Chapter One: Ether theoretic accounts of the experiments of Trouton and Noble

1.0 Introduction: condensers, contractions, and confusion

In the introduction to part one, I already discussed some of the incompatibilities between the ether theoretic accounts of the experiments of Trouton and Noble due to Larmor and Lorentz and the relativistic accounts of these experiments. The main emphasis of this chapter will be on discrepancies among different ether theoretic accounts. These discrepancies, I think, illustrate just how difficult it is to come to terms with these experiments without the benefit of Laue's relativistic mechanics which is tailor-made for this task.

Let me give a brief preview of what I think are the most perplexing discrepancies between the discussions of Trouton, Larmor, and Lorentz of the Trouton-Noble experiment. All three agree that, if one does not assume the Lorentz-FitzGerald contraction hypothesis, there will be a net turning couple on a charged condenser moving through the ether. They even agree on its size. However, they do not agree on its direction. Trouton believes the electromagnetic energy of the condenser will have its lowest value if the plates are perpendicular to the direction of motion. He concludes that the turning couple will try to put the plates at right angles with their velocity. Larmor believes that the electromagnetic energy will have its lowest value if the plates are parallel to the direction of motion. He concludes that the turning couple will try to put the plates in the direction of their velocity. Lorentz agrees with Larmor on this last point. However, his theory also vindicates Trouton's conclusion that the electromagnetic energy will have its lowest value if the plates are perpendicular to the direction of motion. Lorentz never actually did this calculation, and never commented on this rather counter-intuitive state of affairs in his theory.

Larmor and Lorentz agreed that, if one does assume the Lorentz-FitzGerald contraction hypothesis, there will be no net turning couple. However, the role of the contraction in Larmor's account is very different from its role in Lorentz's account. According to Larmor (although he only sketched the argument he thought would justify this claim), the contraction ensures that the electromagnetic energy of the condenser is independent of the orientation of the plates with respect to the direction of motion. In Lorentz's theory, the electromagnetic energy does depend on the orientation of the plates with respect to the direction of motion, even with the Lorentz-FitzGerald contraction (although Lorentz never did this calculation). The contraction hypothesis enters into Lorentz's account of the Trouton-Noble experiment through one of the hypotheses from which Lorentz in 1904 wanted to derive the contraction, viz. the hypothesis that motion through the ether affects molecular forces in the same way as it affects Coulomb forces. On that assumption, any turning couple coming from the Coulomb forces will be exactly compensated by the turning couple coming from the intermolecular forces preventing the condenser from collapsing under the influence of the Coulomb attraction between its plates. Lorentz never spelled out this explanation of the Trouton-Noble experiment in any detail (the interpretation offered here stems from Laue 1911a) and never commented on the fact that his account is at odds with Larmor's. These brief comments will already convey how confusing the situation concerning the Trouton-Noble experiment was in the ether theory of the early years of this century. It will take a serious effort on the part of the reader, I am afraid, to get it all straight.

Here is how I will proceed. In Section 1.1, I will briefly go over the experiments themselves and the conclusions drawn from them by the experimenters. This section will also cover Larmor's view of the Trouton experiment. To follow Larmor's reasoning for the Trouton-Noble experiment, one needs a clear understanding of how ether-theorists like Larmor and Lorentz exploited the Lorentz invariance of Maxwell's equations through a calculational device involving what Lorentz called "corresponding states." I will introduce this notion in section 1.2 and explain its relation to the notion of rest frames, familiar from special relativity. As an example of applying this strategy of corresponding states, I will give a simplified version of the 'forces'-account of the Trouton-Noble experiment due to Laue (1912b). We will then be ready to tackle Larmor's 'energy'-account of the Trouton-Noble experiment (see section 1.3). In section 1.4, I will present Lorentz's account of both the Trouton and the Trouton-Noble experiments in terms of the condenser's so-called electromagnetic momentum. I will show how Lorentz's account of these experiments should be seen against the background of the debate over the fate of Newton's third law in his theory in the period 1895–1904. I will also show how Lorentz's account can be used to amend Larmor's account of the Trouton-Noble experiment so as to make it compatible with both Lorentz's theory and special relativity.

1.1 Moving condensers and torsion balances

Trouton's original experiment was suggested by G.F. FitzGerald in the fall of 1900. Trouton was FitzGerald's assistant at Trinity College in Dublin at the time. The story of this first experiment is told in a paper that Trouton published in April 1902 (Trouton 1902; see also Warwick 1992). FitzGerald, Trouton tells us, thought that a condenser moving through the ether should experience an impulse when it is charged or discharged. Trouton designed an experiment to detect this effect. The results of this experiment were negative. Trouton does not spend much time discussing this result in his paper. Instead, he goes on to suggest that one should look for a turning couple on a carefully insulated charged condenser moving through the ether rather than for an impulse upon charging or discharging the condenser. Trouton pursued this idea, first in Dublin and then, together with Noble, in London, where Trouton became a physics professor at University College in 1903. The results of these experiments were also negative.

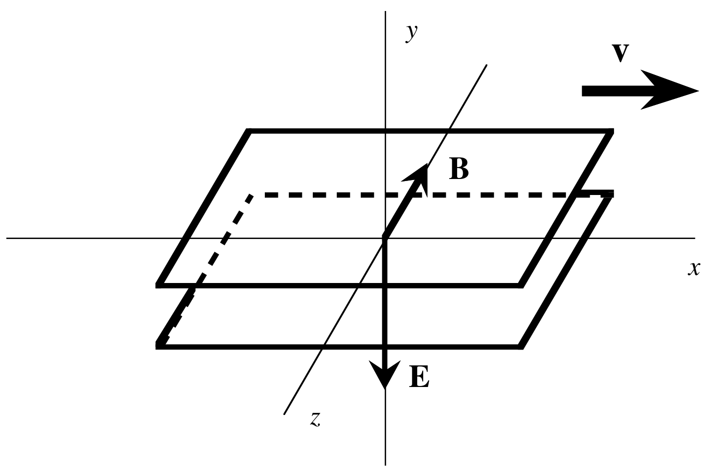

Figure 1.1 Moving charged condenser.

1.1.1 FitzGerald and the Trouton experiment. Fig. 1.1 illustrates the situation FitzGerald and Trouton considered. A charged condenser is moving through the ether. Let A be the area of its plates and let d be the distance between them. Suppose the condenser is moving with velocity v in the x-direction of a chosen reference frame with its plates parallel to the direction of motion. It will be convenient to assume that the dielectric constant for the dielectric between the plates is ε_0, the dielectric constant for vacuum, i.e., that it is just as if there were only ether between the plates. Suppose the top plate carries a positive charge $+Q$, and the bottom plate a negative charge $-Q$. If we ignore edge effects, there will be a homogeneous electromagnetic field between the condenser plates, and no field outside. As indicated in Fig. 1.1, the electric field **E** points in the direction of the negative y-axis of the co-moving Galilean reference frame shown in the figure, whereas the magnetic field **B** points in the direction of the negative z-axis. Both the presence and the direction of the **B**-field can be understood by looking upon the two charged plates as representing two opposite currents. On the basis of this simple picture one can also understand that there would be no **B**-field if the condenser were moving with its plates perpendicular to the velocity (see Fig. 1.3 below). In that case the two currents cancel one another. One has to be more careful in order to find the magnitude of both the **E**- and the **B**-field (see section 1.4). As we will see, Trouton was not very careful in this respect.[1]

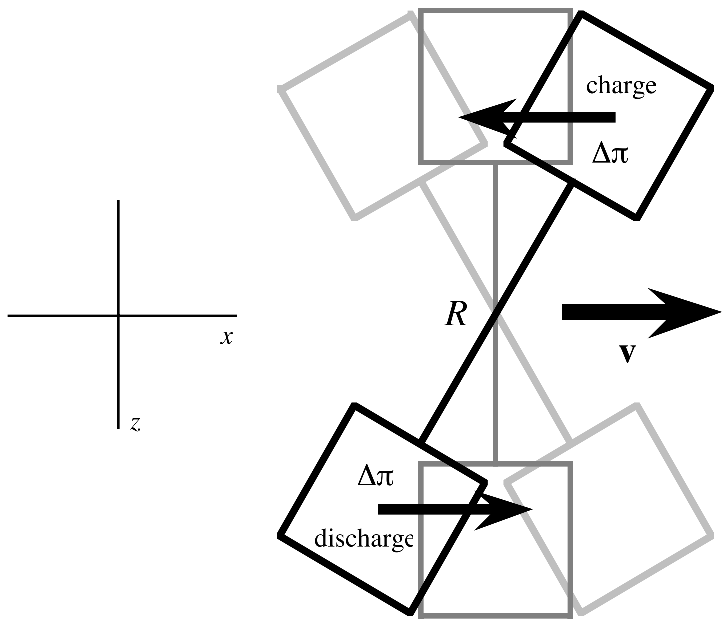

Figure 1.2 Trouton's experimental design to detect an impulse
upon charging or discharging a moving condenser (seen from above).

When, in the fall of 1900, FitzGerald suggested to Trouton that one might be able to detect the earth's motion through the ether with the help of a condenser, his reasoning, according to Trouton, went as follows. When a condenser is charged while it is at rest in the ether, there will only be an electric field. However, when the condenser is charged while moving through the ether, there will also be a magnetic field. Where is the energy to build up the magnetic field coming from? FitzGerald thought that it would come from a decrease in the kinetic energy of

[1] These preliminary remarks are meant only to fix the reader's intuition about the physics involved and are not meant as either physically or historically fully accurate statements about the experiments I want to discuss. To avoid a possible misconception on the part of those readers approaching this subject matter with strong late 20th century intuitions in physics, I want to emphasize that in pre-relativistic electrodynamics it was commonly and tacitly assumed—the only exception I am aware of being Poincaré (see chapter three)—that different inertial observers measure the same **E**- and **B**-fields, i.e., the same disturbances in the ether. In modern terms, all inertial observers measure the **E**- and **B**-fields of a frame at rest in the ether. Likewise, a current is understood to be motion of charges with respect to the ether, not motion with respect to the observer measuring the current.

the condenser. He therefore expected an impulse, a change of momentum $\Delta\mathbf{p}$, upon charging or discharging the condenser.

Fig. 1.2 schematically shows the experimental setup that Trouton designed to measure the effect. Trouton constructed a torsion balance with two condensers in it.[2] The system is hooked up to the torsion wire at R. The torsion wire (not shown in the figure) is perpendicular to the plane of the paper. In other words, it is parallel to the y-axis of the co-moving Galilean reference frame shown on the left (the plane of the paper coincides with the xz-plane of this coordinate system). The idea was to get the torsion balance to oscillate in its proper mode by charging and discharging the condensers at appropriate time intervals. The electrical wiring was such that the condenser could be charged and discharged without disturbing the motion of the torsion balance other than through the effect predicted by FitzGerald.[3]

Trouton did not find any effect. In the paper he published on the experiment, he considers two possible responses to this negative result. Either the energy for the magnetic field is supplied in some other way or there is some compensating effect (Trouton 1902, p. 562). Trouton—probably under Larmor's influence (see below)—seems to favor the first option. At the beginning of his paper, before he has even given FitzGerald's answer to the question where the energy for the magnetic field is coming from, he notes: "If we attribute it to the electric generator, say a battery, there is no difficulty indeed" (Trouton 1902, pp. 557–558). FitzGerald had been of a different opinion. Trouton reports:

> On the last opportunity I had of discussing the matter with Professor FitzGerald, preliminary experiments had been made, giving as far as they went negative results: the final results not being completed till after Science had to deplore the grievous loss it sustained at his death. FitzGerald, on that occasion, made a remark which, as well as I remember, was to the effect that should the negative results then obtained be sustained by further work, he would attribute the non-occurrence of any observable effect to the same general cause as produced the negative results in Michelson and Morley's experiments on the relative motion of the Earth and the ether by means of the interference of light. (Trouton 1902, p. 562)

As Trouton goes on to explain, "the same general cause" refers to the Lorentz-FitzGerald contraction. In the next paragraph, Trouton elaborates on FitzGerald's suggestion: "From some such cause [i.e., the Lorentz-FitzGerald contraction] a diminution of the electrostatic energy might be brought about [...] just sufficient in amount to provide the energy required for the magnetic field" (ibid., pp. 562–563). It is not entirely clear whether this elaboration is Trouton's or FitzGerald's. However, no matter whose idea it was, it is hard to see how it could be made to work.

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[2] By the time Trouton made his actual measurements he had only one working condenser left. The others all broke down under the voltage of 1200 Volts that Trouton was using (Trouton 1902, pp. 558–559). The role of one of the condensers in Fig. 2.2 was thus reduced to that of a balance-weight.

[3] See Trouton's own drawing of his apparatus (Trouton 1902, p. 560).

1.1.2 Larmor on the Trouton experiment. Larmor got involved in Trouton's experiment at an early stage (see Warwick 1992 for details). He is the only one to get an acknowledgment in Trouton's 1902 paper. Larmor also became the editor of FitzGerald's scientific papers that were published later in 1902, less than two years after FitzGerald's death. Larmor included Trouton's 1902 paper in this volume and added an interesting note himself (Larmor 1902). FitzGerald's original suggestion is dismissed in one short paragraph at the end of this note. According to Larmor no such effect was ever to be expected. Larmor was very interested though in the new experiment that Trouton had proposed. I will return to Larmor's discussion of this new experiment in section 1.3. At this point, I just want to look at his discussion of Trouton's original experiment.

In the last paragraph of his commentary on Trouton's paper, Larmor offers the following simple argument to establish that the energy to build up the magnetic field cannot come from the kinetic energy of the condenser. Larmor writes:

> If the condenser AB is held *absolutely fixed* while it is being charged, any impulsive torque there might be could do no work; yet the condenser gets its energy. This seems by itself sufficient to negative the suggestion that the energies of charge and discharge [...] have to do directly with mechanical forces (Larmor 1902, p. 569; italics in the original).

I take it that by 'absolutely fixed,' Larmor means 'fixed with respect to the laboratory,' i.e., not freely suspended on a torsion wire as in the actual experiment. The alternative reading 'fixed with respect to the ether' does not seem to make sense, since the problem only arises for a moving condenser. At first glance, Larmor's argument is a gross non-sequitur. It is perfectly consistent to maintain that the energy for building up the magnetic field in the case where the condenser is fixed to the laboratory comes from an ever so slight decrease of the kinetic energy of the earth as a whole.

Andrew Warwick and John Stachel[4] have both suggested (different) more charitable reconstructions of the argument Larmor offers in this passage. On the reading suggested above, Larmor missed a very obvious point, viz. that if the condenser is not freely suspended in the laboratory, a decrease in its kinetic energy would have to be accompanied by a (tiny) decrease in the kinetic energy of the earth. Warwick and Stachel do not believe Larmor could have overlooked such an obvious point, and suggest that his reductio is, in fact, to the absurdity of the notion that the earth's kinetic energy would decrease in the experiment. Unfortunately, Larmor fails to spell out exactly why he believed this was so absurd (if indeed he held this belief). Warwick and Stachel offer different reconstructions of the reasons behind Larmor's conjectured belief.

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[4] Private communications.

Warwick points out that Larmor believed that it was impossible to extract energy from an object's motion through the ether, except maybe a very small amount, proportional to some higher power of v/c, the ratio of the object's velocity with respect to the ether and the velocity of light. Otherwise, Larmor believed, the whole universe would have long come to rest in the ether. If FitzGerald were right, the energy extracted from the motion of the condenser and the earth through the ether in the Trouton experiment would be of order v^2/c^2. Larmor's belief that this would be impossible is illustrated by his attitude toward the new experiment Trouton proposed. In a passage that I will analyze in more detail in section 2.3, a passage written before the experiment was even performed, Larmor writes: "Thus the energy of motion of the Earth through the æther is available for mechanical work to an unlimited extent, unless [...] the FitzGerald-Lorentz contraction is a fact" (Larmor 1902, p. 568).[5] Larmor appears to be quite confident that the result of the experiment will be negative. This would seem to support Warwick's interpretation of Larmor's response to Trouton's original experiment. There is an important difference though between Larmor's responses to these two experiments. In the case of the new experiment Trouton suggests, Larmor offers a detailed explanation (viz. the FitzGerald-Lorentz contraction) to explain why it will be impossible to extract energy from the earth's motion through the ether in this experiment. In the case of Trouton's original experiment, we find no such thing. Instead, Larmor simply dismisses FitzGerald's idea out of hand ("This seems by itself sufficient to negative the suggestion ..."). This suggests that Larmor had some other reason for believing FitzGerald's idea to be absurd.

Stachel has suggested such a reason. If the fully isolated system of the earth and the condenser were to change its velocity upon charging or discharging the condenser, we would have a blatant violation of a basic theorem in mechanics according to which the center of mass of a fully isolated system cannot change its state of motion. It seems very plausible to me that this violation of the center of mass theorem is indeed the absurdity that Larmor sensed in FitzGerald's proposal. In that case, it has to be said that he would prove to be dead on. However, the connection between the Trouton experiment and the center of mass theorem would prove to be considerably more complex than Larmor, given the extreme brevity of his dismissal of FitzGerald's idea, can possibly have realized at the time.

The center of mass theorem is closely related to Newton's third law, the principle that action equals reaction, which, in turn, is closely related to the conservation of momentum. When Larmor wrote his comment on FitzGerald's idea, the status of momentum conservation in its

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[5] The following passage in Trouton's paper may actually reflect discussions between Trouton and Larmor over this issue: "Should this turning moment be proved to operate, instead of being masked by some compensating effect, it would open up a road leading to illimitable possibilities, for it would at once remove from the category of utter hopelessness the idea of mankind ever being able to utilize the vast store of energy in the Earth's motion through space" (Trouton 1902, p. 564).

various guises in theories positing a stationary ether, such as the theories of Lorentz and Larmor, had been the subject of some serious debate, notably between Lorentz and Poincaré. In 1902, the situation was unclear at best.[6] Moreover, even if we set aside the doubts about the universal validity of momentum conservation that were not uncommon at the time, the connection between the Trouton experiment and the center of mass theorem is far from straightforward. That this could have appeared to be otherwise to Larmor in 1902 is only because he was blissfully ignorant of two complicating factors to be introduced in the years just ahead.

First, Abraham (1903) introduced the notion of electromagnetic momentum. When the role this quantity plays in the Trouton experiment is taken into account, it looks as if momentum conservation is violated if the effect predicted by FitzGerald does *not* occur. As we will see in section 1.4, this is the conclusion that Lorentz reached in 1904. Second, Einstein (1905b, 1906) introduced the equivalence of mass and energy. When both electromagnetic momentum and the inertia of energy are taken into account, Larmor's intuition proves to be right, and momentum conservation rules out the effect predicted by FitzGerald. No one, to my knowledge, least of all Larmor, ever drew attention to this rather complicated state of affairs.

Whatever the correct interpretation of these comments by Larmor I quoted—whether it is simply the gross non-sequitur it appears to be at first sight or whether there is some sound physical intuition behind it—they seem to have sealed the fate of the experiment FitzGerald had suggested to Trouton. Both Larmor and Trouton were convinced that the effect predicted by FitzGerald could not possibly occur. However, Trouton, probably with Larmor's help, had already thought of a more promising way to detect etherdrift with the help of condensers.

1.1.3 The Trouton-Noble experiment. As was noticed above, there will be no magnetic field when the condenser is moving with its plates perpendicular to the velocity. More generally, drawing on the simple picture of the moving condenser as two opposite currents, Trouton found that the magnetic field will be proportional to cos θ, where θ is the angle between the plates of the condenser and its velocity. In Fig. 1.3 the extreme cases ($\theta = 0$ and $\theta = \pi/2$) are shown. Trouton refers to these two cases as "edgewise" and "flatwise," respectively (Trouton 1902, p. 563); Larmor uses "longitudinal" and "transverse," instead (Larmor 1902, p. 568).

Trouton asked where the energy for the magnetic field is coming from when we charge a moving condenser in the 'flatwise' position with no **B**-field, then disconnect the condenser from the power supply, and rotate it over 90 degrees to the 'edgewise' position with **B**-field. FitzGerald presumably would have answered "from the condenser's kinetic energy," but given

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[6] I will return to this issue in section 1.4 and chapter two.

the negative result of Trouton's original experiment and Larmor's alleged reductio of FitzGerald's reasoning this possibility was not seriously entertained.[7] Trouton's answer was that the rotation must be resisted by a turning couple, so that we must do work to turn the condenser into a state with a stronger **B**-field.

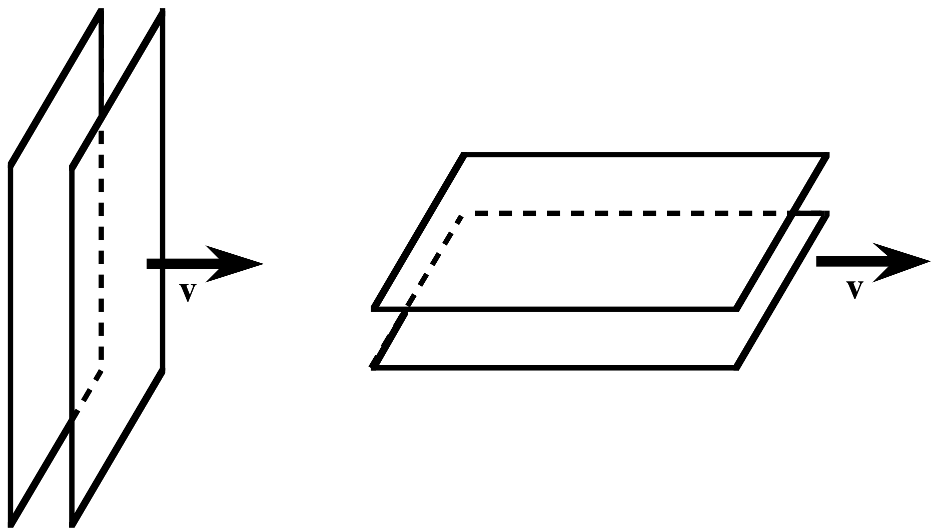

Figure 1.3 On the left: condenser moving "flatwise" (Trouton), "transversal" (Larmor), $\theta = \pi/2$.
On the right: condenser moving "edgewise" (Trouton), "longitudinal" (Larmor), $\theta = 0$.

Trouton also calculated how big this turning couple would be. He followed essentially the same path as the one followed by Larmor that I briefly described in the introduction. Like Larmor, Trouton only considered the electromagnetic energy of the moving condenser; and like Larmor, he tacitly assumed that only the electromagnetic part of the condenser's energy can depend on the angle between the condenser's plates and its velocity. The way in which Trouton actually calculated the electromagnetic energy of the condenser is different though from the way in which Larmor calculated this quantity. Their results contradict each other and are also at odds with both Lorentz's theory and special relativity. From the perspective of Lorentz's theory and special relativity, Trouton derives an expression for the electromagnetic energy to order β^2 from expressions for the electromagnetic field that are valid only to order β (see Eqs. 1.69–1.74). When this is corrected for, the discrepancy between Trouton's result and Larmor's result is seen to have the same origin as the discrepancy between Larmor's result and the result found by Lorentz and in special relativity.

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[7] Larmor in his commentary on Trouton's paper implicitly rules out another possible answer, viz. that the temperature of the condenser drops slightly when it is rotated from the 'flatwise' to the 'edgewise' position. Larmor notices that the process is reversible (Larmor 1902, p. 568). Hence, a drop in temperature would lead to conflicts with the second law of thermodynamics.

Trouton used the well-known expression for the energy density u of an arbitrary electromagnetic field, which in modern notation reads

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0^{-1} B^2, \tag{1.1}$$

to find an expression for the electromagnetic energy of a moving condenser as a function of the angle θ. Since the field in the moving condenser is homogeneous, we can just multiply u by the volume V of the condenser to obtain the electromagnetic energy U stored in it. For the condenser at rest, the electromagnetic energy U' is given by:

$$U' = \frac{1}{2}\varepsilon_0 E'^2 \mathsf{V}, \tag{1.2}$$

which is just the first term of Eq. 1.1 multiplied by V. Trouton assumed the electric field $\mathbf{E}$ in the moving condenser to be the same as the electric field $\mathbf{E}'$ in the same condenser at rest carrying the same charge. Drawing on the analogy between the charged moving condenser and two currents of opposite sign, Trouton found that the magnitude B of the magnetic field is given by (Trouton 1902, p. 558; Trouton and Noble 1903, p. 168):[8]

$$B = \beta\, E'/c \cos\theta. \tag{1.3}$$

Inserting Eq. 1.3 into the second term of Eq. 1.1, and using the relation $\mu_0^{-1}/c^2 = \varepsilon_0$ and the expression for U' in Eq. 1.2, one sees that the magnetic field gives a contribution $\beta^2\, U' \cos^2\theta$ to the electromagnetic energy of the moving condenser. So, Trouton arrives at (Trouton 1902, p. 564; Trouton and Noble 1903, p. 168):

$$U(\theta) = U'\,(1 + \beta^2 \cos^2\theta). \tag{1.4}$$

Hence, to rotate the condenser clockwise from some angle $\theta+\Delta\theta$ to some slightly smaller angle θ (for which U will be bigger according to Eq. 1.4), we have to do an amount of work $\Delta W = U(\theta) - U(\theta+\Delta\theta)$.

In Fig. 1.1.4 two forces of equal size $F(\theta)$ are drawn that we have to exert to overcome the turning couple $T(\theta)$ resisting the rotation from $\theta+\Delta\theta$ to θ. These forces form a turning couple $F(\theta)d$ of the same size as $T(\theta)$. The work done by these two forces is given by:

$$\Delta W = 2\, F(\theta)\, \frac{1}{2} d\, \Delta\theta = T(\theta)\, \Delta\theta\, . \tag{1.5}$$

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[8] It follows directly from Maxwell's equations that the expressions Trouton uses for the fields $\mathbf{E}$ and $\mathbf{B}$ only hold to first order in β (cf. Eq. 1.65 and Eq. 1.69).

Using $\Delta W = U(\theta) - U(\theta+\Delta\theta)$, we find for $T(\theta)$:

$$T(\theta) = \frac{\Delta W}{\Delta\theta} = \frac{U(\theta) - U(\theta+\Delta\theta)}{\Delta\theta}\,. \tag{1.6}$$

In the limit $\Delta\theta \rightarrow 0$, Eq. 1.6 turns into:

$$T(\theta) = -\frac{dU(\theta)}{d\theta}. \tag{1.7}$$

Inserting Eq. 1.4 into Eq. 1.7, we find that the turning couple is given by (Trouton 1902, p. 564; Trouton and Noble 1903, p. 169):

$$T(\theta) = U'\,\beta^2 \sin 2\theta, \tag{1.8}$$

where I used the relation $2 \sin\theta \cos\theta = \sin 2\theta$.

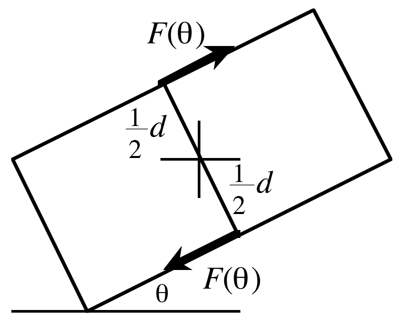

Figure 1.4 Work done upon rotating a moving condenser.

Larmor, Lorentz, and Laue all found a turning couple of the same size as Trouton, but working in the opposite direction. According to Trouton and Noble the turning couple tries to put the plates perpendicular to their velocity ('flatwise;' the position without a magnetic field). According to Larmor, Lorentz, Laue and every one who has dealt with the experiment since, the turning couple tries to put the plates parallel to their velocity ('edgewise;' the position with a magnetic field).[9] For the experiment Trouton and Noble performed to detect the turning couple its direction does not matter.

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[9] Understandably, Trouton and Noble did not accept Larmor's conclusion that the turning couple actually tries to get the condenser to rotate from a position with no magnetic field to a position with a magnetic field. After all, the whole experiment was based on the idea that the turning couple would work in the opposite direction, providing the energy for the magnetic field if we were to rotate the condenser from the 'flatwise' to the 'edgewise' position. Consequently, Trouton and Noble did not accept the expression Larmor gave for the electromagnetic energy (see section 1.3, Eq. 1.20 and Eq. 1.32). According to Larmor, the state with a magnetic field (θ=0) is less energetic than the state without a magnetic field ($\theta=\pi/2$). In the 1903 paper by Trouton and Noble, Trouton's own 1902 expression for the condenser's energy is used (see Eq. 1.4). In a footnote, they

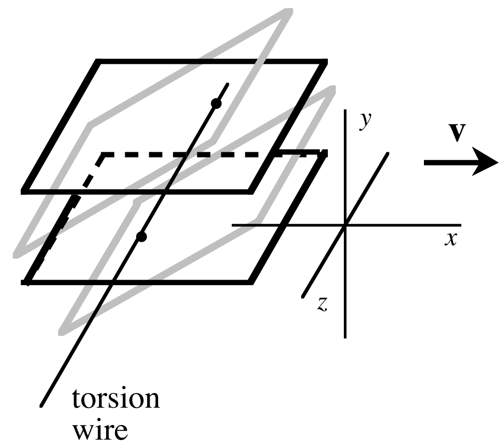

Figure 1.5 Trouton and Noble's experimental design to detect a turning couple on a charged moving condenser (seen from above).

Fig. 1.1.5 schematically shows the experimental setup Trouton and Noble used for their experiment. As in Trouton's original experiment, Trouton and Noble had a condenser suspended on a torsion wire. Trouton and Noble had the system oscillating in its proper mode while the condenser was uncharged. They then charged the condenser—again, the electrical wiring was such that this could be done without disturbing the system's motion[10]—and studied whether this had any effect on the damping process of the oscillation as one would expect if there really were a turning couple acting on a charged condenser. "There is no doubt that the result is a purely negative one," Trouton and Noble write in the conclusion of their paper (Trouton and Noble 1903, p. 181).[11] They suggest that maybe the energy of the electric field depends on the orientation of the condenser too, in such a way that the sum of electric and magnetic energy is independent of the orientation. This explanation is reminiscent of FitzGerald's—or Trouton's—explanation of the original experiment. Again, it is hard to see how such an explanation could be made to work.

As with the Michelson-Morley experiment, there is the remote possibility that the negative result of the Trouton-Noble experiment is due to the fact that the net velocity of the condenser with respect to the ether happened to be zero or very small at the time the experiment was done.

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inform the reader that Larmor arrived at a different result (Trouton and Noble 1903, p. 165, footnote; cf. Larmor 1902, p. 568, footnote). As we will see in section 1.4, Lorentz's theory vindicates Trouton and Noble in that the electromagnetic energy of the moving condenser does have its minimum in the 'flatwise' position with no magnetic field. At the same time, however, Lorentz's theory predicts that the turning couple will be in the direction Larmor predicted (see the discussion following Eq. 1.42 in section 1.4).

[10] See Trouton and Noble's own drawing of their experimental setup (Trouton and Noble 1903, p. 167; reproduced in Miller 1981, p. 69).

[11] Trouton and Noble claimed that their experiment put an upper limit of about 1.5 km/s on the velocity of the earth with respect to the ether. Michelson and Morley only claimed an upper limit of 5 km/s on the basis of their famous 1887 experiment. The velocity of the earth in its orbit around the sun is 30 km/s.

A large portion of Trouton and Noble's paper is actually devoted to astronomical considerations that would make such a conspiracy extremely unlikely (Trouton and Noble 1903, pp. 169–176).

1.2 Corresponding states and rest frames; application to the Trouton-Noble experiment

1.2.1 Electrostatics in moving frames of reference, with and without the Lorentz-FitzGerald contraction. The most intuitive way of calculating the turning couple on a moving condenser is probably through the forces acting on the condenser. Before I go through Larmor's derivation in terms of energy (section 1.3) and Lorentz's derivation in terms of momentum (section 1.4), I will therefore present a simplified version of a derivation due to Laue in terms of forces (Laue 1912b). This simplified derivation will only give a turning couple half the size found in more rigorous derivations. In section 2.4, I will show how one gets the other half (see Eqs. 2.116–2.130).

The simplified version of Laue's argument to be given in this section not only serves to provide a simple intuitive account of the Trouton-Noble experiment. Its main purpose, in fact, is to introduce the concept of "corresponding states." All derivations that we will look at in sections 1.3 and 1.4 exploit the Lorentz invariance of Maxwell's equations in one way or another. The ether theoretic way of looking upon such calculations is rather different from the relativistic way of looking upon them. Whenever Lorentz is dealing with the state of a system in motion, he introduces a so-called "corresponding state" of a system at rest in the ether. Other ether theorists, such as Abraham and Larmor, also used Lorentz's device. From a modern relativistic point of view, a corresponding state is just the state of the moving system in its rest frame. By the end of this section, the reader will hopefully feel comfortable switching back and forth between thinking in terms of corresponding states and thinking in terms of rest frames (cf. Laue 1912b, pp. 176–177).

The basic result I will use in this section is what the reader will recognize as Planck's relativistic transformation law for forces:

$$\mathbf{F} = \mathrm{diag}(1, 1/\gamma, 1/\gamma)\,\mathbf{F}' \qquad\qquad \gamma \equiv 1\Big/\sqrt{1-\beta^2}\ . \tag{1.9}$$

In Fig. 1.6 this relation is illustrated. I will go over both the relativistic and Lorentz's ether theoretic way of looking upon Eq. 1.9 and Fig. 1.6. All other diagrams in this section can likewise be understood in these two different ways. From the relativistic point of view, the rectangle on the left in Fig. 1.6 represents some static charge distribution in its rest frame. A charge Q at some point P' experiences a Coulomb force $\mathbf{F}'$; the rectangle on the right represents the same charge distribution in a frame in which it is moving at a velocity v in the direction of the positive x-axis of the chosen reference frame.[12] In this frame the charge distribution is shorter by a factor γ in the direction of motion ($l = l'/\gamma$). The relation between the force **F** on the charge Q at P and the force **F**′ on that same charge Q at the corresponding point P' , is given by Eq. 1.9.[13]

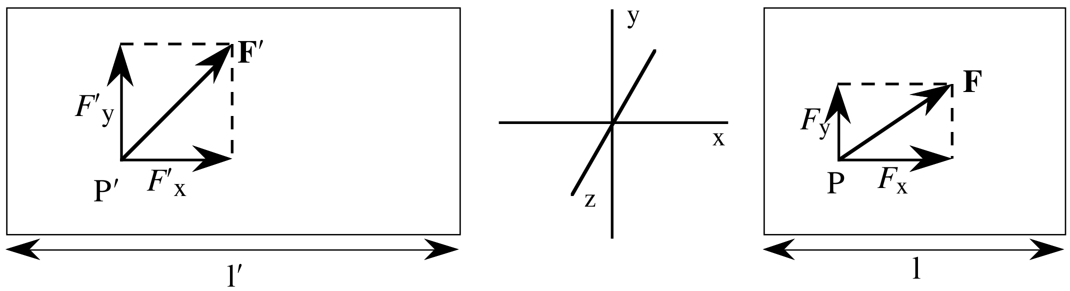

Figure 1.6 Forces on a moving static charge distribution.

As far as I know, Eq. 1.9 was first derived by Lorentz in 1895 in the context of his treatment of electrostatics in (Galilean) reference frames in uniform motion with respect to the ether (Lorentz 1895, sections 19–23, pp. 31–37).[14] Suppose we have a static charge distribution moving through the ether, such as the charge distribution on the right of Fig. 1.6. Lorentz wanted to calculate the forces experienced by charges in such charge distributions. For static charge distributions at rest in the ether this problem is easily solved with the help of Maxwell's equations. The problem of a moving charge distribution, even a static one in uniform motion, is harder to handle. In a frame at rest in the ether, in which Maxwell's equations hold, the case of a moving distribution is considerably more complicated than the case of a distribution at rest. And in a Galilean frame moving along with the moving charge distribution, Maxwell's equations no longer hold and get replaced by equations of a more complicated form.[15] Lorentz's strategy was to introduce a set of auxiliary quantities such that the equations for electrostatics in the Galilean co-moving frame would be the same as the equations for electrostatics in a frame at rest in the ether. For instance, instead of the x-coordinate of the Galilean co-moving frame Lorentz introduced the auxiliary quantity $x' = \gamma x$. The idea was to solve the problem in terms of these auxiliary quantities and then to rewrite the solution in terms of the real quantities of the

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[12] In order to bring out the effects more clearly, I picked $\beta = .75$, which gives $\gamma \approx 1.5$. In the actual etherdrift experiments, such as the Michelson-Morley experiment and the Trouton-Noble experiment, β was assumed to be of the order of 10^{-4}.

[13] For those readers who like to think in terms of active rather than passive transformations: the system on the right is obtained by Lorentz boosting the system on the left to a velocity $v = .75c$ in the positive x-direction.

[14] Later on his book (Lorentz 1895, sections 91–92, pp. 123–125; see Lorentz et al. 1952, pp. 5–7), Lorentz used the result in a well-known plausibility argument for the Lorentz-FitzGerald contraction hypothesis. See, e.g., Miller 1981, pp. 32-34. I will discuss this argument and the derivation of Eq. 1.9 in section 3.2.

[15] The partial derivative $\partial/\partial t$ gets replaced by $\partial/\partial t - v\, \partial/\partial x$, and the current density $\rho\mathbf{u}$ gets replaced by $\rho(\mathbf{u}+\mathbf{v})$.

co-moving Galilean frame. Since the auxiliary quantities obey the equations of electrostatics in a system at rest in the ether, solving the problem in terms of the auxiliary quantities boils down to solving a problem in electrostatics in a frame at rest in the ether. Not surprisingly, this problem in a frame at rest in the ether is just to find the forces in the charge distribution on the left of Fig. 1.6. So, the upshot is the following strategy for calculating forces in static charge distributions moving through the ether. Suppose we want to find the force **F** on some charge Q at point P in the static charge distribution on the right of Fig. 1.6 in a Galilean frame moving through the ether at a velocity v in the positive x-direction. Here is how we proceed. *(i)* We stretch out the charge distribution by a factor γ in the x-direction. *(ii)* We treat this stretched out charge distribution as being at rest in the ether. *(iii)* We calculate the force $\mathbf{F}'$ on the same charge Q at the corresponding point P' in this stretched out charge distribution at rest. *(iv)* We use Eq. 1.9 to get from $\mathbf{F}'$ to **F**.

I want to make three comments on this 1895 derivation of Lorentz's. First of all, it should be mentioned that, in Lorentz's 1895 book, the treatment of electrostatics in moving frames is completely separate from the very similar treatment of optics in moving frames involving the auxiliary quantity 'local time' (Lorentz 1895, sections 56–83, pp. 82–114). In 1899 Lorentz set out to give a unified treatment of all electromagnetic phenomena in moving frames of reference, a project he finished in 1904 (Lorentz 1899, 1904b). It is only after this synthesis that Lorentz starts referring to situations such as the ones shown in Fig. 1.6 as 'corresponding states' and to the strategy for dealing with electrostatics in moving frames as an application of 'the theorem of corresponding states.' In 1895 these locutions are reserved for optics.[16]

Secondly, I want to stress that Lorentz's 1895 derivation of Eq. 1.9 is completely independent of the Lorentz-FitzGerald contraction hypothesis. Eq. 1.9 holds irrespectively of any assumptions about what happens to the system on the left of Fig. 1.6 when it is given the same velocity as the system on the right. According to the contraction hypothesis the system on the left, when set in motion, will actually turn into the system on the right, but Eq. 1.9 also holds under the assumption that it will simply retain its shape. Lorentz's more general 'theorem of corresponding states' of 1904 also applies whether or not one assumes a physical contraction of the system under consideration. In 1904 the Lorentz-FitzGerald contraction is an integral part of Lorentz's own theory, but we can still use the theorem of corresponding states without it.[17]

[16] See chapter three for a more detailed discussion of this development and for references to the extensive secondary literature on this topic.

[17] Abraham (1903), for instance, used it to find the electromagnetic mass of his rigid spherical electrons. See, e.g., Miller 1981, pp. 55–61.

Finally, I want to draw attention to the fact that Lorentz is tacitly assuming that an observer at rest in the ether and an observer in the Galilean co-moving frame will agree upon the dimensions of the moving charge distribution. It was only after 1905, that Lorentz fully realized that to a co-moving observer a situation in a moving frame would always appear to be the corresponding state in a frame at rest (Lorentz 1916, pp. 223–230). In the example of Fig. 1.6, for instance, the moving charge distribution on the right would appear to a co-moving observer as the charge distribution on the left. Still, Lorentz maintained that this was only so because of the way all measuring instruments are affected by their motion through the ether. For a co-moving observer equipped with measuring devices that would not be disturbed by motion through the ether the dimensions of the moving charge distribution would be the same as for an observer at rest in the ether. In the example of Fig. 1.6, for instance, such an observer would be dealing with the charge distribution on the right rather than with the one on the left. It will be helpful to call this latter observer a “Galilean co-moving observer,” and the co-moving observer whose measuring instruments always show him the corresponding state a “Lorentzian co-moving observer.”

In summary, Fig. 1.6 and all other figures in this section are to be looked upon as follows. On the right, we have a moving system as observed by both an observer at rest (at rest with respect to the ether in the ether theory, at rest with respect to some inertial frame in relativity), and a Galilean co-moving observer. On the left, we have (in ether theoretic terms) the corresponding state of that system at rest in the ether, or (in relativistic terms) the moving system in its rest frame. To put it differently: on the left we have (in relativistic and post 1905 ether theoretic terms) the moving system as observed by a Lorentzian co-moving observer.

1.2.2 Application: the turning couple of the Coulomb forces on a moving charged condenser. We will be concerned with two different systems of the type shown in Fig. 1.6: a moving condenser undergoing the Lorentz-FitzGerald contraction and a moving condenser *not* undergoing the Lorentz-FitzGerald contraction. These two systems are shown in Fig. 1.7 and Fig. 1.8, respectively.

On the right, the systems are drawn as observed by either an observer at rest or a Galilean co-moving observer; on the left, they are shown as they appear in their rest frame, i.e., to a Lorentzian co-moving observer. Notice the shaded lines on the left that indicate how the systems on the left are obtained through stretching the systems on the right by a factor γ in the x-direction. In this way, the parallelogram representing the contracted moving condenser on the right in Fig. 1.7 turns into the rectangle representing the uncontracted condenser at rest on the left. Likewise, the rectangle representing the uncontracted moving condenser on the right in Fig. 1.8 turns into the parallelogram representing the stretched out condenser at rest on the left.

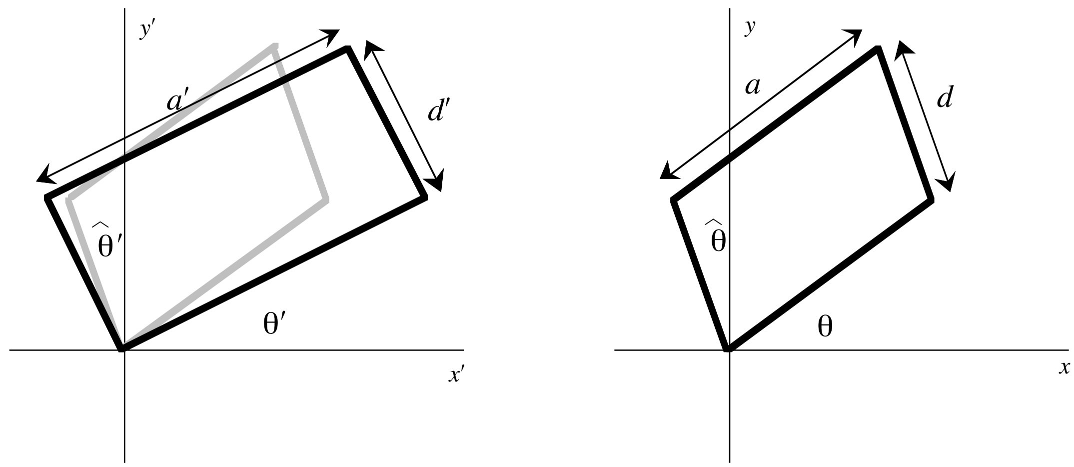

Figure 1.7 Moving condenser (with the Lorentz-FitzGerald contraction).

Fig. 1.7 represents the case of an ordinary moving plate condenser both in the ether theories of Lorentz and Larmor featuring the Lorentz-FitzGerald contraction, and in the special theory of relativity. The only difference is that in the ether theory the contraction is thought of as a dynamical effect, whereas in special relativity it is purely kinematical (see chapter two for discussion of this distinction).

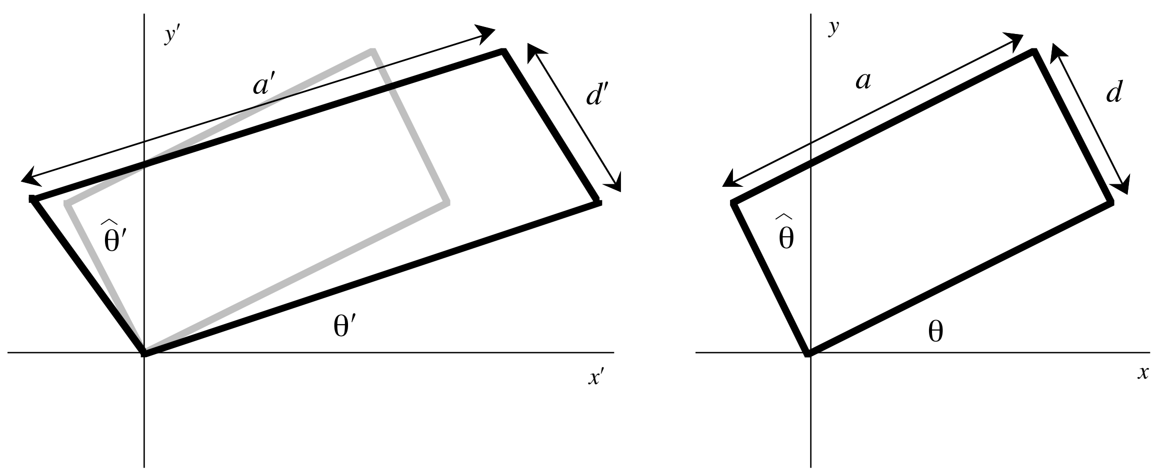

Figure 1.8 Moving condenser (without the Lorentz-FitzGerald contraction).

The system shown in Fig. 1.8, on the other hand, is rather odd from a relativistic point of view. It represents a squashed condenser that happens to be a regular rectangular shaped one in a reference frame in which it is moving in the x-direction and tilted at an angle θ with respect to the velocity v. Moreover, the (no Lorentz-FitzGerald contraction) ether theoretic and the relativistic accounts of what happens when we change the orientation of the moving system with respect to its velocity are very different. According to the ether theory the moving condenser will still be represented by the rectangle shown on the right of Fig. 1.8: that rectangle will just be tilted at a different angle θ. Consequently, the parallelogram on the left representing the corresponding state at rest in the ether will not only be tilted at a different angle θ', but will have a different shape as well. According to relativity theory, on the other hand, the rectangle on the right of Fig. 1.8 representing the squashed condenser in motion will turn into parallelogram the moment we change the orientation of the condenser with respect to its velocity. However, on the left of Fig. 1.8, in the squashed condenser's rest frame, it will always be represented by the same parallelogram, only tilted at different angles θ'.

Despite these differences of interpretation between relativity theory and ether theory with or without the Lorentz-FitzGerald contraction, quantities belonging to the systems in motion on the right of Fig. 1.7 and Fig. 1.8 can always be obtained from the corresponding quantities belonging to the systems at rest on the left through a Lorentz transformation, both in relativity theory and in ether theory.

Before I compute the forces acting on the plates of the condenser with the help of Eq. 1.9, I want to write down some relations that we will need later on. First, compare the lengths a and a' of the plates and the distances d and d' between them for the condensers shown in Fig. 1.7 and Fig. 1.8. For both figures, we have:

$$\begin{aligned} a' \cos\theta' &= \gamma\, a \cos\theta & \qquad d' \sin\widehat{\theta}' &= \gamma\, d \sin\widehat{\theta} \\ a' \sin\theta' &= a \sin\theta & \qquad d' \cos\widehat{\theta}' &= d \cos\widehat{\theta} \end{aligned} \tag{1.10}$$

In Fig. 1.7 (with the Lorentz-FitzGerald contraction) we have ($\theta' = \widehat{\theta}'$, $\theta \neq \widehat{\theta}$), whereas in Fig. 1.8 (without the Lorentz-FitzGerald contraction) we have ($\theta' \neq \widehat{\theta}'$, $\theta = \widehat{\theta}$).

From Eq. 1.10, we can easily derive expressions for the ratios a/a' and d/d'. In terms of the unprimed angles, these ratios are given by:

$$\frac{a}{a'} = \sqrt{\frac{1-\beta^2}{1-\beta^2 \sin\theta}}\,; \qquad \frac{d}{d'} = \sqrt{\frac{1-\beta^2}{1-\beta^2 \cos\widehat{\theta}}}\,. \tag{1.11}$$

In terms of the primed angles, these same ratios are given by:

$$\frac{a}{a'} = \sqrt{1-\beta^2 \cos\theta'}\,; \qquad \frac{d}{d'} = \sqrt{1-\beta^2 \sin\widehat{\theta}'}\,. \tag{1.12}$$

With the help of Fig. 1.9 and Fig. 1.10, I will calculate the moments of the Coulomb forces on the plates of a moving condenser, both when it does and when it does not undergo the

Lorentz-FitzGerald contraction.[18] We need the moments of these forces with respect to the torsion wire, the only axis of rotation for the moving condenser in the Trouton-Noble experiment. I have chosen a Galilean co-moving frame in which the z-axis coincides with the torsion wire (cf. Fig. 1.5). I will assume that the Coulomb forces on a plate of the condenser can all be represented by one Coulomb force on the center of mass of the plate.[19]

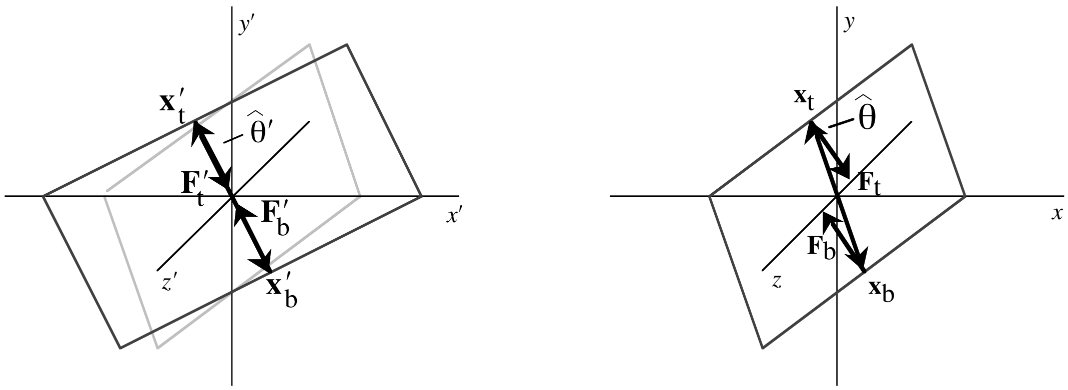

Figure 1.9 Forces on a charged moving condenser (with the Lorentz-FitzGerald contraction).

I will look at the case with the Lorentz-FitzGerald contraction first. In this case the condenser at rest (on the left of Fig. 1.9) has the familiar rectangular shape. Elementary electrostatics tells us what the net forces $\mathbf{F}'_t$ and $\mathbf{F}'_b$ on top and bottom plate of the condenser at rest will be. They are equal and opposite, and perpendicular to the plates. With the help of Eq. 1.9, we can then find the forces $\mathbf{F}_t$ and $\mathbf{F}_b$ on top and bottom plate of the moving condenser (on the right of Fig. 1.9). The turning couple on the moving condenser is just the sum of the moments of these forces with respect to the z-axis:

$$\mathbf{T} = \mathbf{x}_t \times \mathbf{F}_t + \mathbf{x}_b \times \mathbf{F}_b = 2\,\mathbf{x}_t \times \mathbf{F}_t, \tag{1.13}$$

where $\mathbf{x}_t$ and $\mathbf{x}_b$ are the position vectors of the centers of mass of the top and the bottom plate in the co-moving Galilean frame.[20] Applying Eq. 1.9, we find that:

$$\begin{aligned} \mathbf{F}_t &= \mathrm{diag}\left(1,\ {}^1\!/\!_\gamma,\ {}^1\!/\!_\gamma\right)\mathbf{F}'_t \\ &= F'\left(\sin\hat{\theta}', -\frac{1}{\gamma}\cos\hat{\theta}', 0\right), \end{aligned} \tag{1.14}$$

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[18] I am grateful to Hans Montanus for catching an error in an earlier version of this calculation.

[19] It turns out that the forces at the edges of the condenser give a turning couple of the same size as the turning couple we are about to compute here (see section 2.4, Figs. 2.9–2.10).

[20] From a relativistic point of view, we should use the position vectors in the Lorentz frame in which the condenser is moving, i.e., instead of $\mathbf{x}$, we should use $\mathbf{x} + \mathbf{v}t$. Since $\mathbf{F}_t = -\mathbf{F}_b$, this does not make any difference.

where F' is the size of the forces $\mathbf{F}_t'$ and $\mathbf{F}_b'$. For $\mathbf{x}_t$ we can write:

$$\begin{aligned} \mathbf{x}_t &= \frac{d}{2}(-\sin\hat{\theta}, \cos\hat{\theta}, 0) \\ &= \frac{d'}{2}(-\frac{1}{\gamma}\sin\hat{\theta}', \cos\hat{\theta}', 0), \end{aligned} \tag{1.15}$$

where in the second line Eq. 1.10 was used. Inserting Eq. 1.14 and Eq. 1.15 into Eq. 1.13, and using that $\gamma^{-2} = 1 - \beta^2$, we find for the turning couple:

$$\begin{aligned} \mathbf{T} &= (0, 0, 2\,(x_t F_{y_t} - y_t F_{x_t})) \\ &= (0, 0, -\,d' F' \beta^2 \sin\hat{\theta}' \cos\hat{\theta}'). \end{aligned} \tag{1.16}$$

So, the turning couple $\mathbf{T}$ points in the direction of the negative z-axis, which means that it will try to align the plates with the direction of motion. So, the direction of the turning couple is the opposite of the one calculated by Trouton. The size of the turning couple is half the size found by Trouton. To see this, notice that, with the help of some well-known relations from elementary electrostatics, $F'd'$ can be rewritten as:

$$F'\,d' = \tfrac{1}{2}Q\,E'\,d' = \tfrac{1}{2}Q\,(V'/d')\,d' = \tfrac{1}{2}Q\,V' = U', \tag{1.17}$$

where E' is the electric field in the condenser at rest, and V' is the potential difference between the plates in the condenser at rest. In the last step, the well-known expression for the energy content of a charged condenser in its rest frame was used: $U' = 1/2\ Q\ V'$. When Eq. 1.17 is inserted into Eq. 1.16, the z-component of Eq. 1.16 starts to look very similar to Eq. 1.8. In fact, the only difference, apart from the factor 1/2 I already mentioned, is the angle. This difference can be neglected. The difference between the various angles I distinguished in Fig. 1.7 and Fig. 1.8 is of order β^2. Since the turning couple itself is of order β^2, picking one angle rather than another only makes a difference of order β^4, which is completely negligible. Therefore, I will simply write θ in Eq. 1.16. For θ, one can pick any of the angles labeled in Fig. 1.7 and Fig. 1.8. So, we get a turning couple

$$\mathbf{T} = (0, 0, -\tfrac{1}{2}U'\,\beta^2 \sin 2\theta), \tag{1.18}$$

which is a turning couple half the size of the one found by Trouton and working in the opposite direction (cf. Eq. 1.8).

I now turn to the case without the Lorentz-FitzGerald contraction. This case is slightly more complicated than the case with the Lorentz-FitzGerald contraction, because the condenser at rest corresponding to the uncontracted moving condenser does not have the usual rectangular shape. Its plates are not exactly opposite to one another but slightly dislocated (see Fig. 1.10). Elementary electrostatics does not provide us with ready-made equations for this case. Fortunately, all we need is that the forces on top and bottom plate point in the direction indicated on the left of Fig. 1.10. Whatever the size of those forces, one easily convinces oneself that their line of work goes through the origin of the chosen reference frame.

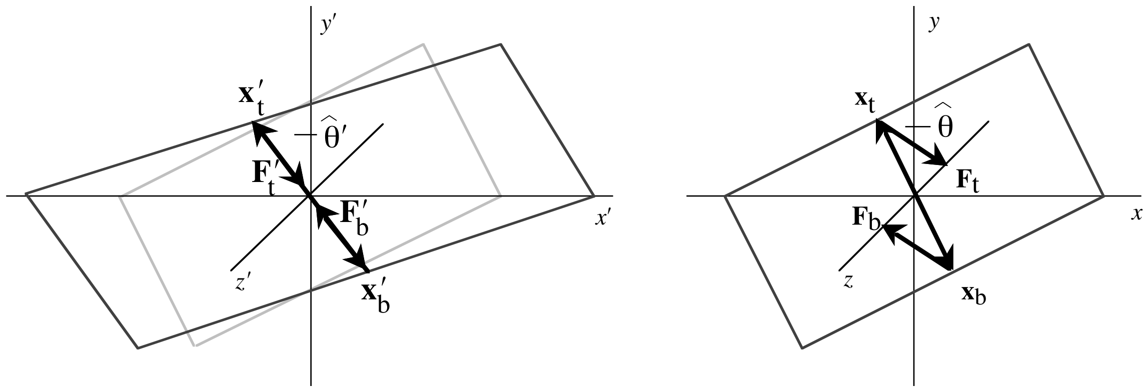

Figure 1.10: Forces on charged moving condenser (without the Lorentz-FitzGerald contraction).

From this point onwards, we can just copy line by line the derivation for the case with the Lorentz-FitzGerald contraction. The turning couple **T** on the condenser on the right of Fig. 1.10 will be equal to 2 $\mathbf{x}_t \times \mathbf{F}_t$ (cf. Eq. 1.13). Eq. 1.14 for $\mathbf{F}_t$ and Eq. 1.15 for $\mathbf{x}_t$ hold no matter whether $\mathbf{x}_t$ and $\mathbf{F}_t$ refer to Fig. 1.9 or to Fig. 1.10. Hence, we can just copy Eq. 1.16 for the turning couple. Now, at this point we are dealing with a quantity of order β^2. So, for F' and d' for the case without contraction we may just as well take F' and d' for the case with contraction. This means that we can again replace $F'd'$ by U' (see Eq. 1.17). For the angle, we can take any of the angles labeled in Fig. 1.7 or Fig. 1.8. The upshot of all of this is that to order β^2 we get the same turning couple **T** of Eq. 1.18 with and without the Lorentz-FitzGerald contraction. This result is in accordance with Lorentz's account of the Trouton-Noble experiment, but contradicts Larmor's account.

1.3 Larmor's 1902 'Energy'-account of the Trouton-Noble experiment

1.3.1 Extracting energy from the earth's motion through the ether, "unless the FitzGerald-Lorentz contraction is a fact." In section 1.1, I looked at Larmor's cryptic comments on Trouton's original experiment in the note he added to the reprint of Trouton's 1902 article in FitzGerald's scientific papers. I now turn to Larmor's discussion, in that same note, of the new proposal Trouton made, the proposal that would lead to the Trouton-Noble experiment. Unfortunately, Larmor's discussion of this proposal is as cryptic as his discussion of the Trouton experiment.[21] Nonetheless, I think it is possible to give a very plausible reconstruction of Larmor's argument. The conclusion of the argument is clear enough: without the Lorentz-FitzGerald contraction the electromagnetic forces exert a turning couple on a moving condenser, with the Lorentz-FitzGerald contraction they do not. My reconstruction of the argument with which Larmor tried to establish this conclusion looks rather convincing. However, as with the Trouton experiment, matters turned out to be more complicated than Larmor realized (see sections 1.4 and 2.4)[22]

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[21] Larmor has a reputation for being an opaque writer. Both Buchwald and Darrigol preface discussions of Larmor's work with comments to this effect. Buchwald writes: "Larmor was not gifted, to say the least, with stylistic clarity. Indeed, his is probably the most difficult of contemporary scientific locutions to decipher" (Buchwald 1985, pp. 141–142). Darrigol approvingly cites these remarks by Buchwald and writes: "Whereas Lorentz was known for his clarity and directness, Larmor's writings were notoriously difficult to read" (Darrigol 1994a, p. 299).

[22] Before I give my reconstruction of Larmor's argument, I need to address a more general historiographical issue. Andrew Warwick (private communication) has essentially dismissed my analysis of Larmor's argument because I do not take into account the tradition in which Larmor worked, which is the Maxwellian tradition in electromagnetic theory in late 19th century Britain and Ireland (see, e.g., Warwick 1991, 1992; Buchwald 1985; Hunt 1991). I am aware of the fact that I look at Larmor's analysis of the experiments of Trouton and Noble through Lorentzian spectacles. However, contrary to Warwick, I claim that, *in the case at hand*, this is a perfectly respectable approach. There can be no question that Lorentz heavily influenced Larmor. Larmor had read Lorentz 1895 as early as April 1895 (Warwick 1991, p. 54). In his relentless but convincing analysis of the development of Larmor's theory, Darrigol writes: "Larmor's claim that his work remained largely independent [after he had read Lorentz 1895] should not be accepted. As we will see, the dramatic improvement of his theory, from a rough and partially misconceived scheme to a precise deductive theory, owed much to Lorentz's insights" (Darrigol 1994a, p. 316). Most importantly for my purposes, Darrigol writes: "I believe that Larmor's and Lorentz' interpretations of the "corresponding states" were identical. For a different view, cf. Warwick [1991], p. 63" (Darrigol 1994a, p. 320, footnote 130). Since there is a lot of confusion in the secondary literature about Lorentz's own interpretation of the theorem of corresponding states, it is important to point out that I fully agree with Darrigol's understanding of Lorentz. It is not clear to me exactly what Warwick's reading of Lorentz is, but in the passage Darrigol is referring to, Warwick attributes the following interpretation of corresponding states to Larmor: "An observer moving through the ether with the earth did not measure the *real* ether fields (**E**, **B**)—the fields that would be measured by an observer stationary in the ether—but rather [the Lorentz transformed fields]" (Warwick 1991, p. 63). Warwick cites Larmor 1904 in this context. I have been unable to find anything in this paper that would support Warwick's claim. As we will see in chapter three, the interpretation Warwick attributes to Larmor is a depressingly common mis-interpretation of Lorentz's own pre-1905 usage of corresponding states. This circumstance and the fact that Warwick has so far failed to produce any textual evidence for his claim about Larmor lead me to believe that Darrigol is probably right and that Lorentz and Larmor had the same interpretation of corresponding states. This means that, at least for the time being, I

The basic relation Larmor uses in his account of the Trouton-Noble experiment is the relation between the potential difference V between the plates in a moving condenser (contracted or uncontracted) and the potential difference V' between the plates of a corresponding condenser at rest in the ether, stretched out by a factor γ in the direction of motion (see Figs. 1.7–1.8):

$$V = \gamma\, V'. \tag{1.19}$$

This relation also holds relativistically. From Eq. 1.19 Larmor infers that the energy U a Galilean co-moving observer would need to charge the condenser is related to the energy U' an observer at rest in the ether would need to charge the corresponding stretched out condenser at rest through:

$$U = U'/\gamma. \tag{1.20}$$

This relation is not vindicated by special relativity (see Eqs. 1.40–1.42, Eqs. 1.68–1.77, and Eq. 2.132).

Eq. 1.20 is all that is needed for a qualitative understanding of Larmor's reasoning. Without the Lorentz-FitzGerald contraction U, it turns out, depends on θ, the angle between the plates of the moving condenser and its velocity, whereas with the Lorentz-FitzGerald contraction it does not. Larmor tacitly assumed that only the electromagnetic energy stored in the condenser can depend on θ. This is a very natural assumption given that Larmor and other ether theorists expected that, as a rule, a Galilean principle of relativity would obtain, electrodynamics being the exception to that rule. From this assumption it follows that the turning couple is the derivative of the energy U with respect to θ (cf. Eq. 1.7 and Eq. 1.33 below). Hence, without the Lorentz-FitzGerald contraction we expect a turning couple, with the Lorentz-FitzGerald contraction we do not.

Comparing Figs. 1.7 and 1.9 (for the case with the Lorentz-FitzGerald contraction) to Figs. 1.8 and 1.10 (for the case without the Lorentz-FitzGerald contraction), we can easily see why the θ-dependence of V and U is so different in these two cases. Suppose we rotate the moving condensers in Figs. 1.9 and 1.10 around the z-axes of the chosen reference frames from $\theta = \theta_1$ to $\theta = \theta_2$.

First, look at the drawings for the case without the Lorentz-FitzGerald contraction (Figs. 1.8 and 1.10). The rectangle on the right, representing the moving condenser, retains its shape as θ

can proceed with my admittedly Lorentzian analysis of Larmor's argument without having to immerse myself in the Maxwellian milieu first.

goes from θ_1 to θ_2. The parallelogram on the left, however, representing the corresponding condenser at rest, changes its shape as θ' accordingly goes from θ_1' to θ_2'.

In the case with the Lorentz-FitzGerald contraction (Figs. 1.7 and 1.9), the situation is just the reverse. The parallelogram on the right, representing the moving condenser, changes its shape as θ goes from θ_1 to θ_2, whereas the rectangle on the left, representing the corresponding condenser at rest, retains its shape as θ' accordingly goes from θ_1' to θ_2'.

The potential difference V' and the energy U' of a condenser at rest depend on its shape (on the area of its plates and the distance between them, for instance). Hence, in the case without the Lorentz-FitzGerald contraction, V' and U', and thereby, according to Eq. 1.19 and Eq. 1.20, V and U depend on θ, whereas in the case with the Lorentz-FitzGerald, V' and U', and, consequently, V and U are independent of θ.[23]

I want to take a closer look at Eq. 1.19 and Eq. 1.20. For Eq. 1.19 it will be helpful to adopt the relativistic perspective on the situation for a moment. For a Galilean co-moving observer the potential difference V between the plates of the moving condenser is the same as for an observer at rest in the frame in which the ether is at rest and in which the condenser is moving at a velocity v in the x-direction. The potential difference V' between the plates of the corresponding condenser at rest in the ether is the same as the potential difference between the plates of the moving condenser in its rest frame, i.e., the potential difference for a Lorentzian co-moving observer (cf. section 1.2). So, V and V' are related through a Lorentz transformation. The potential difference between the plates of a condenser is more explicitly written as $\phi_t - \phi_b$, where ϕ_t is the potential of the top plate and ϕ_b is the potential of the bottom plate. In special relativity, the potential ϕ, divided by c, becomes the first component of the four-vector potential $A^\mu \equiv (\phi/c, \mathbf{A})$, where $\mathbf{A}$ is the ordinary (three-) vector potential. In the condenser's rest frame, the (three-) vector potential can be taken to vanish ($\mathbf{A}' = 0$). So, $A'^\mu = (\phi'/c, 0, 0, 0)$. In a frame in which the ether is at rest and in which the condenser is moving at a velocity v in the direction of the positive x-axis, the four-vector potential will consequently be $A^\mu = (\gamma\phi'/c, \gamma\beta\phi'/c, 0, 0)$. Hence, $\phi = \gamma\phi'$ and $V = \gamma V'$, which is just Eq. 1.19.

Larmor does not derive Eq. 1.19 in the note in FitzGerald's scientific papers. He calls it a "known electrodynamic result," and refers to his 1900 book *Æther and matter* for a derivation

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[23] Langevin gave a similar argument on the basis of the relation $L = L'/\gamma$ between the Lagrangians for the electromagnetic fields in the corresponding systems in rest and in motion (Langevin 1905b). To infer the absence of a turning couple from this orientation independence of the Lagrangian for the field, one needs to assume that the orientation of the condenser can only make a difference for the electromagnetic part of the system. Langevin, like Trouton and Larmor before him, tacitly made this assumption.

An argument along the lines of these arguments for the Trouton-Noble experiment can be worked out for the Michelson-Morley experiment (see chapter three).

(Larmor 1902, p. 567).[24] As far as I know, the relation was first derived by Lorentz in 1895 in the course of his calculations on electrostatics in moving reference frames that I discussed in section 1.2 (Lorentz 1895, p. 37). Larmor does not refer to Lorentz when he derives the relation, even though he quotes extensively from Lorentz's 1895 book elsewhere in *Æther and matter*.[25] For our purposes it is not necessary to look at either Larmor's or Lorentz's derivation of Eq. 1.19.

What is important for our purposes, though, is to see how Larmor got from Eq. 1.19 to Eq. 1.20. Larmor writes:

> Now the condenser is charged by transferring the charges into the plates against the electric force; the energy required for this operation is half the charge Q multiplied by the potential difference between the plates. (Larmor 1902, p. 568)

On the basis of this statement, one would expect Larmor to use the relation $U = \gamma U'$ rather than $U = U'/\gamma$ (see Eq. 1.19 and Eq. 1.20). In the actual calculations, however, Larmor seems to use the latter rather than the former relation. So, he does *not* simply assume that the relation $U' = 1/2\ Q\ V'$, which holds for an ordinary plate condenser at rest, also holds for a moving condenser. Rather, he assumes that for a moving condenser the relation is

$$U = \frac{1}{2}(1 - \beta^2)\, Q\, V. \qquad (1.21)$$

This is the result we find when we actually do the calculation suggested by Larmor in the sentence quoted above. The energy U needed to charge a moving condenser ($+Q$ on the top plate and $-Q$ on the bottom plate) can be written as:

$$U = \int_0^Q dW(q)\,, \qquad (1.22)$$

---

[24] The reference is to Larmor 1900, section 96, p. 153. There is a typo in *Æther and matter* at this point that gets copied in Larmor's 1902 paper. Larmor writes that we should multiply the potential difference in the system at rest by some quantity ε to obtain the potential difference in the corresponding system in motion, i.e., that $V = \varepsilon V'$. The quantity ε is defined as $(1 - v^2/C^2)^{-1}$, where C is Larmor's notation for the velocity of light in vacuo. Hence, $V = \varepsilon V'$ would be $V = \gamma^2 V'$. In the list of *corrigenda* at the beginning of *Æther and matter*, this mistake was already corrected: "for ε read $\varepsilon^{1/2}$" (Larmor 1900, p. *xxvii*). Larmor would eventually correct the typo in his 1902 paper as well (Larmor 1929, p. 226). When Larmor actually derived the expression for the turning couple he gives in his paper, he either used the correct relation or made another error canceling this one, for there is no trace of the spurious factor $\varepsilon^{1/2}$ in the end result.

[25] Larmor, in fact, included (with some minor omissions) a translation (Larmor 1900, pp. 185–186) of the plausibility argument that Lorentz gave for the Lorentz–FitzGerald contraction (Lorentz 1895, pp. 123–125), an argument that was based on Lorentz's calculations for electrostatics in moving frames earlier in his book (Lorentz 1895, pp. 19–23). See sections 1.2 and 3.2.

where *dW(q)* is the amount of work needed to transfer an infinitesimal positive charge *dq* from the bottom plate to the top plate when there already is a charge +*q* on the top plate and a charge –*q* on the bottom plate. For *dW(q)* we can write:

$$dW(q) = \int_{\text{bottom}}^{\text{top}} \mathbf{F}_{\text{on } dq} \cdot d\mathbf{s}. \tag{1.23}$$

To charge the moving condenser a battery moving along with the condenser is used. It therefore seems reasonable to assume that we can use a co-moving Galilean frame to evaluate the integral in Eq. 1.23.[26]

Another worry at this point is whether the integral in Eq. 1.23 is independent of the path along which we choose to transport the charge *dq* from the bottom to the top plate. This worry will be dispelled in the course of evaluating the integral.

Once again, compare the situation in the moving frame to its corresponding state at rest in the ether (see Figs. 1.7 and 1.10). To the process of charging the plates of the condenser moving through the ether, there corresponds the process of charging a stretched out condenser at rest in the ether; and to every path between the plates of the moving condenser in a co-moving Galilean frame, there corresponds a path between the plates of the stretched out condenser at rest in the ether. At any point in the two corresponding processes of charging the condenser in motion and the one at rest, the relation between the forces experienced by a charge *dq* at corresponding points of a pair of such paths is given by Eq. 1.9. Furthermore the relation between the infinitesimal segments $d\mathbf{s}$ and $d\mathbf{s}'$ of the two paths is given by:

$$d\mathbf{s} = \text{diag}(1/\gamma, 1, 1)\ \ d\mathbf{s}'. \tag{1.24}$$

With the help of Eq. 1.9 and Eq. 1.24, Eq. 1.23 can be rewritten as:

$$dW(q) = \int_{\text{bottom}}^{\text{top}} \left(\text{diag}(1, 1/\gamma, 1/\gamma)\ \mathbf{F}'_{\text{on } dq}\right) \cdot \left(\text{diag}(1/\gamma, 1, 1)\ \ d\mathbf{s}'\right), \tag{1.25}$$

where ‘top’ and ‘bottom’ now refer to the plates of the stretched out condenser at rest. From Eq. 1.25, it follows that there is a simple relation between the quantity *dW(q)* for the moving condenser and the corresponding quantity *dW′(q)* for the stretched out condenser at rest:

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26 Perhaps the easiest way to see that we actually have to be a little more careful (see sections 1.4 and 2.4) is through invoking the equivalence of mass and energy. When we charge a moving condenser, we increase both its rest mass and its kinetic energy. Larmor, in effect, fails to take into account the increase in kinetic energy.

$$dW(q) = \frac{1}{\gamma}\int_{\text{bottom}}^{\text{top}} \mathbf{F}'_{\text{on }dq} \cdot d\mathbf{s}' = dW'(q)\big/\gamma. \tag{1.26}$$

Since the work done when we transport a charge dq from the bottom to the top plate of a condenser *at rest*—or between any two points in an arbitrary static charge distribution *at rest*, for that matter—is independent of the path we choose, it follows from Eq. 1.26 that this will also be the case for a condenser—or some arbitrary static charge distribution—in motion. When Eq. 1.26 is inserted into Eq. 1.22, we arrive at Eq. 1.20:

$$U = \frac{1}{\gamma}\int_0^Q dW'(q) = U'\big/\gamma. \tag{1.27}$$

Using $U' = 1/2QV'$ and $V = \gamma V'$ (Eq. 1.19), one easily verifies that Eq. 1.27 implies Eq. 1.21.

Eq. 1.27 holds no matter whether we assume that a moving condenser undergoes the Lorentz-FitzGerald contraction or not. Consider the case without the Lorentz-FitzGerald contraction. In that case the energy U' and the potential difference V' will depend on θ, since the shape of the condenser at rest corresponding to the one in motion will depend on θ. Larmor looks at the special cases $\theta = 0$ and $\theta = \pi/2$. For $\theta = 0$, the condenser at rest will have plates of area $A' = \gamma A$ at a distance $d' = d$ apart; for $\theta = \pi/2$, it will have plates of area $A' = A$ at a distance $d' = \gamma d$ apart (see Eq. 1.10). The energy U' and the potential difference V' of a condenser at rest are proportional to d'/A'. From these proportionalities, the relation $V = \gamma V'$ (Eq. 1.19) and Larmor's relation $U = U'/\gamma$ (Eq. 1.20), it follows that:

$$\frac{U_{\theta=0}}{U_{\theta=\pi/2}} = \frac{V_{\theta=0}}{V_{\theta=\pi/2}} = \left(\frac{d'}{A'}\right)_{\theta=0}\left(\frac{A'}{d'}\right)_{\theta=\pi/2} = \frac{d}{\gamma A}\,\frac{A}{\gamma d} = 1-\beta^2. \tag{1.28}$$

So, according to Larmor, the potential difference and the electromagnetic energy of a moving condenser that does not undergo the Lorentz-FitzGerald contraction are smaller for $\theta = 0$ than for $\theta = \pi/2$.[27] Immediately after the sentence quoted above Larmor writes:

> As it is charged at one potential difference [i.e., $V_{\theta=\pi/2}$] and discharged at another [i.e., $V_{\theta=0}$], there is energy remaining over of the amount estimated above [i.e., $\beta^2 U_{\theta=\pi/2}$]; and as the process is reversible, this energy must be mechanically available. Thus the energy of motion of the Earth through the æther is available for mechanical work to an unlimited extent, unless the potential difference in the condenser is independent of its orientation; that is, by accepted electrodynamics, unless the FitzGerald-Lorentz contraction is a fact (Larmor 1902, p. 568).

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[27] As I pointed out in section 1.1, this is rather counter-intuitive: the position without a magnetic field ($\theta = \pi/2$) turns out to have lower electromagnetic energy than the position with a magnetic field ($\theta = 0$).

This is Larmor's rather cryptic statement of the argument I spelled out in great detail above. If "the FitzGerald-Lorentz contraction is a fact," the ratio d'/A' will be independent of θ, the right hand side of Eq. 1.28 will be 1, and the argument given above no longer goes through. In other words, if "the FitzGerald-Lorentz contraction is a fact," it will not be possible to extract energy from the earth's motion through the ether via turning couples on charged condensers.

2.3.2 The turning couple without the contraction hypothesis. Assuming that only the electromagnetic energy of a moving condenser depends on θ, we can infer from Eq. 1.28 that, if the Lorentz-FitzGerald contraction were not "a fact," there would be a turning couple acting on a moving condenser trying to align its plates with the direction of motion. In this respect Larmor's result disagrees with the result found by Trouton and agrees with the result found by Lorentz. To conclude this section, I will show that the derivation suggested by Larmor gives a turning couple of the same size as the one found by Trouton and Lorentz. The reader who is prepared to take this on faith can move on to the discussion of Lorentz's account in section 1.4 without loss of continuity.

To find the turning couple on a moving condenser that does not undergo the Lorentz-FitzGerald contraction, Larmor needs an expression for U explicitly showing its θ-dependence. As I mentioned above, U, which, according to Larmor, is equal to U'/γ, depends on θ via the shape of the corresponding stretched out condenser at rest in the ether for which we have to evaluate U'. To make this θ-dependence explicit, I will derive a relation between U' and U_0, the energy stored in a condenser of the same rectangular shape as the one in motion, and carrying the same charge, but being at rest in the ether. Hence, U_0 will not depend on θ. The θ-dependent shape of the stretched out condenser at rest will in general differ from the θ-independent shape of the moving condenser in three ways. The distance between the plates will be different, the area of the plates will be different, and the plates of the stretched out condenser will not be exactly opposite to one another. This last difference does not play a role when $\theta = 0$ and $\theta = \pi/2$, the special cases Larmor looked at. If we assume that the area of the plates is very large compared to the distance between the plates, this complication can be neglected for arbitrary θ. The parallelogram-shaped condenser can then be replaced by an ordinary rectangular-shaped one with plates of area A', a distance d' apart. Since the energy stored in an ordinary plate condenser for some fixed charge is proportional to the distance of the plates and inversely proportional to the area of the plates, the relation between U' and U_0 is given by:

$$U' = \frac{d'A}{dA'} U_0 \,. \tag{1.29}$$

Combining Eq. 1.20 and Eq. 1.29, we arrive at:

$$U = \frac{1}{\gamma}\frac{d'A}{dA'}\,U_0\ . \tag{1.30}$$

Now use Eq. 1.11 for the ratios d'/d and $a/a' = A/A'$. Since we are dealing with the case without the Lorentz-FitzGerald contraction, we have $\theta=\widehat{\theta}$. So, the θ-dependence of U is given by:

$$U(\theta) = \sqrt{1-\beta^2}\,\sqrt{\frac{1-\beta^2\cos^2\theta}{1-\beta^2\sin^2\theta}}\;U_0. \tag{1.31}$$

When all terms smaller than of order β^2 are neglected, Eq. 1.31 becomes:

$$U(\theta) = U_0\,(1-\beta^2\cos^2\theta) \tag{1.32}$$

The expression for $U(\theta)$ derived by Trouton had a plus rather than a minus sign in the factor between parentheses (see Eq. 1.4[28]). Hence, Trouton and Larmor disagree over which angle minimizes the energy (i.e., $\theta = \pi/2$ or $\theta = 0$), and, consequently, over the direction of the turning couple on the moving condenser (i.e., counterclockwise or clockwise).

According to Larmor we have to do an amount of work ΔW to rotate the condenser counterclockwise from some angle θ to some slightly bigger angle $\theta+\Delta\theta$. This amount of work ΔW will be equal to $T(\theta)\Delta\theta$ (cf. Fig. 1.4), where $T(\theta)$ is the size of the turning couple acting on the condenser. On the other hand, $\Delta W = U(\theta+\Delta\theta) - U(\theta)$. In the limit $\Delta\theta \rightarrow 0$, we obtain (cf. Eq. 1.7):

$$T(\theta) = \frac{dU(\theta)}{d\theta}\ . \tag{1.33}$$

Inserting Eq. 1.32 into Eq. 1.33, we find that the size of the turning couple is given by:[29]

$$T(\theta) = U_0\,\beta^2\sin 2\theta\ , \tag{1.34}$$

in agreement with the results found by Trouton (see Eq. 1.8) and Lorentz (see Eq. 1.39[30]).

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[28] U' in Eq. 1.4 is actually what I called U_0 here.

[29] Cf. Larmor 1902, p. 568. Notice that Larmor's θ is the complement of the angle I call θ. Since $\sin 2(\pi/2-\theta) = \sin 2\theta$, this makes no difference.

[30] U_0 in Eq. 1.34 and U' in Eq. 1.39 only differ by something in the order of β^2.

1.4 Lorentz's 1904 'Momentum'-account of the experiments of Trouton and Noble

1.4.1 Lorentz on the experiments of Trouton and Noble. Both the Trouton and the Trouton-Noble experiments are discussed in Lorentz's famous 1904 paper.[31] Trouton's original experiment is dealt with in the last section and does not play any role in the body of the paper.[32] The Trouton-Noble experiment, on the other hand, is discussed in some detail in the introduction and is presented as one of two new second order etherdrift experiments that partly motivated the paper.[33] Lorentz's approach to the problem of a condenser moving through the ether is very different from the approach taken by Larmor.[34] It is cast in terms of a quantity for which Abraham (1903) had introduced the term "electromagnetic momentum." In modern notation, the electromagnetic momentum **G** of some arbitrary charge distribution is defined as:

$$\mathbf{G} = \int d^3x \;\; \varepsilon_0 \, \mathbf{E} \times \mathbf{B} \, , \tag{1.35}$$

where **E** and **B** are the fields generated by the charge distribution. The integration stretches out over all space.

Fig. 1.11 shows the electromagnetic momentum **G** of a charged moving condenser. It turns out that we only need the electromagnetic momentum up to first order in β to account for the Trouton and Trouton-Noble experiments, so we do not have to distinguish between the case with and the case without the Lorentz-FitzGerald contraction. To first order in β, the moving

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31 Lorentz 1904b, pp. 172–173, p. 190 (Trouton-Noble), pp. 194–197 (Trouton).

32 This section was omitted when Lorentz's paper was reprinted in Blumenthal 1913, a well-known collection of papers on relativity (Miller 1981, p. 391). Presumably, this was not just to save space. Lorentz thought that, in principle, Trouton should have found a positive effect, and that the reason he had not was that his apparatus had not been sensitive enough. Clearly, it would be awkward for an anthology of papers on the principle of relativity to include a passage implying that more accurate measurements might actually produce a violation of the principle.

33 The other one was an experiment first performed by Rayleigh (1902) and repeated by Brace (1904) to see whether the Lorentz-FitzGerald contraction would cause a body to become doubly refracting. The new theory Lorentz put forward in his paper was further motivated by the criticism of Poincaré (1900a) of the 1895 version of the theory (Lorentz 1904b, pp. 173–174).

34 As far as I know, Lorentz never explicitly mentioned that his explanation of the Trouton-Noble experiment is incompatible with Larmor's. He certainly knew about Larmor's explanation though. Both in Lorentz 1904a, p. 259, and in Lorentz 1904b, p. 194, he mentions the reprint of Trouton's 1902 paper in FitzGerald's collected papers which has Larmor's note attached to it. Moreover, the Lorentz Archives contain a document dated October 1902 in which Lorentz checks some calculations in both Trouton's and Larmor's papers (Archief H.A. Lorentz, Rijksarchief Noord-Holland, Haarlem, The Netherlands, No. 266). The document is entitled: "Aantekeningen en berekeningen betreffende de electromagnetische lichttheorie, het experiment van Trouton en de krachten op een bolvormig electron" ("Notes and calculations concerning the electromagnetic theory of light, Trouton's experiment and the forces acting on a spherical electron"). Unfortunately, this document offers no clue as to how Lorentz saw the relation between Larmor's explanation of the Trouton-Noble experiment and his own.

condenser can be represented by a rectangle in both cases, and the electromagnetic momentum **G** is parallel to the plates.

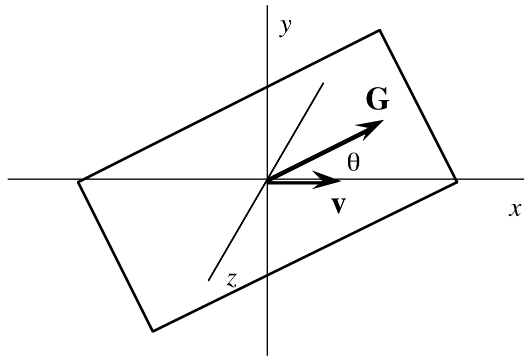

Figure 1.11 Electromagnetic momentum **G** of the field of a moving condenser.

In the coordinate system shown in Fig. 1.11, the same as the one I used in sections 1.2 and 1.3, the electromagnetic momentum is given by (cf. Lorentz 1904b, p. 173):[35]

$$\mathbf{G} = 2\,(U'/c)\,\beta\cos\theta\,(\cos\theta, \sin\theta, 0), \tag{1.36}$$

where U' is the electromagnetic energy the condenser would have if it carried the same charge while being at rest in the ether. This expression plays a central role in Lorentz's account of both the Trouton and the Trouton-Noble experiments.

In the case of the Trouton experiment, Lorentz simply used Eq. 1.36 in conjunction with the Newtonian law of conservation of momentum. When the condenser is charged, he argued, it gains electromagnetic momentum **G**, hence it should experience a change in ordinary momentum of –**G**. When it is discharged, it loses electromagnetic momentum **G**, and should therefore experience a change in ordinary momentum of +**G**. Lorentz goes on to show that Trouton's apparatus was not sensitive enough to detect this effect. In the section of his paper devoted to the Trouton experiment, Lorentz writes:

> I take this opportunity for mentioning an experiment that has been made by Trouton at the suggestion of FitzGerald, and in which it was tried to observe the existence of a sudden impulse acting on a condenser at the moment of charging or discharging; for this purpose the condenser was suspended by a torsion balance, with its plates parallel to the earth's motion. For forming an estimate of the effect that may be expected, it will suffice to consider a condenser with aether as dielectricum. Now if the apparatus is charged there will be (§ 1) an electromagnetic momentum
>
> $$\mathbf{G} = \frac{2U}{c^2}\,\mathbf{w}.$$ [36]

[35] The expression given by Lorentz is somewhat different because he used a slightly different coordinate system. Eq. 1.36 will be derived below (see Eqs. 1.63–1.67).

> (Terms of the third and higher orders are here neglected). This momentum being produced at the moment of charging, and disappearing at that of discharging, the condenser must experience in the first case an impulse –**G** and in the second an impulse +**G**.
>
> However Trouton has not been able to observe these jerks [sic].
>
> I believe it may be shown (though his calculations have led him to a different conclusion) that the sensibility of the apparatus was far from sufficient for the object Trouton had in view. (Lorentz 1904b, pp. 194–196)

As we will see below, Lorentz's argument for the Trouton experiment is a-typical of his general approach. Usually, Lorentz is far more cautious about the interpretation of the quantity **G** as a form of momentum.

In the case of the Trouton-Noble experiment, one arrives at the expression Lorentz gives for the turning couple through the following simple argument. As we will see below, Lorentz's own reasoning was more complicated. For the moment, however, consider the electromagnetic angular momentum $\mathbf{L} = \mathbf{x} \times \mathbf{G}$,[37] associated with the electromagnetic momentum **G**, where **x** is the position vector of the geometrical center of the condenser with respect to a reference frame at rest in the ether. When we want to change the angular momentum, we need to apply an external turning couple $\mathbf{T}_{\text{ext}} = d\mathbf{L}/dt$. This turning couple is just the opposite of the turning couple **T** the condenser experiences from the field between its plates. Hence,

$$\mathbf{T} = -\frac{d\mathbf{L}}{dt} = -\frac{d}{dt}\left(\mathbf{x} \times \mathbf{G}\right). \qquad (1.37)$$

Since $d\mathbf{G}/dt = 0$ (see Eq. 1.36) and $d\mathbf{x}/dt = \mathbf{v}$, Eq. 1.37 reduces to:

$$\mathbf{T} = -\,\mathbf{v} \times \mathbf{G} \qquad (1.38)$$

From Eq. 1.38 and Eq. 1.36, it is immediately clear that we only need **G** up to first order in β to find the turning couple: β^3-terms in **G** (there are no β^2-terms) would only make a difference of order β^4 in **T**. The Lorentz-FitzGerald contraction is an effect of order β^2. It follows that, according to Lorentz and contrary to what Larmor thought (see section 1.3), the same turning couple will be present no matter whether the condenser undergoes the Lorentz-FitzGerald contraction or not.

---

[36] Lorentz actually uses Gothic letters to represent the momentum vector **G** and the velocity vector **w**. To first order in β, the electromagnetic energy U in the moving condenser is equal to the electromagnetic energy U¢ of the condenser at rest (see, e.g., Eq. 1.42 and Eq. Eq. 1.73 below).

[37] The electromagnetic momentum **G** is a space integral over the electromagnetic momentum density **g**. Likewise, the electromagnetic angular momentum **L** is a space integral over the electromagnetic angular momentum density $\mathbf{l} = \mathbf{x} \times \mathbf{g}$. I want to emphasize that at this point I only want to show that on the basis of Abraham's interpretation of the vector **g** as electromagnetic momentum density, one arrives at the correct expression for the turning couple through a very simple intuitive argument. Let me reassure the reader that I am aware of the fact that this argument does not constitute a derivation of the result. Derivations will be given later, in Eqs. 1.43–1.62 below and in Eqs. 2.44–2.76 in section 2.2.

Inserting Eq. 1.36 into Eq. 1.38, we recover the by now familiar expression for the turning couple (cf. Eq. 1.8 (Trouton) and Eq. 1.34 (Larmor)):

$$\mathbf{T} = (0, 0, -v\, G_y) = (0, 0, -U'\, \beta^2 \sin 2\theta). \tag{1.39}$$

Compare the argument given above with what Lorentz himself has to say about the Trouton-Noble experiment in the introduction of his 1904 paper:

> In the second place [Lorentz has just mentioned the experiments of Rayleigh and Brace] Trouton and Noble4) [the corresponding footnote gives the reference to Trouton and Noble 1903] have endeavoured to detect a turning couple acting on a charged condenser, whose plates make a certain angle with the direction of translation. The theory of electrons, *unless it be modified by some new hypothesis* [my italics], would undoubtedly require the existence of such a couple. In order to see this, it will suffice to consider a condenser with aether as dielectricum. It may be shown that in every electrostatic system, moving with a velocity **w**1),[38] there is a certain amount of "electromagnetic momentum". If we represent this, in direction and magnitude, by a vector **G**, the couple in question will be determined by the vector product2) [a reference to Lorentz 1904a, section 21a]
>
> $$[\mathbf{G}\,.\,\mathbf{w}] \tag{1}$$
>
> Now, if the axis of z is chosen perpendicular to the condenser plates, the velocity **w** having any direction we like, and if U is the energy of the condenser, calculated in the ordinary way, the components of **G** are given3) [a reference to Lorentz 1904a, section 56c] by the following formulae, which are exact up to the first order:
>
> $$G_x = \frac{2U}{c^2}\, w_x, \quad G_y = \frac{2U}{c^2}\, w_y, \quad G_z = 0.$$
>
> Substituting these values in (1), we get for the components of the couple, up to terms of the second order,
>
> $$\frac{2U}{c^2}\, w_y w_z, \quad -\frac{2U}{c^2}\, w_x w_z, \quad 0.$$
>
> These expressions show that the axis of the couple lies in the plane of the plates, perpendicular to the translation. If α is the angle between the velocity and the normal to the plates, the moment of the couple will be $Uw^2 \sin 2\alpha / c^2$; it tends to turn the condenser into such a position that the plates are parallel to the Earth's motion.
>
> In the apparatus of Trouton and Noble the condenser was fixed to the beam of a torsion balance, sufficiently delicate to be deflected by a couple of the above order of magnitude. No effect could however be observed. (Lorentz 1904b, pp. 172–173))

The equations given in this passage are easily seen to be equivalent with to Eq. 1.36 and Eqs. 1.38–1.39. It may seem, therefore, at first sight, that Lorentz's argument is not all that different from the argument I gave above. This impression is misleading. As can be gathered from the two references to his article for the *Encyklopädie der Mathematischen Wissenschaften* (Lorentz

[38] In the corresponding footnote, Lorentz informs his readers that he will use Gothic letters to represent vectors. I will continue to use bold type face instead.

1904a), the argument offered in the passage quoted above is intended only as a convenient short-cut for a rigorous derivation to be found in that earlier article. Before I elaborate on this, I want to make two other comments on the passage quoted above.

First, notice the clause I italicized: "unless it be modified by some new hypothesis." As one would expect , Lorentz, in the course of his paper, introduces some new hypotheses that allow him to explain the negative results of various ether drift experiments. He never explicitly says exactly which hypotheses are needed in the case of the Trouton-Noble experiment, but on the basis of what he does say, it is possible, I think, to give a unique reconstruction of the explanation he had in mind.[39] Lorentz briefly returns to the Trouton-Noble experiment in section 11 of his paper, after he has finished his exposition of his new theory for the electrodynamics in moving frames of reference:[40]

> It is easily seen that the proposed theory can account for a large number of facts. [...]
>
> As to the experiments of Trouton and Noble, their negative result becomes at once clear, if we admit the hypotheses of § 8. It may be inferred from these and from our last assumption (§ 10) that the only effect of the translation must have been a contraction of the whole system of electrons and other particles constituting the charged condenser and the beam and thread of the torsion balance. Such a contraction does not give rise to a sensible change of direction. (Lorentz 1904b, pp. 189–190)

One of the assumptions Lorentz makes in section 8 of his paper (ibid., p. 183) is that the relation $\mathbf{F} = \mathrm{diag}(1, 1/\gamma, 1/\gamma)\, \mathbf{F}'$ (Eq. 1.9) for Coulomb forces also holds for molecular forces. This is all that is needed to understand the negative result of the Trouton-Noble experiment in Lorentz's theory. Consider the Coulomb forces on the plates of a condenser at rest in the ether. Those forces try to pull the plates of the condenser together. Molecular forces exactly opposite to these Coulomb forces prevent this from happening. When the condenser is set in motion, the Coulomb forces and molecular forces are assumed to change in exactly the same way. If the Coulomb forces on the moving condenser give rise to a turning couple, the molecular forces on the moving condenser give rise to a turning couple of equal size working in the opposite direction. There will be no net turning couple.[41]

Notice that the picture painted by Lorentz is very different from the picture painted by Larmor. According to Larmor there is no turning couple on a moving condenser having

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[39] Laue would be the first to clearly state this reconstruction (Laue 1911a, p. 136). I am grateful to A. J. Kox for pointing out to me that what has come to be known as Lorentz's explanation of the null result of the Trouton-Noble experiment, is, strictly speaking, Laue's interpretation of Lorentz's explanation.

[40] See chapter three for a discussion of this theory. Once we have a clear grasp of this theory, it will also become clear why Lorentz phrased his account of the Trouton-Noble experiment the way he did (see section 4.1).

[41] As I mentioned above, this interpretation of Lorentz's reasoning is due to Laue: "In the co-moving system, the electrostatic forces are canceled by the molecular cohesion, otherwise the condenser would not be in equilibrium. At every point, therefore, the sum of the electric and the molecular forces is zero. If the two types of forces transform in the same way to other frames of reference, their sum will remain zero in all frames and there will be no cause for rotation" (Laue 1911a, p. 136).

undergone the Lorentz-FitzGerald contraction, whereas according to Lorentz there are two turning couples that cancel one another.

That leads me to my second comment. From what has been said so far about Lorentz's account, it is already clear that from a Lorentzian point of view, we cannot accept the simple relation $U = U'/\gamma$ between the energy U of the moving condenser and the energy U' of the corresponding condenser at rest that formed the basis of Larmor's account of the Trouton-Noble experiment. As we saw in section 1.3, the derivation of this relation involves the assumption that in charging a moving condenser by transferring charges from the bottom to the top plate no work is needed to build up momentum. According to Lorentz, however, electromagnetic momentum is created when charging a moving condenser. This will cost some energy U^G. This energy can be written as:

$$U^G = \int_0^Q \frac{d\mathbf{G}}{dt} \cdot \; d\mathbf{s} = \int_0^Q dG_x \, v = v \int_0^Q \frac{dG_x}{dq} \, dq \; = v \; G_x \Big|_0^Q . \tag{1.40}$$

Inserting Eq. 1.36 into Eq. 1.40, we find that, to order β^2, U^G is given by:

$$U^G = 2 \; U' \; \beta^2 \cos^2 \theta. \tag{1.41}$$

This energy should be added to the energy $U = U'/\gamma \approx U' \; (1 - 1/2\beta^2)$ which, according to Larmor, is needed to charge the moving condenser. Hence, in Lorentz's theory, the energy needed to charge the moving condenser is given (to order β^2) by:

$$U = U' \; (1 - \tfrac{1}{2}\, \beta^2 + 2\, \beta^2 \cos^2 \theta) \; . \tag{1.42}$$

This relation is vindicated by special relativity (see Eqs. 1.74–1.77 and Eq. 2.132).

Notice that it follows from Eq. 1.42 that the electromagnetic energy of a moving condenser has a maximum for $\theta = 0$. So, on the basis of Trouton and Larmor's energy arguments we would expect that the turning couple tries to put the plates of the condenser at right angles to the direction of motion (the position in which there is no magnetic field). However, the conclusion of Lorentz's argument in terms of electromagnetic momentum is that the turning couple tries to align the plates with the direction of motion (see Eq. 1.39). From these observations it is immediately clear that the tacit assumption in Trouton and Larmor's energy arguments has to be false in Lorentz's theory: the electromagnetic energy cannot be the only part of the condenser's energy that depends on θ. I will return to this in chapter two (see section 2.4, Eq. 2.132).

I now return to the relation between the argument in the passage I quoted from the introduction of Lorentz 1904b and the argument I gave in Eqs. 1.37–1.39. These equations and Eqs. 1.40–1.42 are misleading in that I treated the vector **G** as one would treat any sort of momentum: a change of **G** gives rise to an impulse, there is angular momentum $\mathbf{L} = \mathbf{x} \times \mathbf{G}$ associated with **G**, changes in **L** are accompanied by a turning couple, work is done when **G** is changed, etc.

Lorentz is more careful about the interpretation of this vector **G**. When Lorentz calculates forces or moments of forces—be it on a moving condenser or on a moving charged ellipsoid representing a moving electron in his theory—he proceeds as follows. The input essentially consists of three ingredients: Maxwell's equations, the equation for the Lorentz force, and some rudimentary Newtonian mechanics.[42] With the help of these ingredients Lorentz then derives an expression for the force or the moment of the force at hand. This expression will involve the vector **G**. At that point Lorentz will inform his readers that this quantity behaves like momentum in certain respects, and that Abraham therefore introduced the name electromagnetic momentum for it (see, e.g., Lorentz 1904a, pp. 162–163; 1916, pp. 30–33). However, with the exception of his discussion of the Trouton experiment and his electron model in this 1904 paper,[43] Lorentz is careful not to commit himself to such an interpretation. Consequently, the above derivation of Eq. 1.39 for the turning couple, and the derivation of Eq. 1.42 for the condenser's electromagnetic energy, which are *based* on the interpretation of **G** as a form of momentum, are not to be found in Lorentz's own work. For Lorentz, these expressions can only be justified by a derivation from the three ingredients I mentioned (Lorentz force, Maxwell's equations, a little Newtonian mechanics).

In the encyclopedia article referred to in the introduction of Lorentz 1904b, Lorentz, in fact, offered such a derivation of the equation $\mathbf{T} = -\,\mathbf{v} \times \mathbf{G}$ (Eq. 1.38) that he used to evaluate the turning couple of the Coulomb forces on the condenser in the Trouton-Noble experiment (Lorentz 194a, p. 191, p. 259). One might be inclined to take the position that such derivations can be discarded as scaffolding we no longer need once we recognize that the expression was to be expected on the basis of the new interpretation of its terms. I want to emphasize that this is not the position that Lorentz took.

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[42] Lorentz is very cautious about using Newtonian mechanics in calculations in electrodynamics. It is largely because of this caution, that Lorentz's account of the Trouton-Noble experiment carries over unproblematically to special relativity, whereas Larmor's account does not.

[43] After deriving an expression for the electromagnetic momentum of the electron (see section 3.4), Lorentz writes: "every change in the motion of a system will entail a corresponding change in the electromagnetic momentum and will therefore require a certain force, which is given in direction and magnitude by [$\mathbf{F} = -d\mathbf{G}/dt$]" (Lorentz 1904b, p. 184). In his lectures at Columbia University in New York in 1906, however, Lorentz gave a derivation of this relation of the kind outlined here (Lorentz 1916, pp. 26–33).

Lorentz's reservations with respect to the notion of electromagnetic momentum should be viewed against the background of his discussions with Poincaré over the fate of Newton's third law in his theory. We need to look at this issue a little more closely (I already touched upon it briefly in section 1.1). It is clear that Newton's principle of the equality of action and reaction is hard to reconcile with the notion of a stationary ether which acts on matter (via the Lorentz force electromagnetic fields exert on charged particles), yet is never acted upon. Lorentz clearly stated this obvious difficulty in his widely read monograph of 1895. After discussing the problem of how to make sense of forces acting on a stationary ether and concluding that the easiest way to solve the problem would simply be never to apply the notion of force to the ether at all, Lorentz wrote, in an often quoted passage:

> It is true that this conception violates the principle of the equality of action and reaction—because we do have grounds for saying that the ether *exerts* forces on ponderable matter—but nothing, as far as I can see, forces us to elevate that principle to the rank of a fundamental law of unlimited validity. (Lorentz 1895, p. 28; italics in the original)[44]

Poincaré strongly objected to this aspect of Lorentz's theory, especially to the violations of the center of mass theorem it entails. He even made it the topic of his contribution to a *Festschrift* for Lorentz on the occasion of the 25th anniversary of his doctorate (Poincaré 1900b). Lorentz remained unconvinced. In a letter to Poincaré in response to the latter's paper, he reiterated his point that "any theory which can explain the Fizeau experiment" (by which, I take it, Lorentz meant any theory positing a stationary ether) will violate the reaction principle, but added: "Must we, in truth, worry ourselves about it?"[45] The introduction by Abraham (1903) of electromagnetic momentum was an important step in resolving the issue, but it did not settle the dispute between Lorentz and Poincaré. A full resolution of the problem had to wait upon the advent of relativity theory. As Darrigol recently pointed out: "in 1906, Einstein showed that Poincaré's paradoxes [having to do with violations of the center of mass theorem in Lorentz's theory] could be solved only if a revolutionary assumption was made: the mass of a macroscopic body had to depend on its energy content" (Darrigol 1994b, p. 3).[46] In chapter

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[44] The translation of the last part of this quotation is from Darrigol 1994b, p. 32; see also Miller 1981, p. 43.

[45] Lorentz to Poincaré, January 20, 1901. Translation from Darrigol 1994b, p. 54. Lorentz's letter is quoted in full in Miller 1986, pp. 70–71. Lorentz also discussed these issues in his lectures at Columbia University in 1906 (Lorentz 1916, pp. 30–33).

[46] I am indebted to John Stachel for providing me with a preprint of this paper and for stressing its importance for a proper analysis of the Trouton experiment. To my knowledge, Darrigol has been the first to look into the connection between a paper by Einstein (1906) on the inertia of energy and Poincaré's contribution to the Lorentz *Festschrift* (Poincaré 1900b). There is no doubt in my mind that his insightful account of the debate over the fate of Newton's third law in Lorentz's theory (including his discussion of its importance for relativity theory) will soon supersede older accounts of this episode such as Miller 1981, pp. 41–45; 1986, pp. 68–72.

Darrigol gives another important reference in this context (ibid., pp. 39–41): Wien 1898, pp. xii–xiii. Wien's position appears to be somewhere in between the position of Lorentz and Poincaré. He agrees with two I will return to this issue, in the context of a relativistic analysis of the Trouton experiment.[47]

A particularly vivid description of the difficulties concerning the reaction principle that physicists were facing around the turn of the century was given a few years later by Planck (1908) in a lecture entitled "Comments on the principle of action and reaction in general dynamics" that he delivered at the *Versammlung Deutscher Naturforscher und Ärzte* in Cologne on September 23, 1908. The short non-technical paper Planck published based on this talk turns out to be one of the most important references in a seminal paper by Laue on relativistic mechanics, a paper in which the Trouton-Noble experiment plays a very prominent role (Laue 1911a). That makes it all the more appropriate to include an extensive quotation from Planck's paper at this point, a passage in which the author reminisces about the state of mind of the physics community in the early years of the century with regard to the reaction principle. At the appropriate juncture in chapter two, I will quote from Planck's assessment, in that same lecture, of how relativity theory helped clarify the situation. This is what Planck has to say about the attitude of the physics community toward the reaction principle before 1905:

> As is well-known, the real content of the Newtonian principle of the equality of action and reaction is the theorem of the constancy [read: conservation] of the quantity of motion or of the momentum of motion; I therefore want to talk about this principle only in the sense of that theorem, and, more specifically, about its relevance for general dynamics, which not only includes mechanics in a more restricted sense, but also electrodynamics and thermodynamics.
>
> Many of us will still have memories of the stir it caused, when Lorentz, in laying the foundations of an atomistic electrodynamics on the basis of a stationary ether, denied Newton's third axiom absolute validity, and it could not have failed to happen that this circumstance was turned into a serious objection against Lorentz's theory, as was done, for instance, by Poincaré. A calmness of sorts only returned when it became clear, especially through the investigations of Abraham, that the reaction principle could be saved after all, in its full generality at that, if only one introduces, besides the mechanical quantity of motion, the only kind known at that point, a new quantity of motion, the electromagnetic kind. Abraham made this notion even more plausible by a comparison between the conservation of the quantity of motion and the conservation of energy. Just as the energy principle is violated if one does not take electromagnetic energy into account and satisfied if one does introduce this form of energy, so is the reaction principle violated if one only considers the mechanical quantity of motion but satisfied as soon as one also takes into account the electromagnetic quantity of motion.
>
> However, this comparison, incontestable in and of itself, leaves one essential difference untouched. In the case of energy, we already knew a whole series of different kinds—kinetic energy, gravitation [sic], elastic energy of deformation, heat, chemical energy—so it does not

---

Poincaré that the violation of the center of mass theorem is a serious problem for Lorentz's theory, but he agrees with Lorentz that ultimately the reaction principle may nonetheless have to be given up.

The connection between Einstein 1906 and Poincaré 1900b suggests another possible connection. One wonders whether the passage from Lorentz 1895 that I quoted above was somehow in the back of Einstein's mind when, years later, he wrote the following often quoted sentence in *The meaning of relativity*: "It is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum), which acts itself, but which cannot be acted upon" (Einstein 1922, pp. 55–56). This is not the place to pursue this possible connection any further.

[47] We will see that in order for the Trouton experiment to satisfy both the relativity principle and momentum conservation, the energy the condenser gains (loses) when it is (dis-)charged must have mass (see section 2.5).

> constitute a fundamental innovation if one adds electromagnetic energy to these different forms of energy as yet another form. In the case of the quantity of motion, by contrast, we only knew one kind so far: the mechanical kind. Whereas energy had constituted a universal physical concept all along, the quantity of motion had so far been a typically mechanical concept and the reaction principle had been a typically mechanical theorem. Consequently, its generalization, while recognized to be necessary, was bound to be experienced as a revolution of a fundamental nature, through which the up to that point relatively simple and uniform concept of the quantity of motion acquired a considerably more complicated character. (Planck 1908, pp. 215–216)

Planck may have exaggerated the difficulties physicists were experiencing with the notion of electromagnetic momentum somewhat for rhetorical purposes (he goes on to show that the idea of putting energy and momentum on equal footing is a very natural one in relativity theory), but his reminiscing in this passage may help to understand Lorentz's attitude toward electromagnetic momentum, an attitude that otherwise might seem overly cautious.

1.4.2 Lorentz's derivation of the expression for the turning couple of the Coulomb forces in the Trouton-Noble experiment. I will give derivations of the relation $\mathbf{T} = -\mathbf{v} \times \mathbf{G}$ (Eq. 1.38) and of Eq. 1.36 for $\mathbf{G}$, which—except for the use of modern notation—are faithful to Lorentz's own derivations. Nowhere in this calculation will we need the interpretation of $\mathbf{G}$ as a form of momentum.

To calculate the turning couple $\mathbf{T}$ on a static charge distribution moving through the ether at a constant velocity $\mathbf{v}$, Lorentz starts from an equation which in modern notation reads (cf. Lorentz 1904a, p. 164):

$$\mathbf{T} = \int \mathbf{x} \times \mathbf{f} \; d^3x, \tag{1.43}$$

where $\mathbf{f}$ is the Lorentz force density associated with the charge distribution's self-field. The integration ranges over all space and is to be carried out in a frame of reference at rest in the ether. For $\mathbf{f}$, we have:

$$\mathbf{f} = \rho\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1.44}$$

where ρ is the charge density and $\mathbf{E}$ and $\mathbf{B}$ are the self-fields of the charge distribution. With the help of two of Maxwell's equations,

$$\mathrm{div}\;\mathbf{E} = \rho/\varepsilon_0\,, \qquad \mathrm{curl}\;\mathbf{B} = \mu_0 \rho\,\mathbf{v} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}, \tag{1.45}$$

the charge density ρ can be eliminated from Eq. 1.44:

$$\mathbf{f} = \varepsilon_0 \mathbf{E}\,\mathrm{div}\,\mathbf{E} + \mu_0^{-1}\,\mathrm{curl}\,\mathbf{B} \times \mathbf{B} - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B}\ . \tag{1.46}$$

With the help of another one of Maxwell's equations, viz. curl $\mathbf{E} = -\ \partial\mathbf{B}/\partial t$, the last term on the right hand side of Eq. 1.46 can be written as:

$$-\,\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} = -\frac{\partial}{\partial t}\,\varepsilon_0\,\mathbf{E} \times \mathbf{B}\ -\ \varepsilon_0\,\mathbf{E} \times \mathrm{curl}\,\mathbf{E}. \tag{1.47}$$

Now divide $\mathbf{f}$ into two parts, a part $\mathbf{f}_1$ involving spatial derivatives and a part $\mathbf{f}_2$ involving a time derivative. The first part can be written as:

$$\mathbf{f}^1 = \varepsilon_0\big(\mathbf{E}\,\mathrm{div}\,\mathbf{E} + \mathrm{curl}\,\mathbf{E} \times \mathbf{E}\big) + \mu_0^{-1}\big(\mathbf{B}\,\mathrm{div}\,\mathbf{B} + \mathrm{curl}\,\mathbf{B} \times \mathbf{B}\big)\ , \tag{1.48}$$

where I used another one of Maxwell's equations, viz. div $\mathbf{B} = 0$, to make the structure of the expression for $\mathbf{f}^1$ symmetric in $\mathbf{E}$ and $\mathbf{B}$. The second part can be written as:

$$\mathbf{f}^2 = -\frac{\partial}{\partial t}\,\varepsilon_0\,\mathbf{E} \times \mathbf{B} = -\frac{\partial \mathbf{g}}{\partial t}\,, \tag{1.49}$$

where I introduced the electromagnetic momentum density $\mathbf{g} \equiv \varepsilon_0\,\mathbf{E} \times \mathrm{B}$. The right hand side of Eq. 1.48 is just the divergence of the so-called *Maxwell stress tensor*.[48] This is most easily shown in terms of the components of the equation. I will only go through the algebra for $\mathbf{f}^1(\mathbf{E})$, the part of $\mathbf{f}^1$ depending on $\mathbf{E}$. Because of the symmetry of Eq. 1.48 the expression for $\mathbf{f}^1(\mathbf{B})$ will have exactly the same structure. The i-th component of $\mathbf{f}^1(\mathbf{E})$ can be written as (summation over repeated indices being understood):

$$f_i^1(\mathbf{E}) = \varepsilon_0(E_i\,\partial_k E_k + \varepsilon_{ijk}(\varepsilon_{jlm}\partial_l E_m)E_k)\ , \tag{1.50}$$

where ε_{ijk} equals plus one when ijk is an even permutation of (1,2,3), minus one for any odd permutation, and zero otherwise. Using

$$-\,\varepsilon_{jik}\,\varepsilon_{jlm} = -\,\delta_{il}\,\delta_{km} + \delta_{im}\,\delta_{kl}\ , \tag{1.51}$$

where δ_{ij} is the Kronecker delta, we can rewrite Eq. 1.50 as:

---

[48] See, e.g., Pauli 1921, p. 85.

$$
\begin{aligned}
f_i^1(\mathbf{E}) &= \varepsilon_0 \left(E_i\, \partial_k E_k - \partial_i E_k\, E_k + \partial_k E_i\, E_k \right) \\
&= \varepsilon_0\, \partial_k \left(E_i E_k - \tfrac{1}{2} \delta_{ik} E^2 \right) .
\end{aligned}
\tag{1.52}
$$

A similar expression obtains for $\mathbf{f}^1(\mathbf{B})$. Hence, the components of $\mathbf{f}^1$ can be written as:

$$
f_i^1 = \partial_k \left\{ \varepsilon_0 \left(E_i E_k - \tfrac{1}{2} \delta_{ik} E^2 \right) + \mu_0^{-1} \left(B_i B_k - \tfrac{1}{2} \delta_{ik} B^2 \right) \right\} . \tag{1.53}
$$

The quantity between curly brackets is just the Maxwell stress tensor T_{ik}. Adding Eq. 1.53 for the components of $\mathbf{f}^1$ to the components of $\mathbf{f}^2$ in Eq. 1.49, we finally obtain:

$$
f_i = f_i^1 + f_i^2 = \partial_k T_{ik} - \dot{g}_i . \tag{1.54}
$$

This relation is an instance of a much more general relation from relativistic mechanics that we will have occasion to use in section 2.4 (see Eq. 2.117).

For now, I return to the turning couple $\mathbf{T}$ exerted on the static moving charging distribution by the force $\mathbf{f}$. This turning couple can be divided into a contribution from $\mathbf{f}^1$ and a contribution from $\mathbf{f}^2$:

$$
\mathbf{T} = \mathbf{T}^1 + \mathbf{T}^2 = \int d^3x\ \mathbf{x} \times \mathbf{f}^1 + \int d^3x\ \mathbf{x} \times \mathbf{f}^2 . \tag{1.55}
$$

The contribution from $\mathbf{f}^1$ vanishes. Consider the i-th component of $\mathbf{T}^1$:

$$
T_i^1 = \int d^3\mathrm{x}\ \varepsilon_{ijk}\, x_j f_k^1 = \int d^3x\ \varepsilon_{ijk}\, x_j\, \partial_l T_{kl} . \tag{1.56}
$$

Integration by parts gives:

$$
T_i^1 = \int d^3x\ \varepsilon_{ijk}\, \partial_l \left(x_j\, T_{kl} \right) - \int d^3x\ \varepsilon_{ijk}\, \partial_l\, x_j\, T_{kl} . \tag{1.57}
$$

The first term vanishes on account of Gauss's theorem. In the second term, substitute $\partial_l x_j = \delta_{lj}$ and carry out the summation over l. This leaves us with $\varepsilon_{ijk} T_{kj}$ which vanishes since it is a contraction of a quantity anti-symmetric in j and k and a quantity symmetric in j and k. This concludes the proof that $\mathbf{T}^1 = 0$.

Now substitute Eq. 1.49 for $\mathbf{f}^2$ into the $\mathbf{T}^2$-part of Eq. 1.55. Replacing the differential quotient by the corresponding difference quotient in the limit where Δt goes to zero, we obtain (cf. Lorentz 1904a, p. 191):

$$\mathbf{T} = -\int d^3x\ \mathbf{x} \times \frac{\partial \mathbf{g}(t,\ \mathbf{x})}{\partial t} = \lim_{\Delta t \to 0} -\frac{1}{\Delta t}\int d^3x\ \ \mathbf{x} \times \big(\mathbf{g}(t{+}\Delta t,\mathbf{x}) - \mathbf{g}(t,\mathbf{x})\big). \tag{1.58}$$

Since we are dealing with a moving *static* charge distribution, the only time dependence of $\mathbf{g}$ is through the change of position:

$$\mathbf{g}(t{+}\Delta t,\mathbf{x}) = \mathbf{g}(t,\mathbf{x}{-}\mathbf{v}\Delta t). \tag{1.59}$$

Now rewrite the integrand in Eq. 1.58 as follows:

$$\big(\mathbf{x}{-}\mathbf{v}\Delta t\big) \times \mathbf{g}(t,\mathbf{x}{-}\mathbf{v}\Delta t)\ +\ \mathbf{v}\Delta \mathrm{t} \times \mathbf{g}(t,\mathbf{x}{-}\mathbf{v}\Delta t)\ -\ \mathbf{x} \times \mathbf{g}(t,\mathbf{x})\ . \tag{1.60}$$

The integral over the first term of Eq. 1.60 cancels the integral over the third. So, letting Δt go to zero, we are left with:

$$\mathbf{T} = -\int d^3x\ \ \mathbf{v} \times \mathbf{g}\ . \tag{1.61}$$

When finally the electromagnetic momentum $\mathbf{G}$ is introduced as the integral over the electromagnetic momentum density $\mathbf{g}$, Eq. 1.38 is recovered which we wrote down earlier simply on the basis of the interpretation of the quantity $\mathbf{G}$ as momentum:

$$\mathbf{T} = -\,\mathbf{v} \times \mathbf{G}. \tag{1.62}$$

I will now derive Eq. 1.36 for the electromagnetic momentum $\mathbf{G}$ of a condenser moving through the ether. Since the fields in the moving condenser can be treated as homogeneous we can simply multiply the electromagnetic momentum density $\mathbf{g}$ with the volume V of the moving condenser to obtain the electromagnetic momentum $\mathbf{G}$. To find an expression for $\mathbf{g} = \varepsilon_0(\mathbf{E} \times \mathbf{B})$, we use Lorentz's theorem of corresponding states. Look back at Fig. 1.9. The fields $\mathbf{E}$ and $\mathbf{B}$ in the moving condenser represented by the parallelogram on the right are related to the field $\mathbf{E}'$ in the corresponding condenser at rest represented by the rectangle on the left through a Lorentz transformation. Remember that, relativistically, the system on the left is just the moving system on the right in its rest frame. Since $\mathbf{B}' = 0$ in this case, the Lorentz transformation for the electromagnetic field reduces to:

$$\mathbf{E} = \mathrm{diag}(1,\ \gamma,\ \gamma)\,\mathbf{E}', \qquad \mathbf{B} = \frac{1}{c^2}\,\mathrm{diag}(1,\ \gamma,\ \gamma)\,\mathbf{v} \times \mathbf{E}'\ . \tag{1.63}$$

The electric field in the condenser at rest is given by:

$$\mathbf{E}' = E'\,(\sin\theta', -\cos\theta', 0). \tag{1.64}$$

Since we only need **G** to first order in β, we can set $\gamma = 1$ in Eq. 1.63 and $\theta' = \theta$ in Eq. 1.64. So, to first order in β, **E** and **B** are given by:[49]

$$\mathbf{E} = E'\,(\sin\theta, -\cos\theta, 0), \quad \mathbf{B} = E'\,(0, 0, -(v/c^2)\cos\theta). \tag{1.65}$$

Inserting Eq. 1.65 into the expression for the electromagnetic momentum density **g**, we find:

$$\mathbf{g} = \varepsilon_0\,\mathbf{E}\times\mathbf{B} = \varepsilon_0\,(E_y B_z, -E_x B_z, 0) = \varepsilon_0\,\mathrm{v/c^2}\,E'^2\,(\cos^2\theta, \sin\theta\cos\theta, 0). \tag{1.66}$$

Now use that $\varepsilon_0 E'^2 = 2u'$, where u' the energy density of the field in the condenser at rest, that $\mathbf{G} = \mathbf{g}\,\mathsf{V}$, and that, to first order in β, $\mathsf{V} = \mathsf{V}'$. In this way we recover Eq. 1.36:

$$\mathbf{G} = 2\,(U'/c)\,\beta\cos\theta\,(\cos\theta, \sin\theta, 0), \tag{1.67}$$

where $U' = u'\,\mathsf{V}'$ is the energy of the field in the condenser at rest. Inserting Eq. 1.67 into Eq. 1.62, we recover Eq. 1.39 for the turning couple of the Coulomb forces in the Trouton-Noble experiment. I want to emphasize once again that in the derivation spanning Eqs. 1.43–1.67, we did not have to invoke the interpretation of **G** as electromagnetic momentum at any point, as I had to do in the derivation I gave in Eqs. 1.37–1.39.

1.4.3 The electromagnetic energy in a moving condenser according to Lorentz's theory. Eq. 1.42 for the electromagnetic energy U of the moving condenser can be derived in the same way as Eq. 1.67 for the electromagnetic momentum. We do not have to invoke the interpretation of **G** as electromagnetic momentum, as I had to do in the derivation I gave in Eqs. 1.40–1.42.

The energy density for any electromagnetic field is given by

$$u = \tfrac{1}{2}\,\varepsilon_0\,E^2 + \tfrac{1}{2}\,\mu_0^{-1}\,B^2. \tag{1.68}$$

For the fields **E** and **B** of the condenser we can use Eq. 1.63 and Eq. 1.64. We cannot use Eq. 1.65, however, since we now need **E** and **B** to order β^2. For the same reason we cannot assume that Eq. 1.64 for $\mathbf{E}'$ will be valid in the case where we do not assume the Lorentz-FitzGerald contraction. The following derivation is valid only in the context of a theory incorporating the Lorentz-FitzGerald contraction.[50] From Eq. 1.63 and Eq. 1.64 it follows that:

---

[49] Trouton used these values for the **E** and **B** fields to compute the electromagnetic energy of the condenser (see Eq. 1.3 and Eq. 1.4; cf. Eq. 1.69 below).

[50] On the basis of the interpretation that I gave earlier of the various terms in Eq. 1.42, it is, in fact, clear that the result, if not the derivation, holds for the case without the Lorentz-FitzGerald contraction as well. Notice,

$$\mathbf{E} = E' (\sin\theta', -\gamma\cos\theta', 0), \quad \mathbf{B} = E' (0, 0, -\gamma\,(v/c^2)\cos\theta'). \tag{1.69}$$

Inserting Eq. 1.69 into Eq. 1.68, we obtain:

$$u = \frac{1}{2}\,\varepsilon_0\, E'^2\,(\sin^2\theta' + \gamma^2\cos^2\theta') + \frac{1}{2}\,\mu_0^{-1}/c^2\,\beta^2\,\gamma^2\,E'^2\,\cos^2\theta'. \tag{1.70}$$

Now use that $\mu_0^{-1}/c^2 = \varepsilon_0$, and that $1/2\ \varepsilon_0 E'^2 = u'$, the electromagnetic energy density of the corresponding condenser at rest. Eq. 1.70 then turns into:

$$u = u'\,(\sin^2\theta' + \gamma^2\cos^2\theta' + \beta^2\,\gamma^2\cos^2\theta'). \tag{1.71}$$

Inserting $1 = \gamma^2(1-\beta^2)$ in the first term on the right hand side, we can rewrite Eq. 1.71 as:

$$u = \gamma^2 u'\,(1 - \beta^2\sin^2\theta' + \beta^2\cos^2\theta'). \tag{1.72}$$

To get from the energy density u to the energy U itself, we can just multiply Eq. 1.72 by the volume V of the moving condenser. This volume will be a factor γ smaller than the volume V' of the corresponding condenser at rest. Hence,

$$U = \gamma U'\,(1 - \beta^2\sin^2\theta' + \beta^2\cos^2\theta'), \tag{1.73}$$

where $U' = u'\mathsf{V}'$. To order β^2, $\gamma = 1 + \frac{1}{2}\beta^2$ and $\theta' = \theta$, so that Eq. 1.73 reduces to Eq. 1.42:

$$U = U'\,(1 - \frac{1}{2}\beta^2 + 2\,\beta^2\cos^2\theta)\,. \tag{1.74}$$

The interpretation of the energy U as the sum of the energy U'/γ found by Larmor (see Eqs. 1.19–1.27) and the energy U^G needed to build up the electromagnetic momentum $\mathbf{G}$ of the moving condenser (see Eqs. 1.40–1.42) not only holds to order β^2, but exactly. Inserting the exact expressions for $\mathbf{E}$ and $\mathbf{B}$ in Eq. 1.69 into $\mathbf{g} = \varepsilon_0 \mathbf{E} \times \mathbf{B}$ (see Eq. 1.66) and multiplying by the volume V'/γ of the contracted condenser, we find that

$$\mathbf{G} = 2\,(U'/c)\,\beta\cos\theta'\,(\gamma\cos\theta', \sin\theta', 0). \tag{1.75}$$

To order β^2, $\gamma = 1$ and $\theta' = \theta$, so that Eq. 1.75 reduces to Eq. 1.67 and Eq. 1.36. Inserting the x-component of Eq. 1.75 into Eq. 1.40 for U^G, we find that:

$$U^G = 2\,U'\gamma\beta^2\cos^2\theta'. \tag{1.76}$$

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however, that in that case Eq. 1.42 does not give the full θ-dependence of U explicitly, since U' will still depend on θ (see Eq. 1.29 and Eq. 1.32).

Adding Larmor's U'/γ to U^G, we arrive at

$$\begin{aligned} U &= \frac{U'}{\gamma} + 2U'\gamma\beta^2\cos^2\theta' \\ &= \gamma U'\left(1 - \beta^2 + 2\beta^2\cos^2\theta'\right) \\ &= \gamma U'\left(1 - \beta^2\sin^2\theta' + \beta^2\cos^2\theta'\right), \end{aligned} \tag{1.77}$$

which is exactly the total electromagnetic energy of the condenser given in Eq. 1.73.

So, we have two derivations for the expression for the electromagnetic energy of a moving charged condenser in Lorentz's theory, one (Eqs. 1.75–1.77, the exact version of the derivation in Eqs. 1.40–1.42) in which the physical interpretation of **G** as momentum plays a crucial role, and one (Eqs. 1.68–1.74) in which this interpretation does not play any role. Lorentz, as far as I know, never did either of these calculations, although the latter derivation is perfectly in line with his general approach. As a consequence, it is idle speculation how he would have responded to the contradiction between this expression for the electromagnetic energy in a moving charged condenser and the expression $U = U'/\gamma$, which formed the basis for Larmor's account of the Trouton-Noble experiment. I will not get into such speculation.