is acquired, minus what is sold, minus what is used in the production process, plus what is produced must be positive. Given these constraints, and given the prices of all commodities (determined by an auctioneer explained below), every agent uses linear optimization to determine what and how much to sell, what and how much to buy, and what and how much to produce in order to achieve maximum possession.

Buy: Once an agent has optimized its plan, it will look for a partner that has what it is looking for, and they will exchange q units of product l for $q \times p_l$ units of money, where p_l is the current value of product l.

Sell: Once an agent has optimized its plan, it will look for a partner that has the money to buy what the agent wants to sell, and they will exchange q units of product k for $q \times p_k$ units of money, where p_k is the current value of product k.

Produce: After the exchange of products and money, the agent has all the resources required for the production plan and it can transform the input commodities into its output.

Update: An agent executes all of these steps, and the state of the agent and the agents engaged in trade are updated.

Unlike cells in cellular automata, the agents cannot be updated synchronously, because one agent's action will change the state of its partners with whom it engages in trade. Therefore, the agents perform sequentially. All the agents are randomly ordered, and in turn they go through the above list of actions. When everybody has had a turn, a new randomly ordered list is created for the next iteration. A complete sequence of actions of all agents' defines one iteration. The random list of agents generated anew for each iteration prevents any bias due to specific ordering of the agents.

Suppose the input matrix and output matrix are I and O. Dimensions of these matrices are $n \times m$, meaning that there are m production processes and that the economy consists of n products, with $m, n \in \mathbb{N}$. As in previous examples, the first product is labour and the second product represents one unit of money. Now expand these matrices by adding the matrices M and $\mathbb{I}$:

$$
\begin{aligned}
A &= \begin{pmatrix} I & | & M & | & \mathbb{I} \end{pmatrix} \\
B &= \begin{pmatrix} O & | & \mathbb{I} & | & M \end{pmatrix}
\end{aligned}
$$

where $\mathbb{I}$ is the identity matrix of size n, and M is the square matrix of size n with all elements equal to 0 except for the second row which is equal to $p = \{p_1, p_2, \ldots p_n\}$ indicating the quantities of commodity 3 that need to be paid for each of the products. The addition of these two matrices to I and O represent all possible actions involving production, buying and selling. Just as columns in I and O represent the input and output for production processes, the columns in M and $\mathbb{I}$ represent input and output for buying products. When we write $z = \{z_1, z_2, \ldots z_{m+2n}\}$ as the vector of all actions of an agent, that is, z consists of elements $z_1, \ldots z_m$ to indicate the activities regarding the m production processes, $z_{m+1}, \ldots, z_{m+n+1}$ to indicate the quantities of products that need to be bought, and $z_{m+n+2}, \ldots, z_{m+2n}$ the quantities of products that need to be sold, then $A \cdot z$ lists the quantities of products required for the execution of vector z and $B \cdot z$ lists the quantities of products generated by vector z. The quantities $A \cdot z$ and $B \cdot z$ are named the input and output, respectively, of the vector of action z. Likewise, when the vector $p = \{p_1, p_2, \ldots p_n\}$ lists the prices for each of the products, then $p \cdot (B - A)$ is a vector that gives the profit of each of the actions and $p \cdot (B - A) \cdot z$ equals the profit generated by activity z.

Every agent's behaviour is described by

$$\max_{z \in \mathbb{Q}^{m+2n}} profit(z) = \max_{z \in \mathbb{Q}^{m+2n}} p \cdot (B - A) \cdot z$$

under the conditions that:

- a positive balance is maintained both in production and in trade,
- the total activity per turn per agent is capped and
- consumers meet their basal metabolic rate; i.e. the minimum level of consumption necessary for them to remain alive and productive.

The prices $p = \{p_1, p_2, \ldots p_n\}$ mentioned above are determined by an auctioneer who attempts to find a set of prices such that, for those products for which there is a shortage, the production process becomes profitable (cost of input lower than price of output). For the production processes of products for which there is a surplus, prices are set such that these can be manufactured, but without profit (cost of input equals price of output). To a certain extent such prices are similar to Sraffa's market clearing prices based on the cost of input. When dealing with a shortage, the value of the output is such that it enables the sector to buy the required input and produce with a profit. Through this, agents are encouraged to reduce shortages and surpluses. The presence of surplus and shortage is determined as follows: the auctioneer keeps track of what input is required during the last w iterations (often $w = 50$). Subsequently, the auctioneer attempts to keep in store the required input for those w iterations. There are more advanced models of agent-based market dynamics, see for example Tesfatsion (2007), but these focus on all aspects of procurement. As such, much effort goes into the bottom-up determination of the market price through a cyclical process of matching offers and bids, which is much more than we need in this paper.

A Sample Run

The above described model of an artificial chemistry-based economy is illustrated here by a presentation of a typical simulation run. The agent population consists of 15 producers with random skills (the know-how to execute particular transformations given by the technology matrices), and 15 consumers. All agents are connected to all other agents such that the spatial configuration is irrelevant for the time being. The technology matrices are comparable to those in Table 3, that is they deal with the same products but have slightly different output coefficients to allow a surplus production such that the basal metabolic rate of consumers can be satisfied.

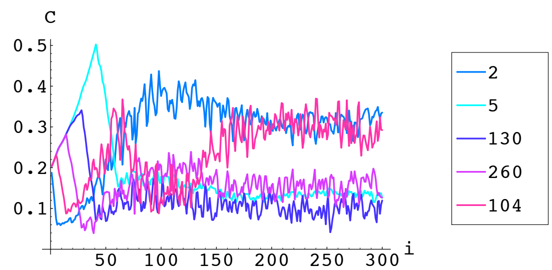

Figure 1: Product concentrations C per iteration i of the commodities 2, 5, 130, 260 and 104.

Figure 1 graphs the resulting product concentrations, where a concentration of a product is the proportion of the total product population. At the start of the simulations each of the agents was supplied with equal amount of each of the products. Initially only the stock levels of the labour (product 2) is reduced due to the basal metabolic rate of the consumers, and thus its concentration goes down while the other concentrations go up. This phase includes merely consumption, and thus the concentrations change linearly. However, the reduced stock of labour leads to a profitable production of this "product", which in turn leads to reduced levels of the consumable (commodity 104) and subsequently of other commodities involved. Agents do not necessarily have access to the required inputs, depending on the random order in which they are allowed to act for example, and thus the production of commodities appears a little erratic. However, this simple economy converges to a more or less stable state which corresponds to the Leontief stable state.

The next figure graphs the price dynamics for one of the commodities (product 104), and its close relation to the stock levels (see Figure 2). The blue line indicates the price, the red line indicates a surplus or a shortage in stock levels. Appropriate stock levels depend on the past activity of the agents. Note that a higher price does not guarantee an immediate reaction in stock levels, but that every time stock levels do become positive this is preceded by an increased price.

The temporarily higher prices trigger agents to engage in

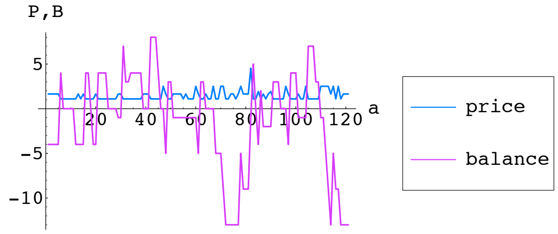

Figure 2: Price development P and balance B of product 104 graphed per agent's action a.

production activity if they possess the appropriate skills and if they have access to the required inputs. Initially the agents are using the surplus of stocks present at the start of the simulation and no manufacturing takes place. Once stocks are depleted, one by one the different sectors and thus the agents that compose the sectors have to come into action (see Figure 3). The figure shows the number of agents per iteration that are involved (either heavily or just a little) in a particular production process.

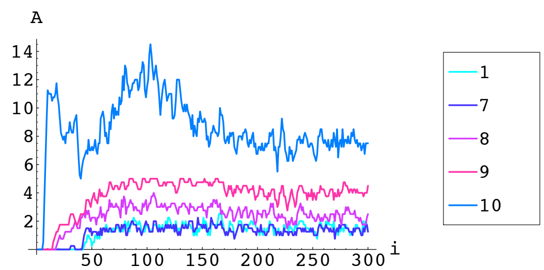

Figure 3: Number of agents A per iteration i participating in the production of the different commodities of Figure 1. The numbers indicating the different production processes correspond to the columns of the input and output matrices, such as in Table 3.

The peak in the generation of labour (sector 5) around iteration 100 can be explained by the absence of an appropriate record of past activity at start up. First the surplus is consumed and none of these goods have to be generated. When the system begins to run out of intermediates these have to be produced, but not even the stocks to do so are there since appropriate stock levels depend on the past activity levels. With a short period of higher activity the system is able to catch up.

New Products, Waves of Innovation, Evolutionary Dynamics

The actual innovation process of the evolving economy is composed of two steps: first the technology matrices have to be expanded to include the production of the new commodity or the new use of existing products. Subsequently agents have to start using the new technology.

At random times, new technology is generated. With a certain probability this new technology involves the creation of new consumables. If not a consumable, the new technology aims at producing products involved in the existing production process, be it capital or intermediates. In order to make something from which the system can benefit, we introduce new technology (i.e. a tool and the skill to use it) to produce something of which there is currently a shortage, where shortage is defined as in the price mechanism. Once one of the products in short supply is identified, the input for the new tool t can be composed. More precisely, when the product in excess demand p is factorized $p = f_1^{q_1} \cdot f_2^{q_2} \cdot \ldots \cdot f_n^{q_n}$ for some n and $q_. \in \mathbb{N}$, it is clear which factors are required in the input. A random list of products $i_1, \ldots, i_m$ that contain those factors is generated such that the product of these inputs $i = i_1 \cdot i_2 \cdot \ldots \cdot i_m$ will be divisible by the sought after product p, thus $\frac{i}{p} = g \in \mathbb{N}$. There are then two possibilities: the tool t sets the production of the required product p based on the new input i, or the tool requires the different parts $i_1 \cdot i_2 \cdot \ldots \cdot i_m$ and uses these without product i first being assembled. g is considered garbage and ignored.

The entry of new technology is based on Bruckner *et al.* (1989), who describe a general model to study evolutionary processes. The model consists of a countable set of fields $F = \{F_1, F_2, \ldots\}$ with, for each field F_i, a number N_i indicating the number of elements in the field. The state of the whole system is given by the set of occupancy numbers $\{N_1, N_2, \ldots\}$. Changes in occupancy numbers are discrete and occur in the smallest steps possible; fields gain or lose one element. The probabilities of these changes depend on the current state of the system. Bruckner *et al.* show that the Markov process of interactions between fields is capable of generating a wide variety of evolutionary dynamics. In particular, they write that this type of model is capable of simulating the dynamics of evolutionary systems, including the dynamics of technological evolution. A later paper experiments with the parameters for the stochastic economic substitution model and show that realistic substitution dynamics can be obtained (Bruckner *et al.*, 1996).

Innovation by new agent: When new technology becomes available, its establishment is affected by some of the existing technologies. Parameter A_{ij} describes the inclination of technology j to establish technology i by means of a new agent: $W(N_i + 1, N_j | N_i = 0, N_j) = A_{ij} N_j$

Innovation by existing agent: When an agent expands its skills, be it with a new technology or an existing technology, the agent innovates its production process. The choice of additional technology is affected by the existing technologies. Parameter M_{ij} describes the inclination of technology j to establish technology i by means of an existing agent: $W(N_i + 1, N_j | N_j) = M_{ij} N_j$

Growth of existing technology by spontaneous new agent: An increase in the number of agents using technology i independent of the state of the system: $W(N_i + 1) = \phi_i$

Growth of existing technology by new agent: Increase in the number of agents using technology i due to self-reproduction or sponsoring by technology j: $W(N_i + 1, N_j | N_i, N_j) = A_i^{(0)} N_i + A_i^{(1)} N_i N_i + B_{ij} N_i N_j$

Replacement of technology by existing agent: Agents imitate the successful technologies of other agents, and subsequently use these to replace less successful skills. The parameters A_{ij} represent a measure of success and failure. Furthermore, the probability is influenced by the current size of the the technology fields. The larger the field, the more occurrences of the technology in question, and the greater the probability of replacement: $W(N_i + 1, N_j - 1 | N_i, N_j) = A_{ij}^{(0)} N_j + A_{ij}^{(1)} N_i N_j$

Technologies that are not used for a specified length of time are forgotten by the agent, and when all agents lack a specific technology, this technology can be removed from the model. The same applies to products that have become obsolete. Unlike the random replacement of functionality in Jain and Krishna (1999), here skills and products slowly disappear.

The following figures illustrate the results of this process in a typical run. At random times new commodities and/or new production techniques are added to the technology matrices, and subsequently probabilistic rules distribute new and existing skills over the agents. The simulation begins with a small economy such as was illustrated in the previous section, and as time progresses the model constructs novel functionality and elements, analogous to a constructive artificial chemistry. After a 1000 or so iterations the economy consisted of 35 products and 71 production processes.

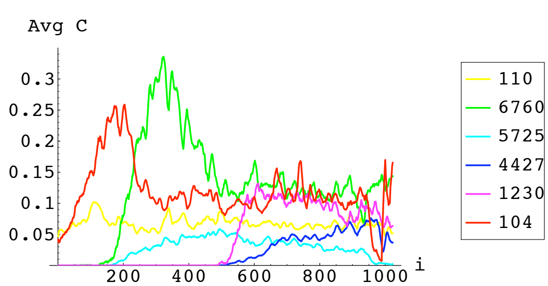

Figure 4: Product concentrations of a selected number of products averaged over a gliding window of length 10 to smooth the curves. Only the first four digits of the product appear in the legend.

Figure 4 illustrates the product concentrations of a selected number of products. Some products are present at the start of the simulations (110 and 104), other products appear later. Product 6760 is a consumable that becomes a serious competitor to the initial consumable, commodity 104. Later it diminishes its importance as other consumables become available. Product 57257200 is only temporarily successful and disappears from the stage. Other new products seem, at least for now, to be adopted more permanently.

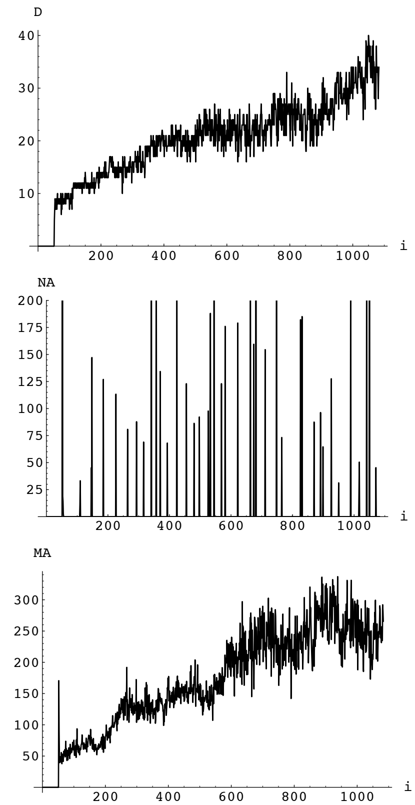

Figure 5: Bedau's measures applied to the economic activity of the present model. The top graph displays the number of active technologies (diversity D), the graph in the middle shows the activity of new technology "NA" expressed in value of output at first application, and the bottom graph displays the mean activity "MA" per active technology .

The innovations introduced to the system by the agents are not without effect. This can be illustrated by a measure developed in Bedau *et al*. (1997). That paper describes a system to classify evolutionary dynamics based on the increase of diversity and the effect the increased diversity has on the average productivity of the system. This measure is capable of distinguishing between a system with truly beneficial adaptive behaviour and a system with merely an increase in diversity. The first is characterized by an increase in average productivity as diversity is non-decreasing, while the latter system displays bounded diversity combined with bounded average productivity. For the application of this measure here, average productivity is defined as the total value of output divided by the number of technologies required to generate the output, and the first results suggest that the artificial economy displays an increased diversity in terms of technology used, while at the same time the average productivity increases. Therefore the system classifies as one with unbounded evolutionary activity (see Figure 5).

We conclude with an illustration of the competition between technologies that produce a single commodity (Figure 6). It concerns a newly introduced commodity, and it is quite successful as a whole row of innovations is triggered by its introduction. 5 alternative production processes jump the band wagon. As in the real economy market shares are far from stable and the most efficient production technique does not necessarily become dominant.

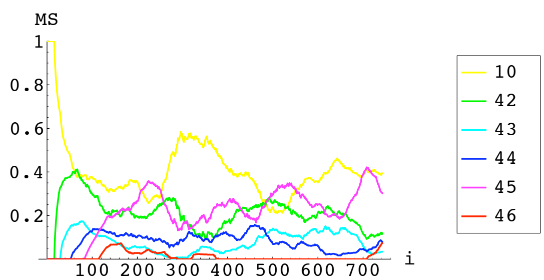

Figure 6: The market share of different technologies to produce one single product (product 7741173440000 in this case) coexist and compete for the market. The graph shows the proportion of the total production of the commodity produced by each of the 6 production techniques. The numbers on the right correspond to the column numbers in the technology matrices.

Conclusions and Perspectives

Though there are still a number of issues that need improving or further exploration, the framework developed in this paper is one with many merits. The main virtue is that this framework demonstrates that it is possible to treat the economic system as a constructive dynamic system, in other words, as a system undergoing continual structural evolution. The framework developed here is capable of dealing with endogenous change, something that is very important in our economy, but that until now has been essentially ignored in research. The application of a constructive dynamic system to the field of economics opens up new opportunities in economics.

In addition, the application of the Bedau measure suggests that the evolutionary economic dynamics generated by the model is appropriately named evolutionary.

The framework is also very useful to complement existing research in evolutionary economics. For example, descriptive approaches on the entry and exit of new enterprises can be assisted by an experiment that can indicate what parameters are responsible for the observed patterns. The model can shed light on why market shares and market size can fluctuate so wildly, and why in the production of one single good there are a variety of technologies being applied.

The result is a broad, abstract framework applicable to the study of evolving economic systems. It has the potential to elucidate many aspects of our economy, from dispersal of technology, to location strategies, to pricing, to the development of higher level organisation.

Acknowledgments

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