Journal of Robotics, Networking and Artificial Life, Vol. 1, No. 2 (September 2014), 125-129

Adaptive Consensus Control of Multi-Agent Systems with Large Uncertainty and Time Delays *

Li Liu

School of Mechanical Engineering
University of Science and Technology Beijing, Beijing 100083, China

Yu Meng**

School of Mechanical Engineering
University of Science and Technology Beijing, Beijing 100083, China

Dongxu Zou

School of Automation and Electrical Engineering
University of Science and Technology Beijing, Beijing 100083, China

Weicun Zhang

School of Automation and Electrical Engineering
University of Science and Technology Beijing, Beijing 100083, China

** *Corresponding author. E-mail:* myu@ustb.edu.cn

Abstract

A weighted multi-model adaptive control (WMMAC) method is proposed to achieve consensus of multi-agent system with large parameter uncertainty and communication delays, in which H^{∞} control is adopted as local control strategy to deal with smaller parameter uncertainty. Moreover a simple and effective weighting algorithm is adopted to assign weights for each local controller, based on which the global control law is obtained as a weighted sum of all the local control at each time instant. The simulation results demonstrate the effectiveness of the proposed method.

Keywords: Adaptive consensus; Multi-agent system; WMMAC; H^{∞} control.

*This work was supported by Major State Basic Research Development Program (973 Program) (No. 2012CB821200), National High-Tech Research and Development Program of China (863 Program) (No. 2011AA060408), Beijing Higher Education Young Elite Teacher Project (No. YETP0362), National Key Technology Research and Development Program of China (No. 2013BAB02B07)

1. Introduction

During the past decades, the study of decentralized control has been focused on multi-agent systems, such as flocking or swarming behaviors [1-3], multi-robot formation control [4,5], and path planning [6] and so on.

The consensus problem, the most important and fundamental issue in the cooperation control of multi-agent system, is of theoretical value and practical significance. For a multi-agent system, consensus means that the states of all agents tend to be identical asymptotically under given protocols (control law) based on the communication networks.

Recently, some researchers have solved the consensus problem of multi-agent systems with time-varying external disturbances and random communication delays [7]. In [8], model uncertainty, i.e., the small parameter uncertainty, was taken into consideration, and a robust H^{∞} controller was designed accordingly.

In this paper, the consensus problem is considered for multi-agent systems with large parameter uncertainty, which will be divided into smaller intervals that can be dealt with by each local H^{∞} controller. External disturbances and communication delays are also taken into account. WMMAC strategy showed its priority of dealing with large parameter uncertainties [9], which consists mainly of two components, i.e. the local controller set and the weighting algorithm. The local controller set was designed according to a local model set which was determined off-line to cover the large uncertainty of system parameters. The weighting algorithm was designed to get weights for each local controller, based on which the global control law is obtained as a weighted sum of all the local control at each time instant.

Finally, simulation results indicate that under the proposed protocol, i.e. the WMMAC strategy, multi-agent systems with large parameter uncertainty can reach the desired consensus performance in the presence of communication delays.

2. Uncertain Multi-Agent Systems

Consider a second-order multi-agent system consisting of n identical agents with the i th one modeled by as the form of (1).

$$\begin{aligned} &\dot{x}_i(t) = Ax_i(t) + B_1\omega_i(t) + B_2u_i(t), \\ &x_i(t) = [x_{i1}(t), x_{i2}(t)]^T, \omega_i(t) \in L_2(0,\infty] \end{aligned} \quad (1)$$

$x_i(t)$ is the state of agent i, while $\omega_i(t)$ represents appropriate external disturbance that belongs to $L_2(0,\infty]$. $u_i(t)$ denotes the control protocol.

If system matrices A, B_1, B_2 are uncertain, then they are supposed to follow the forms of (2).

In (2), A_0, B_{10}, B_{20} are constant matrices and $\Delta A(t), \Delta B_1(t), \Delta B_2(t)$ are time-varying matrices. E and $F_i, i = 1,2,3$ are constant matrices with appropriate dimensions and $\Sigma(t)$ is time-varying uncertain matrix satisfying $\Sigma(t)^T\Sigma(t) \le I$. It is also assumed that (A_0, B_{20}) is stabilizable.

$$\begin{aligned} &A = A_0 + \Delta A(t), B_1 = B_{10} + \Delta B_1(t) \\ &B_2 = B_{20} + \Delta B_2(t) \\ &[\Delta A(t)\ \Delta B_1(t)\ \Delta B_2(t)] = E\ \Sigma(t)[F_1\ F_2\ F_3] \end{aligned} \quad (2)$$

The consensus of multi-agent system is that the states of agents satisfy (3) under control protocol $u_i(t)$.

$$\lim_{t\to\infty}\left(x_i(t) - x_j(t)\right) = \mathbf{0}, \forall i, j \in n \quad (3)$$

3. Weighted Multi-model Adaptive Control

Weighted multi-model adaptive controller is made up of the following components.

3.1. *Model set*

Considering the large uncertainty of plant parameters cannot be dealt with by a single H^{∞} controller, then we need to use multiple models, i.e. a model set, to cover the uncertainty of the plant. The model set is designed as $\Omega = \{M_i \mid i = 1,2,\cdots,N\}$. Obviously, the performance of multi-agent control system depends on the model set and the local control strategy, as well as the right convergence of the weighting algorithm.

3.2. *Controller set*

The controller set C is shown as the form of

$$C = \{C_i \mid i = 1,2,\cdots,N\}$$

in which the controller C_i is designed according to model M_i according to H^{∞} control strategy or other feasible control strategy.

In this paper, robust H^{∞} method is selected as control strategy. Considering the multi-agent system (1), the output is supposed to follow (4). We can reformulate the system (1) as the form of (5) that satisfies (6) with the help of Kronecker product.

$$z_i(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t), i = 1, ..., n \quad (4)$$

$$\begin{aligned} \dot{x}(t) &= (I_n \otimes A)x(t) + (I_n \otimes B_1)\omega(t) + (I_n \otimes B_2)u(t) \\ z(t) &= (L_c \otimes I_m)x(t) \end{aligned} \quad (5)$$

$$\begin{aligned} x(t) &= [x_1(t)^T, x_2(t)^T, ..., x_n(t)^T]^T \in \mathbf{R}^{mn} \\ \omega(t) &= [\omega_1(t)^T, \omega_2(t)^T, ..., \omega_n(t)^T]^T \in \mathbf{R}^{m_1 n} \\ u(t) &= [u_1(t)^T, u_2(t)^T, ..., u_n(t)^T]^T \in \mathbf{R}^{m_2 n} \\ z(t) &= [z_1(t)^T, z_2(t)^T, ..., z_n(t)^T]^T \in \mathbf{R}^{mn} \\ L_c &= \left[L_{c_{ij}}\right] \in \mathbf{R}^{n\times n}, L_{c_{ij}} = \begin{cases} (n-1)/n, i = j \\ -1/n, i \neq j \end{cases} \end{aligned} \quad (6)$$

According to the neighbors' states, the protocol of agent i can be designed as (7) to achieve the above conditions.

$$u_i(t) = K \sum_{j\in N_i(t)} a_{ij}(t)\left[x_i(t-d(t)) - x_j(t-d(t))\right] \quad (7)$$

where $0 \le d(t) \le \bar{d}, \bar{d} > 0$ is the time-varying communication delay, $N_i(t) \ge 0$ is the neighbor set of agent i at time instant t, $a_{ij}(t)$ are adjacency elements of the corresponding interaction graph, and K is an undetermined feedback matrix.

Substituting control protocol (7) into system (5), we obtain

$$\begin{aligned} \dot{x}(t) &= (I_n \otimes A)x(t) + (L_\sigma \otimes B_2K)x(t-d(t)) \\ &+ (I_n \otimes B_1)\omega(t), \ z(t) = (L_c \otimes I_m)x(t) \end{aligned} \quad (8)$$

Finally, the consensus problem of multi-agent system is converted to calculate the feedback matrix K.

Under control protocol (7), system (1) achieves consensus with a given H^∞ index γ, if there are a scalar α>0, a positive define matrix $P \in R^{m\times m}$ and a matrix $Q \in R^{m_2\times m}$ that satisfies

$$\begin{bmatrix} \Psi_{\sigma i} + \Psi_{\sigma i}^T & \alpha\lambda_{\sigma i}B_2Q & B_1 & \bar{d}\Psi_{\sigma i}^T & P \\ \alpha\lambda_{\sigma i}Q^TB_2^T & -\alpha P & 0 & \bar{d}\alpha\lambda_{\sigma i}Q^TB_2^T & 0 \\ B_1^T & 0 & -\gamma^2 I & \bar{d}B_1^T & 0 \\ \bar{d}\Psi_{\sigma i} & \bar{d}\alpha\lambda_{\sigma i}B_2Q & \bar{d}B_1 & -\alpha P & 0 \\ P & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

$$\Psi_{\sigma i} = A_0P + \lambda_{\sigma i}B_2Q \quad (9)$$

where $\lambda_{\sigma i}, i = 1, \cdots, n-1$ is the positive eigenvalues of L_σ, for details please be referred to Reference [8].

If LMI (9) is feasible for the maximum and minimum eigenvalues of all the connected topological graphs, the feedback matrix can be defined by $K = QP^{-1}$.

If LMI (9) is not feasible, then we need to divide the uncertainty of parameters, i.e. $\Delta A(t), \Delta B_1(t), \Delta B_2(t)$ into limited number of smaller intervals, such that for each smaller interval, LMI(9) is feasible.

And then, we have the modified protocol of agent i

$$u_i(t) = \sum_{i=1}^{N} p_i K_i \sum_{j\in N_i(t)} a_{ij}(t)\left[x_i(t-d(t)) - x_j(t-d(t))\right]$$

where $p_i(k)$ can be calculated according to the weighting algorithm described in the next subsection.

3.3. *Weighting Algorithm*

The method based on probability-weighted is that the weighted sum of controller set is taken as the global control input. The weight of controller can be calculated by partition theorem.

Suppose the control output of controller C_i is $u_i(k)$, the weight is $p_i(k)$, the global control input is $u(k)$, the system output is $y(k)$, the output of i th model is $y_i(k)$.

Let the output difference between i th model and plant be $e_i(k) = y(k) - y_i(k)$. Then the weighting algorithm can be designed as in [9].

First, weights of all models are initialized to $\frac{1}{N}$, if there are N models in the model set, i.e.

$$p_i(0) = r_i(0) = 1/N, i = 1, 2, ..., N \quad (10)$$

Then, it is conductive to improve the anti-jamming capability by selecting the cumulative mean squared error as the performance index shown in (11).

$$l_i(k) = 1 + \frac{1}{k}\sum_{j=1}^{k} e_i^2(k) \quad (11)$$

Finally, the computational process of weights is denoted by (12). This method is much simpler than probability-weighted method. The performance indexes are forced to compete at every step in order to make the weights converge.

$$l_{\min}(k) = \min_{i=1,\dots,N} l_i(k)$$
$$r_i(k) = \frac{l_{\min}(k)}{l_i(k)} r_i(k-1)$$
$$p_i(k) = \frac{r_i(k)}{\sum_{i=1}^{N} r_i(k)}$$

(12)

4. Simulation results

In this section, we will illustrate the consensus of multi-agent system under the method of WMMAC.

Considering a multi-agent system of four agents, the parameter matrices are designed as (13).

$$A_0 = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}, B_{10} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{20} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$
$$\Delta A(t) = \begin{bmatrix} 0 & 0 \\ 6(1-e^{-t}) & 0 \end{bmatrix}, \Delta B_1(t) = \mathbf{0}, \Delta B_2(t) = \mathbf{0}$$

(13)

The communication delay is assumed to be constant 0.05. The external disturbances are supposed to be band-limited white noises shown as (14).

$$\begin{aligned}\omega(t) &= \begin{bmatrix} \omega_1(t) & \omega_2(t) & \omega_3(t) & \omega_4(t) \end{bmatrix}^T \\ &= \begin{bmatrix} 2\omega(t) & -1.5\omega(t) & 1.2\omega(t) & 1.8\omega(t) \end{bmatrix}^T\end{aligned} \quad (14)$$

For simplicity, the H^∞ performance index is chosen as $\gamma=1$, the constant scalar $\alpha=0.1$, $\lambda=1$ and all the nonzero weighting factors of adjacency matrix are assumed to be 1. The simplified undirected interaction graph is shown in Fig. 1.

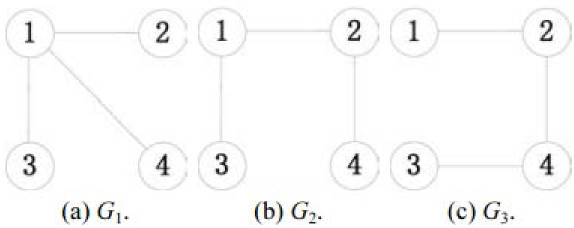

Fig. 1 simplified undirected graph

We can get the maximum and minimum nonzero eigenvalues $\lambda_{\sigma_* i_*} = 0.5858, \lambda_{\sigma^* i^*} = 4$ by calculating the Laplace matrices corresponding to the network graph as shown in Fig. 1.

It is assumed that there are four models in the model set, i.e.

$$A_1 = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 4 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 \\ 8 & 1 \end{bmatrix}$$

(15)

According to the LMI (9), the feedback matrices K_i can be determined as (16) with the help of Matlab LMI toolbox.

$$K_1 = \begin{bmatrix} -2.2365 & -0.3528 \\ 0.1269 & -2.7761 \end{bmatrix}, K_2 = \begin{bmatrix} -1.4127 & 0.2621 \\ 0.4986 & -2.3349 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -2.0759 & 0.9033 \\ 1.4598 & -3.0850 \end{bmatrix}, K_4 = \begin{bmatrix} -2.4147 & 1.3164 \\ 1.9986 & -2.9659 \end{bmatrix}$$

(16)

The simulation results are shown in figures 2-3, in which Fig. 2 represents the trajectories of states x_1 and x_2, Fig. 3 represents the energy relationship between the system output $z(t)$ and the external disturbance $\omega(t)$

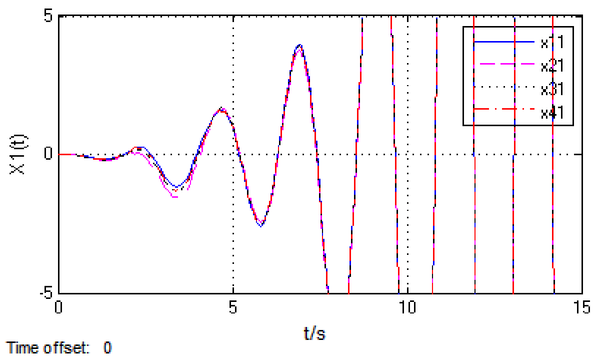

Fig. 2(a) State curve of x_1

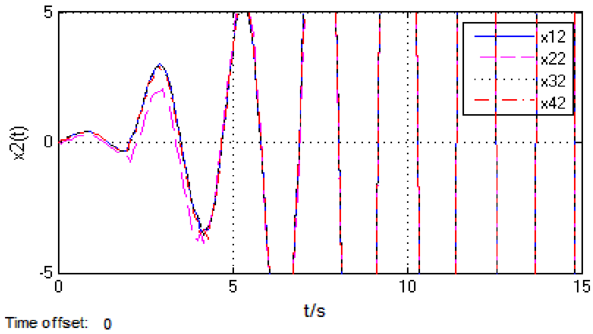

Fig. 2(b) State curve of x_2

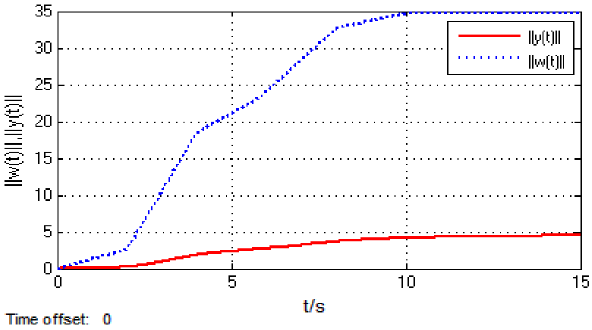

Fig. 3 Energy of $z(t)$ and $\omega(t)$

If the system states are asymptotically stable, the consensus of multi-agent system is all up to the steady-state values of each agent. Therefore, the states of agents are designed to be divergent in order to display the consensus of multi-agent system significantly.

It is easy to say that the multi-agent system achieves consensus according to Fig.2

When the state matrix A changes to

$$A = \begin{bmatrix} 0 & 1 \\ 7.5 & 1 \end{bmatrix}$$

the weights of all models are displayed in Fig.4.

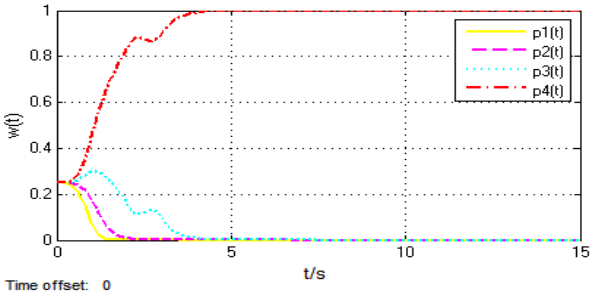

Fig.4 weights of models

In Fig.4, the weight of the 4th model is convergent to 1. Then the controlled quantity of the 4th model is chosen to be the global control input of the multi-agent system.

5. Conclusions

By applying the weighted MMAC methodology, we have addressed the adaptive consensus problem of multi-agent system with large parameter uncertainty and communication delays. According to the states' differences between the plant and each model, we can obtain the weight of each local controller with the weighting algorithm mentioned. Finally, the controller corresponding to the closest model to the plant will be chosen as the right controller.

References

1. R. Olfati-Saber Flocking for multi-agent dynamic systems: algorithms and theory. IEEE Transactions on Automatic Control, 2006, 51(3):401-420.
2. H. G. Tanner, A. Jadbabaie, G. J. Pappas, Flocking in fixed and switching networks. IEEE Transactions on Automatic Control, 2007, 52(5):863-868.
3. H. S. Su, X. F. Wang, W.Yang, Flocking in multi-agent systems with multiple virtual leaders. Asian Journal of Control, 2008, 10(2):238-245.
4. M. Egerstedt, X. Hu, Formation control with virtual leaders and reduced communications. IEEE Transactions on Robotics and Automation, 2001, 17(16):947-951.
5. R. Olfati-Saber, R. M. Murray, Distributed structural stabilization and tracking for formations of dynamic multi-agents. In Proceedings of the 41th IEEE Conference on Decision and Control, 2002, pp. 209-215.
6. F. Lian, Cooperative path planning of dynamical multi-agent systems using differential flatness approach, International Journal of Control, Automation, and Systems, 2008, 6(3): 401-412.
7. W. Ren, R. W. Beard, D. B. Kingston, Multi-agent Kalman consensus with relative uncertainty, Proc. of the American Control Conference, 2005. pp. 1865-1870.
8. Y. Liu, Y. Jia, Robust H∞ consensus control of uncertain multi-agent systems with time delays, International Journal of Control, Automation, and Systems, 2011, 9(6): 1086-1094.
9. W. Zhang, Stable Weighted Multiple Model Adaptive Control: Discrete-Time Stochastic Plant, International Journal of Adaptive Control and Signal Process, 2013, 27(7): 562-581