SYNTHETIC IMAGES OF UNDERWATER SCENES:
A FIRST APPROXIMATION

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ABSTRACT

The creation and rendering of realistic water scenes is one of the challenging tasks in Computer Graphics. To reproduce the illumination and colour inside water bodies an algorithm capable of dealing with media with anisotropic and multiple scattering has to be used. We have chosen the discrete ordinates method to solve the problem of light transport. Both the theoretical basis and the algorithm that has been implemented are described in the paper. A couple of simple images calculated in different waters are presented. Results indicate the relevant role played by the spectral behaviour of the absorption and scattering coefficients in the process of image generation.

Keywords Participating Media, Discrete Ordinates, Ocean Optics, Global Illumination.

1. INTRODUCTION

Research on the rendering of natural scenes, such as clouds, water, trees, terrain, fire, has become increasingly wide spread. In particular, the creation and rendering of realistic water scenes is one of the challenging tasks in Computer Graphics. Most of the work that has been done is concerned with the effects of the reflection and refraction of light on the water surface: shafts of light, caustics…[Shnyi89a], [WattM90a], [Nishi94a]. Great effort has been taken in studying atmospheric conditions to know how much sunlight and skylight reaches the water surface and the problem of wave generation [Fourn86a], [TsoPY87a], [Peach86a]. However, realistic rendering of water scenes requires the transport of light within the water body to be properly handled.

Due its complexity, when considering light transport in water strong simplifications are usually made (single scattering, isotropy, homogeneous media…). Nishita et al. [Nishi93a] have studied the colour of sea surface as viewed from outer space; they include in their model the scattering due to water molecules, but they make an analytic quasi-single scattering approximation. Premoze et al. [Premo00a] have also considered the problem of light transport, but they have centred their work in the simulation of the appearance of the water surface. Therefore, when carrying the light transport they also make important simplifications: they solve a mono-dimensional equation (the radiance in the medium will depend only on the depth). They do not try to simulate the radiance due to scattering; they estimate it using some empirical equations that relate the radiance just below the air-water surface with the light coming from the sun. Tadamura et al. [Tadam95a] have studied the colour of water in applications of lightning design. They are the only ones that have considered the presence of light sources within the water (usually, only illumination due to the sun and the sky is considered). But when performing the light transport they only consider single scattering and no illumination of objects inside the water due to scattering. Jensen et al. [WannJ98a] have generalised the bidirectional Monte Carlo ray-tracing method to scenes containing participating media and have applied it to render scenes with water. They calculate volume photon maps and use them in the rendering stage to estimate the radiance due to scattering within the media. It is a very general method able to consider non-homogeneous media and anisotropic scattering with complex geometries.

With certain media the savings in memory is substantial, compared with some finite element methods, but when applying it to render water the memory increases, specially if the highly-peaked marine scattering behaviour has to be reproduced. It should be pointed out that the behaviour of marine waters is drastically different of that of pure water. This is caused by the presence of several components in these waters (dissolved salts, dissolved organic compounds, plankton…) [Moble94a]. The spectral absorption and scattering coefficients as well as the scattering phase functions are strongly affected by these components: the illumination and colour of the water is consequently determined by their presence.

Our work is focused on the simulation of the transport of light in water in the more general case: non-homogeneous medium, anisotropic scattering and multiscattering. We have applied the method of Languénou et al. [Langu95a] that is based on the discretisation of the participating medium in finite volumes (voxels) and on the use the discrete ordinates method to handle directions. Finite volume element techniques are not as flexible as Monte Carlo ones, but they are undoubtedly faster and more effective for simple scenes. We have generalised the method to the case of objects and light sources inside the medium. The presence of light sources considerably increases the complexity of the light transport problem: the illumination does not only depend on the depth, so pure 3D calculus has to be performed. The illumination of the objects due to direct but also scattered light has to be taken into account. Proper handle of the spectral dependence of the medium characterising parameters and the use of realistic phase functions have to be considered.

The result of our transport calculus in the water body is the radiance distribution within the medium. Knowing the radiance distribution within and leaving the water body is a prerequisite for the solution of many problems such as the synthesis of realistic underwater images, the underwater visibility, the capture of satellite images, the biologic productivity studies, or the thermodynamics of stratified media in submarine environments. From the radiance, all the other magnitudes of interest can be deduced.

The structure of this paper is as follows: in Section 2 the transfer equation to be solved is presented, whereas in Section 3 the method of resolution is analysed. Section 4 presents the application of the algorithm to the specific characteristics of the oceanic medium; in particular, some images showing the dependence of its characterising parameters on the wavelength and the type of water are shown. In Section 5 conclusions and future work are presented.

2. PARTICIPATING MEDIA: THE RADIATIVE TRANSFER EQUATION

As radiation travels through a participating medium it undergoes three kinds of phenomena: absorption, which causes a diminishment of the intensity, emission, which increases intensity and scattering which causes a redirection of energy. There are two types of difficulties when studying the radiation in this kind of media. First of all, emission, absorption and scattering do not only take place in the medium boundaries, but within any point of the medium. A complete solution of the exchange of energy requires knowledge of the physical properties and the intensity of radiation within every point of the medium. A second difficulty comes from the spectral effects when the transfer of radiation from one wavelength to another is possible, making it necessary a detailed spectral analysis.

The equation that governs the transfer of energy in this kind of media is the full radiation transfer equation [Glass95a]:

$$\frac{dL_l(S,\boldsymbol{q},\boldsymbol{j})}{dS} = -a_l(S)L_l(S,\boldsymbol{q},\boldsymbol{j}) + a_l(S)L_{lb}(S,\boldsymbol{q},\boldsymbol{j})$$
$$- \boldsymbol{s}_l(S)L_l(S,\boldsymbol{q},\boldsymbol{j})$$
$$+ \frac{\boldsymbol{s}_l(S)}{4\boldsymbol{p}} \int_{w_i=4p} \Phi_l(S,(\boldsymbol{q}_i,\boldsymbol{j}_i) \to (\boldsymbol{q},\boldsymbol{j}))dw_i \int_{R_l} dl_i F(\boldsymbol{l}_i \to \boldsymbol{l})L_{l_i}(S,\boldsymbol{q}_i,\boldsymbol{j}_i)$$

$$(1)$$

$L_l(S,\boldsymbol{q},\boldsymbol{j})$ is the radiance, ie, the power per unit of projected area perpendicular to the ray per unit of radiation length and solid angle in the direction (θ,φ). The equation gives us its local variation when traversing a distance dS. The meaning of the different terms at the right of the equation is as follows:

- The first term refers to absorption: $a_\lambda(S)$ is the so-called absorption coefficient (the fraction of energy lost per unit length, dimension m^{-1})
- The second term corresponds to internal emission: $L_{\lambda b}$ (S, θ, φ) is the radiant energy emitted, due to spontaneous or stimulated emission
- The third term represents the reduction of the radiance along the propagation direction because of scattering (out-scattering): $\sigma_\lambda(S)$ is the scattering coefficient (dimensions m^{-1})
- The last term accounts for the in-scattering, ie, the increase of radiance along the propagation direction due to the scattering of radiance coming from other directions. We have considered the most general case in which not only radiance coming from other directions but also radiance of other wavelengths contributes to the radiance in the wavelength of interest due to inelastic scattering (fluorescence, for

example). This is why we integrate to all incident directions as well as to all wavelengths (R_λ). $F(\lambda_i \rightarrow \lambda)$ is the efficiency factor which models the transfer of energy from one wavelength to another. $\Phi_\lambda(S, (\theta_i, \varphi_i) \rightarrow (\theta, \varphi))$ is the phase function which describes the angular distribution of the scattered energy.

Another important parameter for characterising the medium is the extinction or attenuation coefficient, which is the sum of the absorption and scattering coefficients:

$$K_\lambda = a_\lambda + \sigma_\lambda \quad (2)$$

The inverse of the attenuation coefficient is called attenuation length (dimension m). In Table1 these and other important parameters characterising a participating medium are summarised. Their spectral dependence should be noted.

a_λ	Absorption coefficient
σ_λ	Scattering coefficient
$K_\lambda = a_\lambda + \sigma_\lambda$	Extinction coefficient
$\Phi_\lambda((\theta_i, \varphi_i) \rightarrow (\theta, \varphi))$	Scattering Phase Function
$\Omega_\lambda = \sigma_\lambda / K_\lambda$	Albedo
$l_\lambda = 1/K_\lambda$	Attenuation length

Table 1. Coefficients characterising a Participating Medium

In this work we consider no internal sources ($L_{\lambda b} = 0$) and only elastic processes, so equation 1 simplifies:

$$\frac{dL_\lambda(S)}{dS} = -K_\lambda L_\lambda(S) + \quad (3)$$

$$\frac{\sigma_\lambda}{4\pi} \int_{w_i = 4\pi} L_\lambda(S, \theta, \varphi) \Phi_\lambda(S, (\theta_i, \varphi_i) \rightarrow (\theta, \varphi)) dw_i$$

This is the so-called Radiative Transfer Equation-RTE-. Boundary conditions have to be added to the equation, basically void conditions (no incoming radiances) or surface reflection conditions:

$$L_\lambda(\theta_r, \varphi_r) = E_\lambda(\theta_r, \varphi_r) +$$

$$\int_0^{2\pi} \int_0^{\pi/2} f_{bsrdf}((\theta_i, \varphi_i) \rightarrow (\theta_r, \varphi_r)) L_\lambda(\theta_i, \varphi_i) \cos\theta_i \sin\theta_i d\theta_i d\varphi_i \quad (4)$$

where E_λ is the energy emitted in the (θ_r, φ_r) direction and f_{brdf} is the bidirectional reflectance function.

3. SOLUTION ALGORITHM

In order to solve the radiative transfer equation, there are basically the following families of methods[6]: analytic methods, zonal methods, Montecarlo methods and flux methods, that include the P-N or spherical harmonics methods and the multiflux or discrete ordinates methods. In Table2 the different families of methods and relevant works are outlined. A good review of these methods can be found in [Pérez97a].

We have chosen the discrete ordinates method to solve the radiative transfer equation because it is general, it poses no restrictions to the medium characteristics and is computationally feasible. It is based on the angular discretization of the solid angle about a location over a finite number of directions.

Analytic	[Blinn82a], [Nishi96a], [Sakas90a]
Zonal	[Rushm87a], [Bhate93a], [Silli95a]
Monte Carlo	[Patta93a], [Blasi93a]
Spherical harmonics	[Kajiy84a], [Bhate92a]
Discrete Ordinates	[Langu95a], [Patmo93a], [MaxNe95a]

Table 2. Different methods to deal with Participating Media

3.1 Angular discretization

The integral over solid angles in equation 3 is replaced by sums over a discrete set of directions. Therefore, the variation of the radiance along direction V_m will be given by (λ subscripts have been omitted for clarity):

$$\mu_m \frac{\partial L_m}{\partial x} + \xi_m \frac{\partial L_m}{\partial y} + \eta_m \frac{\partial L_m}{\partial z} = -K L_m + \frac{\sigma}{4\pi} \sum_{l=1}^{nd} w_l \Phi_{lm} L_l \quad (5)$$

where n_d is the number of discrete directions, (μ_m, ξ_m, η_m) are the director cosines of direction V_m, and w_l is the weight associated to direction V_l, $m, l \in [1, n_d]$ provided that $\sum_{l=1}^{nd} w_l = 4\pi$. Angular discretisation can be uniform or not.

3.2 Spatial discretization

Furthermore, to transform the differential equation into an algebraic one, a spatial discretization is performed: the medium is subdivided into voxels of constant physical properties. In fact, what is computed in the algorithm is not the radiance of the voxel for each direction but the **Source Term** [Siege92a], which in our case reduces to:

$$G_m = \frac{s}{4p} \sum_{l=1}^{n_d} w_l L_l \Phi_{lm} \quad (6)$$

This term represents the gain of radiance in direction m owing to emission and in-scattering. The radiances of the voxel faces and the source term for each of the discrete directions are constant inside the voxel so the RTE (equation 3) can be analytically integrated along a path of length s inside the voxel, obtaining:

$$L_m(s) = L_m(0) e^{-Ks} + \frac{G_m}{K}(1 - e^{-Ks}) \quad (7)$$

which is the basis of the transfer of energy inside the voxel.

3.3 Resolution method

Our resolution method is iterative. First of all, an initialisation step corresponding to the first order of scattering is performed. Then, in the iterative process, each of the iterations corresponds to one scattering. The iterative process follows the one in [Langu95a], but a proper handling of the boundary surfaces and objects has been incorporated.

1. The first step: "loading" the medium and the objects

First of all, the source terms in each voxel are initialised taking into account the contribution of each of the illuminating sources. Direct illumination on the physical boundary surfaces of the medium (seabed, air-water surface…) and on the objects within the water is also computed. These radiances will be incorporated as boundary conditions for next orders of scattering. This way, the contribution of the light reflected by the surfaces is incorporated to the calculation of the scattered light field.

2. The iteration process: computing the multi-scattered light field

Afterwards, for every single direction a complete traversal of the matrix of voxels is carried out:

- A direction V_m $m \in [1, n_d]$ is selected (the phenomenon is additive so the order does not matter)

- The traversal is done beginning in one of the 8 extreme voxels. To carry out the grid traversal, the increment for each axis has to be computed appropriately, following the "energy flux sense": for a direction with direction cosine >0 the sweep should be done increasing voxel indexes and for one with direction cosine <0, inversely

- In each voxel (for the direction being considered):
  - The source term contribution corresponding to the previous order of scattering and the incoming radiances are used to calculate an average radiance in the voxel (assumed constant within the voxel). The incoming radiances will be zero for the first voxel except if one of its incoming faces belongs to a physical surface.
  - This average radiance is used to calculate the out-going radiances. This calculus is done using the equation of energy transfer inside the voxel (equation 7). These radiances will be those of the incoming faces of the adjacent voxels.
  - The average radiance is also used to calculate the contribution to the source terms of the next order of scattering.
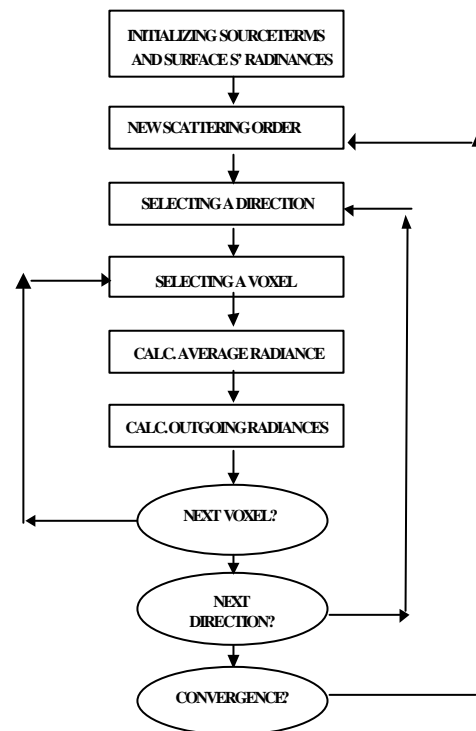

Figure 1. Algorithm's diagram

- Each time a "physical" surface (either belonging to a boundary surface or to an object) is traversed the radiance of the surface is properly

handled. If is the case of an "in-coming" flux, the radiance is stored; if it is an "out-going" flux the reflected radiance is added to the flux (the objects and seabed are considered as having Lambertian properties)

- Once all the voxels have been treated, next direction is considered
- After having considered all the directions, the next order of scattering is considered, initiating again the traversal of the medium for each discrete direction anew

With this iterative process, the energy initially loaded when initialising the voxels' source terms is propagated throughout the medium. When the contributions to the radiances of the boundaries and to the voxels' source terms are negligible (convergence is met), the resolution process is stopped. In Figure 1 the algorithm's diagram is shown.

3.4 Storage of the results and rendering stage

To avoid storing the source terms for each direction, an expansion in spherical harmonics is performed in each voxel:

$$G(\boldsymbol{q},\boldsymbol{j}) = \sum_{n=0}^{\infty} \sum_{m=-l}^{+l} c_{lm} Y_{lm}(\boldsymbol{q},\boldsymbol{j}) \quad (8)$$

where Y_{lm} are the spherical harmonics, related to the associated Legendre polynomials:

$$Y_{lm}(\boldsymbol{q},\boldsymbol{j}) = \sqrt{\frac{2l+1}{4\boldsymbol{p}}\frac{(l-m)!}{(l+m)!}} P_{lm}(\cos\boldsymbol{q}) e^{im\boldsymbol{j}} \quad (9)$$

A ray-tracer adapted to voxelised participating media has been used to obtain the images. Every time a ray travels a distance s, equation 7 is used. The expansion coefficients stored in each voxel are used to interpolate the source terms in new directions when rays are cast in the rendering stage. In our case and opposite to the situation in [Langu95a] we have objects immersed in the participating media.

4. APPLICATION TO THE OCEANIC MEDIUM

To apply the resolution module, the ocean has been chosen as participating medium. In the oceanic medium, electromagnetic radiation interacts as well with the water as with materials dissolved or suspended in it. This makes ocean phenomenologically rich [Spinr94a]. In ocean optics the magnitude they work with is the radiance, which is called in this area "light field", and the equation they solve is the radiative transfer equation. Nevertheless, in ocean optics it is assumed, to simplify, that the radiance is function only of the depth and not of the horizontal position, so that the RTE simplifies. In our method no such assumption is made, and the radiance will be a function of the 3D position, the direction and the wavelength. The ocean is characterised by its inherent optical properties, which are precisely the absorption and scattering coefficients and the phase function.

The specific case we are trying to solve is related to the problem of tracking submarine cables. Our participating media resolution module would be used to validate and fine tune the digital image treatment system to track power cables (such as the ones that comprise the system of electric energy transport between islands). In these systems, cables are located by means of a sequence of images captured by a camera mounted on an AUV (Autonomous Underwater Vehicle). These images are analysed by appropriated digital imaging systems. Due to the difficulties and the expense of obtaining these kind of images, our application could serve as a device to obtain simulated underwater images to study the performance of different digital imaging systems.

Two types of environmental situations were examined: deep clear ocean waters and coastal waters (as posed in [Palow91a]). These environments can be characterised by their inherent optical properties and the spectral varying nature of these properties must
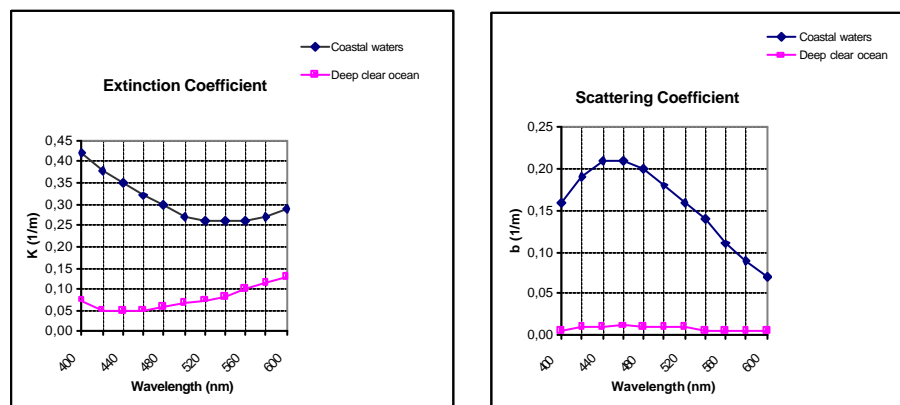

Figure 2. Extinction and scattering coefficients in two types of marine waters

be taken into account (Figure 2). As phase function, a forward-peaked Mie phase function has been used (Figure 3). Because of the different optical properties, visibility in each of the waters is quite different. Visibility in water ranges from one to two attenuation lengths. This means ranges of 6 and 15 meters in deep ocean waters and of 1.6 to 6 meters in coastal waters. Owing to the attenuation of the ambient light coming from the surface it is often necessary to use artificial lights beyond certain depths.

In our simulations we have used an incandescent light in both types of waters. The light source has been placed at a distance of 5 meters from the seabed. A grid of 20x20x20 voxels and a quadrature of 62 directions have been used [McLar63a]. In order to account for the wavelength dependence of the final colour image the visible spectrum is divided up into 11 values that range from 400 to 600 nm in 20 nm increments. Computa-
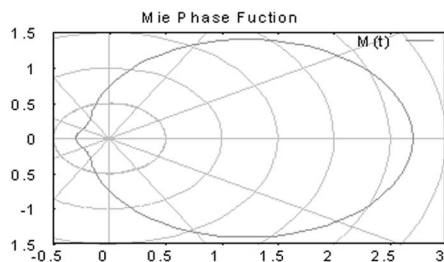

Figure 3. Mie phase function

tions were not performed below 400 nm or above 600 nm because the large degree of attenuation and the lack of experimentally determined input water properties at the spectral extremes. When combining the results of the simulations at different wavelengths in order to obtain a colour image, there is a shift to the blue in the case of deep ocean waters (absorption minimum at 450nm) and to the green in coastal waters (minimum at 550nm). This can be noted in Images 1, 2 and 3 where the same scene with no participating medium (Image1), clear ocean water (Image 2) and coastal water (Image 3) is shown.

5. CONCLUSIONS AND FUTURE WORK

We have extended an existing method to more general situations of light sources and objects within participating media. The method, the discrete ordinate one, allows the consideration of anisotropic and multiple scattering. Both the theoretical basis and the algorithm that has been implemented have been described. The method has been applied to seawaters with an adequate handling of the spectral dependence of their characterising parameters. A couple of simple images calculated in different waters are presented. Results indicate the relevant role played by the spectral behaviour of the absorption and scattering coefficients in the process of underwater image generation.
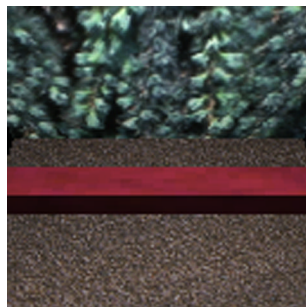

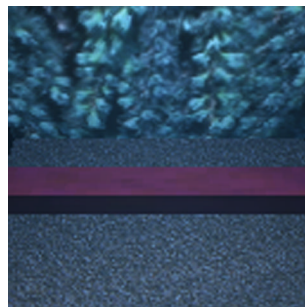Image 1.
Scene without participating medium
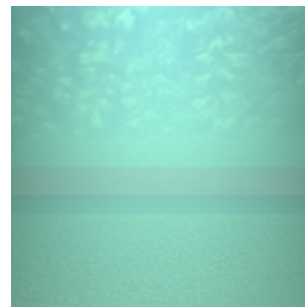
Image 2.
Scene with clear ocean water

Image 3.
Scene with coastal water

This study is part of a wider research line developed by the GIGA (Advanced Computer Graphics Group of the University of Zaragoza) to simulate the behaviour of light in participating media (not only in the visible part of the spectrum). In the case chosen, the visible range, the method developed is used to generate underwater images.

The following improvements could be done in this specific area:
1. Study of different phase functions to accurately reproduce the scattering behaviour of marine waters
2. Study factors such as the nature of the sea bead (algae, sand…) and the spectral radiance on the

water surface, due to the sun and the sky contribution

3. Consideration of possible inelastic phenomena
4. Study of non-homogeneous media.

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