MICRO- AND NANOSCALE FLUID MECHANICS: TRANSPORT IN MICROFLUIDIC DEVICES

This text describes the physics of fluid transport in microfabricated and nanofabricated liquid-phase systems, with consideration of particles and macromolecules. This text brings together fluid mechanics, electrodynamics, and interface science with a focused goal of preparing the modern microfluidics researcher to analyze and model continuum fluid mechanical systems encountered when working with micro- and nanofabricated devices. This text is designed for classroom instruction and also serves as a useful reference for practicing researchers. Worked sample problems are inserted throughout to assist the student, and exercises are included at the end of each chapter to facilitate use in classes.

Brian J. Kirby currently directs the Micro/Nanofluidics Laboratory in the Sibley School of Mechanical and Aerospace Engineering at Cornell University. He joined the school in August 2004. Previously, he was a Senior Member of the Technical Staff in the Microfluidics Department at Sandia National Laboratories in Livermore, California. He was educated at Stanford University and The University of Michigan. Professor Kirby has received numerous research and teaching awards, including the Presidential Early Career Award for Scientists and Engineers (PECASE) and the Mr. and Mrs. Robert F. Tucker Excellence in Teaching Award.

Micro- and Nanoscale Fluid Mechanics

TRANSPORT IN MICROFLUIDIC DEVICES

Brian J. Kirby
Cornell University

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521119030

First published 2010

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Kirby, Brian (Brian J.)
Micro- and nanoscale fluid mechanics : transport in microfluidic devices / Brian Kirby.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-11903-0 (hardback)
1. Microfluidic devices. 2. Microfluidics. 3. Nanofluids. I. Title.
TJ853.4.M53K57 2010
620.1′064–dc22 2009053537

ISBN 978-0-521-11903-0 Hardback

Additional resources for this publication at www.cambridge.org/kirby

Contents

Preface — *page* xv
Nomenclature — xvii

Introduction **1**

1 Kinematics, Conservation Equations, and Boundary Conditions for Incompressible Flow **6**

1.1 Fluid statics 6
1.2 Kinematics of a fluid velocity field 7
1.2.1 Important geometric definitions 7
1.2.2 Strain rate and rotation rate tensors 10
1.3 Governing equations for incompressible flow 13
1.3.1 Conservation of mass: continuity equation 13
1.3.2 Conservation of momentum: the Navier–Stokes equations 14
1.4 Constitutive relations 17
1.4.1 Relation between strain rate and stress 17
1.4.2 Non-Newtonian fluids 19
1.5 Surface tension 20
1.5.1 Definition of surface tension and interfacial energy 20
1.5.2 Young–Laplace equation 20
1.5.3 Contact angle 21
1.5.4 Capillary height 22
1.5.5 Dynamic contact angle 24
1.6 Velocity and stress boundary conditions at interfaces 24
1.6.1 Kinematic boundary condition for continuity of normal velocity 24
1.6.2 Dynamic boundary condition for continuity of tangential velocity 25
1.6.3 Dynamic boundary conditions for stresses 26
1.6.4 The physics of the tangential velocity boundary condition 30
1.7 Solving the governing equations 32
1.8 Flow regimes 33
1.9 A word on terminology and the microfluidics literature 33
1.10 Summary 34
1.11 Supplementary reading 36
1.12 Exercises 36

2 Unidirectional Flow **41**

2.1 Steady pressure- and boundary-driven flow through long channels 41
2.1.1 Couette flow 41
2.1.2 Poiseuille flow 46

2.2 Startup and development of unidirectional flows 49
2.3 Summary 50
2.4 Supplementary reading 51
2.5 Exercises 51

3 Hydraulic Circuit Analysis . . . 60

3.1 Hydraulic circuit analysis 60
3.2 Hydraulic circuit equivalents for fluid flow in microchannels 62
3.2.1 Analytic representation of sinusoidal pressures and flow rates 68
3.2.2 Hydraulic impedance 69
3.2.3 Hydraulic circuit relations 70
3.2.4 Series and parallel component rules 70
3.3 Solution techniques 72
3.4 Summary 74
3.5 Supplementary reading 75
3.6 Exercises 75

4 Passive Scalar Transport: Dispersion, Patterning, and Mixing . . . 79

4.1 Passive scalar transport equation 80
4.1.1 Scalar fluxes and constitutive properties 80
4.1.2 Scalar conservation equation 80
4.2 Physics of mixing 82
4.3 Measuring and quantifying mixing and related parameters 84
4.4 The low-Reynolds-number, high-Peclet-number limit 87
4.4.1 The high-Peclet-number limit 87
4.4.2 The low-Reynolds-number limit 87
4.5 Laminar flow patterning in microdevices 88
4.6 Taylor–Aris dispersion 89
4.7 Summary 91
4.8 Supplementary reading 92
4.9 Exercises 92

5 Electrostatics and Electrodynamics . . . 97

5.1 Electrostatics in matter 97
5.1.1 Electrical potential and electric field 97
5.1.2 Coulomb's law, Gauss's law for electricity in a material, curl of electric field 98
5.1.3 Polarization of matter and electric permittivity 100
5.1.4 Material, frequency, and electric-field dependence of electrical permittivity 102
5.1.5 Poisson and Laplace equations 104
5.1.6 Classification of material types 105
5.1.7 Electrostatic boundary conditions 105
5.1.8 Solution of electrostatic equations 107
5.1.9 Maxwell stress tensor 107
5.1.10 Effects of electrostatic fields on multipoles 108
5.2 Electrodynamics 109
5.2.1 Charge conservation equation 110
5.2.2 Electrodynamic boundary conditions 110
5.2.3 Field lines at substrate walls 112

5.3 Analytic representations of electrodynamic quantities: complex permittivity and conductivity 112
5.3.1 Complex description of dielectric loss 115
5.4 Electrical circuits 116
5.4.1 Components and properties 117
5.4.2 Electrical impedance 119
5.4.3 Circuit relations 119
5.4.4 Series and parallel component rules 120
5.5 Equivalent circuits for current in electrolyte-filled microchannels 122
5.5.1 Electrical circuit equivalents of hydraulic components 122
5.6 Summary 126
5.7 Supplementary reading 127
5.8 Exercises 127

6 Electroosmosis . . . 131

6.1 Matched asymptotics in electroosmotic flow 132
6.2 Integral analysis of Coulomb forces on the EDL 132
6.3 Solving the Navier–Stokes equations for electroosmotic flow in the thin-EDL limit 135
6.3.1 Outer solution 136
6.3.2 Replacing the EDL with an effective slip boundary condition 136
6.3.3 Replacing the Navier–Stokes equations with the Laplace equation: flow–current similitude 137
6.3.4 Reconciling the no-slip condition with irrotational flow 138
6.4 Electroosmotic mobility and the electrokinetic potential 138
6.4.1 Electrokinetic coupling matrix representation of electroosmosis 140
6.5 Electrokinetic pumps 140
6.5.1 A planar electrokinetic pump 140
6.5.2 Types of electrokinetic pumps 143
6.6 Summary 145
6.7 Supplementary reading 145
6.8 Exercises 146

7 Potential Fluid Flow . . . 153

7.1 Approach for finding potential flow solutions to the Navier–Stokes equations 153
7.2 Laplace equation for velocity potential and stream function 154
7.2.1 Laplace equation for the velocity potential 154
7.2.2 No-slip condition 156
7.3 Potential flows with plane symmetry 156
7.3.1 Complex algebra and its use in plane-symmetric potential flow 157
7.3.2 Monopolar flow: plane-symmetric (line) source with volume outflow per unit depth Λ 160
7.3.3 Plane-symmetric vortex with counterclockwise circulation per unit depth Γ 163
7.3.4 Dipolar flow: plane-symmetric doublet with dipole moment κ 165
7.3.5 Uniform flow with speed U 168
7.3.6 Flow around a corner 170
7.3.7 Flow over a circular cylinder 171
7.3.8 Conformal mapping 171

Cambridge University Press
978-0-521-11903-0 - Micro- and Nanoscale Fluid Mechanics: Transport in Microfluidic Devices
Brian J. Kirby
Frontmatter
More information

7.4 Potential flow in axisymmetric systems in spherical coordinates 172
7.5 Summary 173
7.6 Supplementary reading 173
7.7 Exercises 174

8 Stokes Flow . . . 178

8.1 Stokes flow equation 178
8.1.1 Different forms of the Stokes flow equations 179
8.1.2 Analytical versus numerical solutions of the Stokes flow equations 180
8.2 Bounded Stokes flows 180
8.2.1 Hele-Shaw flows 181
8.2.2 Numerical solution of general bounded Stokes flow problems 182
8.3 Unbounded Stokes flows 182
8.3.1 Stokes flow over a sphere in an infinite domain 182
8.3.2 General solution for Stokes flow over a sphere in an infinite domain 187
8.3.3 Flow over prolate ellipsoids 188
8.3.4 Stokes flow over particles in finite domains 189
8.3.5 Stokes flow over multiple particles 189
8.4 Micro-PIV 189
8.4.1 Deterministic particle lag 191
8.4.2 Brownian motion 191
8.5 Summary 191
8.6 Supplementary reading 192
8.7 Exercises 193

9 The Diffuse Structure of the Electrical Double Layer . . . 199

9.1 The Gouy–Chapman EDL 199
9.1.1 Boltzmann statistics for ideal solutions of ions 200
9.1.2 Ion distributions and potential: Boltzmann relation 201
9.1.3 Ion distributions and potential: Poisson–Boltzmann equation 202
9.1.4 Simplified forms of the nonlinear Poisson–Boltzmann equation 203
9.1.5 Solutions of the Poisson–Boltzmann equation 204
9.2 Fluid flow in the Gouy–Chapman EDL 208
9.3 Convective surface conductivity 210
9.4 Accuracy of the ideal-solution and Debye–Hückel approximations 211
9.4.1 Debye–Hückel approximation 212
9.4.2 Limitations of the ideal solution approximation 213
9.5 Modified Poisson–Boltzmann equations 213
9.5.1 Steric correction to ideal solution statistics 213
9.5.2 Modified Poisson–Boltzmann equation 215
9.5.3 Importance and limitations of Poisson–Boltzmann modifications 216
9.6 Stern layer 217
9.7 Summary 217
9.8 Supplementary reading 218
9.9 Exercises 218

10 Zeta Potential in Microchannels . . . 225

10.1 Definitions and notation 225
10.2 Chemical and physical origins of equilibrium interfacial charge 226
10.2.1 Electrochemical potentials 226

Cambridge University Press
978-0-521-11903-0 - Micro- and Nanoscale Fluid Mechanics: Transport in Microfluidic Devices
Brian J. Kirby
Frontmatter
More information

10.2.2 Potential-determining ions 227
10.2.3 Nernstian and non-Nernstian surfaces 230
10.3 Expressions relating the surface charge density, surface potential, and zeta potential 232
10.3.1 Extended interface models: modifications to φ_0 234
10.3.2 Fluid inhomogeneity models: relation between φ_0 and ζ 234
10.3.3 Slip and multiphase interface models: hydrophobic surfaces 236
10.4 Observed electrokinetic potentials on microfluidic substrates 237
10.4.1 Electrolyte concentration 237
10.4.2 pH dependence 238
10.5 Modifying the zeta potential 238
10.5.1 Indifferent electrolyte concentrations 238
10.5.2 Surface-active agents 239
10.5.3 Chemical functionalizations 240
10.6 Chemical and fluid-mechanical techniques for measuring interfacial properties 240
10.6.1 Charge titration 240
10.6.2 Electroosmotic flow 241
10.6.3 Streaming current and potential 242
10.7 Summary 245
10.8 Supplementary reading 246
10.9 Exercises 247

11 Species and Charge Transport 250

11.1 Modes of species transport 250
11.1.1 Species diffusion 250
11.1.2 Convection 250
11.1.3 Relating diffusivity and electrophoretic mobility: the viscous mobility 252
11.2 Conservation of species: Nernst–Planck equations 253
11.2.1 Species fluxes and constitutive properties 253
11.2.2 Nernst–Planck equations 254
11.3 Conservation of charge 256
11.3.1 Charge conservation equation 256
11.3.2 Diffusivity, electrophoretic mobility, and molar conductivity 258
11.4 Logarithmic transform of the Nernst–Planck equations 258
11.5 Microfluidic application: scalar-image velocimetry 259
11.5.1 SIV using caged-dye imaging 259
11.5.2 SIV using photobleaching 259
11.6 Summary 259
11.7 Supplementary reading 261
11.8 Exercises 261

12 Microchip Chemical Separations 265

12.1 Microchip separations: experimental realization 265
12.1.1 Sample injection 266
12.1.2 Resolution 267
12.2 1D Band broadening 268
12.2.1 Analyte transport: quiescent flow, no electric field 268
12.2.2 Transport of analytes: electroosmotic flow and electrophoresis 269

12.3 Microchip electrophoresis: motivation and experimental issues 270
12.3.1 Thermal dissipation 270
12.3.2 Compact, folded, long-pathlength channels 270
12.4 Experimental challenges 270
12.4.1 Pressure-driven flow 271
12.4.2 Analyte band dispersion in turns and expansions 271
12.5 Protein and peptide separation 273
12.5.1 Protein properties 273
12.5.2 Protein separation techniques 273
12.6 Multidimensional separations 275
12.7 Summary 276
12.8 Supplementary reading 276
12.9 Exercises 277

13 Particle Electrophoresis . . . 281

13.1 Introduction to electrophoresis: electroosmosis with a moving boundary and quiescent bulk fluid 281
13.2 Electrophoresis of particles 283
13.3 Electrophoretic velocity dependence on particle size 286
13.3.1 Smoluchowski velocity: large particles, small surface potential 287
13.3.2 Henry's function: effect of finite double layers for small φ_0 287
13.3.3 Large surface potential – effect of counterion distribution 289
13.4 Summary 292
13.5 Supplementary reading 294
13.6 Exercises 295

14 DNA Transport and Analysis . . . 298

14.1 Physicochemical structure of DNA 299
14.1.1 Chemical structure of DNA 299
14.1.2 Physical properties of dsDNA 300
14.2 DNA transport 303
14.2.1 DNA transport in bulk aqueous solution 303
14.3 Ideal chain models for bulk DNA physical properties 308
14.3.1 Idealized models for bulk DNA properties 309
14.3.2 Dependence of transport properties on contour length 320
14.4 Real polymer models 320
14.5 dsDNA in confining geometries 323
14.5.1 Energy and entropy of controlled polymer extension 323
14.5.2 Energy and entropy of confinement for ideal polymers 326
14.5.3 DNA transport in confined geometries 327
14.6 DNA analysis techniques 328
14.6.1 DNA amplification 328
14.6.2 DNA separation 328
14.6.3 DNA microarrays 329
14.7 Summary 329
14.8 Supplementary reading 331
14.9 Exercises 332

15 Nanofluidics: Fluid and Current Flow in Molecular-Scale and Thick-EDL Systems . . . 336

15.1 Unidirectional transport in infinitely long nanochannels 336
15.1.1 Fluid transport 337

15.1.2 Electrokinetic coupling matrix for thick-EDL transport 337
15.1.3 Circuit models for nanoscale channels 344
15.2 Transport through nanostructures with interfaces or nonuniform cross-sectional area 345
15.2.1 1D equilibrium model 346
15.2.2 Large molecule and particle transport 349
15.3 Supplementary reading 350
15.4 Exercises 351

16 AC Electrokinetics and the Dynamics of Diffuse Charge . . . 355

16.1 Electroosmosis with temporally varying interfacial potential 356
16.2 Equivalent circuits 356
16.2.1 The double layer as a capacitor 357
16.3 Induced-charge flow phenomena 363
16.3.1 Induced-charge double layers 363
16.3.2 Flow due to induced-charge double layers – induced-charge electroosmosis 364
16.3.3 Flow due to induced-charge double layers – AC electroosmosis 364
16.4 Electrothermal fluid flow 365
16.5 Summary 367
16.6 Supplementary reading 368
16.7 Exercises 369

17 Particle and Droplet Actuation: Dielectrophoresis, Magnetophoresis, and Digital Microfluidics . . . 373

17.1 Dielectrophoresis 373
17.1.1 Inferring the Coulomb force on an enclosed volume from the electric field outside the volume 375
17.1.2 The force on an uncharged, uniform, isotropic sphere in a linearly varying electric field with uniform, isotropic phase 375
17.1.3 Maxwellian equivalent body for inhomogeneous, spherically symmetric particles 380
17.1.4 Dielectrophoresis of charged spheres 382
17.1.5 Dielectrophoresis of nonspherical objects or objects in nonlinearly varying fields 382
17.1.6 Nonuniform and anisotropic phase effects 385
17.2 Particle magnetophoresis 389
17.2.1 Origin of magnetic fields in materials 390
17.2.2 Attributes of magnetism 391
17.2.3 Magnetic properties of superparamagnetic beads 392
17.2.4 Magnetophoretic forces 392
17.2.5 DC magnetophoresis of spheres – linear limit 393
17.3 Digital microfluidics 393
17.3.1 Electrocapillarity and electrowetting 394
17.4 Summary 395
17.5 Supplementary reading 396
17.6 Exercises 397

APPENDIX A. Units and Fundamental Constants . . . 405

A.1 Units 405
A.2 Fundamental physical constants 406

APPENDIX B. Properties of Electrolyte Solutions **407**
B.1 Fundamental properties of water 407
B.2 Aqueous solutions and key parameters 407
B.3 Chemical reactions, rate constants, and equilibrium 408
B.3.1 Henderson–Hasselbach equation 409
B.3.2 Conjugate acids and bases; buffers 411
B.3.3 Ionization of water 411
B.3.4 Solubility product of weakly soluble salts 412
B.3.5 Ideal solution limit and activity 412
B.3.6 Electrochemical potentials 413
B.4 Effects of solutes 413
B.4.1 Dielectric increments 413
B.5 Summary 415
B.6 Supplementary reading 415
B.7 Exercises 416

APPENDIX C. Coordinate Systems and Vector Calculus **418**
C.1 Coordinate systems 418
C.1.1 3D coordinate systems 418
C.1.2 2D coordinate systems 419
C.2 Vector calculus 420
C.2.1 Scalars, vectors, and tensors 420
C.2.2 Vector operations 423
C.2.3 Del or nabla operations 426
C.2.4 Biharmonic and E^4 operators 431
C.2.5 Vector identities 431
C.2.6 Dyadic operations 433
C.3 Summary 433
C.4 Supplementary reading 433
C.5 Exercises 434

APPENDIX D. Governing Equation Reference **436**
D.1 Scalar Laplace equation 436
D.2 Poisson–Boltzmann equation 437
D.3 Continuity equation 437
D.4 Navier–Stokes equations 438
D.5 Supplementary reading 439

APPENDIX E. Nondimensionalization and Characteristic Parameters **440**
E.1 Buckingham Π theorem 440
E.2 Nondimensionalization of governing equations 440
E.2.1 Nondimensionalization of the Navier–Stokes equations: Reynolds number 440
E.2.2 Nondimensionalization of the passive scalar transfer equation: Peclet number 443
E.2.3 Nondimensionalization of the Poisson–Boltzmann equation: Debye length and thermal voltage 445
E.3 Summary 447
E.4 Supplementary reading 448
E.5 Exercises 448

APPENDIX F. Multipolar Solutions to the Laplace and Stokes Equations 450
F.1 Laplace equation 450
F.1.1 Laplace equation solutions for axisymmetric spherical coordinates: separation of variables and multipolar expansions 450
F.1.2 Systems with plane symmetry: 2D cylindrical coordinates 456
F.2 Stokes equations 458
F.2.1 The Green's function for Stokes flow with a point source 458
F.3 Stokes multipoles: stresslet and rotlet 459
F.4 Summary 461
F.5 Supplementary reading 462
F.6 Exercises 462

APPENDIX G. Complex Functions 465
G.1 Complex numbers and basic operations 465
G.1.1 Arithmetic operations 466
G.1.2 Calculus operations 466
G.2 Using complex variables to combine orthogonal parameters 468
G.3 Analytic representation of harmonic parameters 469
G.3.1 Applicability of the analytic representation 470
G.3.2 Mathematical rules for using the analytic representation of harmonic parameters 470
G.4 Kramers–Krönig relations 471
G.5 Conformal mapping 472
G.5.1 Joukowski transform 472
G.5.2 Schwarz–Christoffel transform 473
G.6 Summary 473
G.7 Supplementary reading 473
G.8 Exercises 474

APPENDIX H. Interaction Potentials: Atomistic Modeling of Solvents and Solutes 475
H.1 Thermodynamics of intermolecular potentials 475
H.1.1 Monopole pair potentials 476
H.1.2 Spherically symmetric multipole pair potentials 477
H.2 Liquid-state theories 478
H.2.1 Integral techniques for concentration profiles 479
H.2.2 Why the direct correlation function does not describe concentration profiles 482
H.2.3 Total correlation functions and the Ornstein–Zernike equation 482
H.3 Excluded volume calculations 483
H.4 Atomistic simulations 483
H.4.1 Defining atomic forces and accelerations 485
H.4.2 Water models 485
H.4.3 Nondimensionalization in MD simulations 490
H.5 Summary 491
H.6 Supplementary reading 492
H.7 Exercises 492

Bibliography 495
Index 505

Preface

This text focuses on the physics of liquid transport in micro- and nanofabricated systems. It evolved from a graduate course I have taught at Cornell University since 2005, titled "Physics of Micro- and Nanoscale Fluid Mechanics," housed primarily in the Mechanical and Aerospace Engineering Department but attracting students from Physics, Applied Physics, Chemical Engineering, Materials Science, and Biological Engineering. This text was designed with the goal of bringing together several areas that are often taught separately – namely, fluid mechanics, electrodynamics, and interfacial chemistry and electrochemistry – with a focused goal of preparing the modern microfluidics researcher to analyze and model continuum fluid-mechanical systems encountered when working with micro- and nanofabricated devices. It omits many standard topics found in other texts – turbulent and transitional flows, rheology, transport in gel phase, Van der Waals forces, electrode kinetics, colloid stability, and electrode potentials are just a few of countless examples of fascinating and useful topics that are found in other texts, but are omitted here as they are not central to the fluid flows I wish to discuss.

Although I hope that this text may also serve as a useful reference for practicing researchers, it has been designed primarily for classroom instruction. It is thus occasionally repetitive and discursive (where others might state results succinctly and only once) when this is deemed useful for instruction. Worked sample problems are inserted throughout to assist the student, and exercises are included at the end of each chapter to facilitate use in classes. Solutions for qualified instructors are available from the publisher at http://www.cambridge.org/kirby. This text is *not* a summary of current research in the field and omits any discussion of microfabrication techniques or any attempt to summarize the technological state of the art.

The text considers, in turn, (a) low-Reynolds-number fluid mechanics and hydraulic circuits; (b) outer solutions for microscale flow, focusing primarily on the unique aspects of electroosmotic flow outside the electrical double layer; (c) inner solutions for microscale flow, focusing on sources of interfacial charge and modeling of electrical double layers; and (d) unsteady and nonequilibrium solutions, focusing on nonlinear electrokinetics, dynamics of electrical double layers, electrowetting, and related phenomena. In each case, several applications are selected to motivate the presentation, including microfluidic mixing, DNA and protein separations, microscale fluid velocity measurements, dielectrophoretic particle manipulation, electrokinetic pumps, and the like.

I select notation with the goal of helping students new to the field and with the understanding that this (on occasion) leads to redundant or unwieldy results. I minimize use of one symbol for multiple different variables, so the radius in spherical coordinates (r) is typeset with a symbol different from the radius in cylindrical coordinates ($\mathscr{r}$) and the colatitudinal angle ϑ in spherical coordinates is distinguished from the polar coordinate in cylindrical coordinates (θ). Because I teach from this text using a chalkboard, I use symbols that I can reproduce on a chalkboard – thus I avoid the use of the Greek letter ν for the kinematic viscosity $\nu = \eta/\rho$, because I am utterly unable to make it distinguishable from the y velocity v. Vectors, though they are placed in boldface to make them stand out, are also written with (admittedly redundant) superscripted arrows to match the chalkboard presentation.

This material is used for a semester-long graduate course at Cornell. Chapters 1, 2, 5, 7, and 8, as well as the appendices, are not covered in class as they are considered review or supplementary material. The remainder of the text is covered in approximately forty-two 50-minute classroom sessions.

I would like to acknowledge a number of people who helped with various aspects of this text. In particular, Dr. Elizabeth Strychalski and Professors Stephen Pope and Claude Cohen at Cornell, Professor Shelley Anna of Carnegie-Mellon University, Professor Kevin Dorfman of the University of Minnesota, Professor Nicolas Green of the University of Southampton, Donald Aubrecht of Harvard University, Professor Sumita Pennathur of UCSB, and Professor Aaron Wheeler of the University of Toronto were kind enough to offer useful suggestions. Professor Amy Herr of the University of California, Berkeley, used a draft of this text for her class during spring 2009; her insight and the feedback from her students were both immensely helpful. Professor Martin Bazant of the Massachusetts Institute of Technology provided materials helpful in completing the bibliography for several of the chapters. The students that have taken my class since 2004 have all contributed to this text in some way, but I would like to thank my student researchers Alex Barbati, Ben Hawkins, Sowmya Kondapalli, and Vishal Tandon in particular for their input, and my student Michael Allen for careful proofreading. Ben Hawkins and Dr. Jason Gleghorn contributed a number of the figures and helped to write material that was included in the chapters on Stokes flow and dielectrophoresis. David J. Griffiths (Reed College) provided files that assisted with typesetting. Gabe Terrizzi created many of the figures; his contributions were immensely helpful. Greg Parker (gparker@chorus.net) designed the cover.

Although many people assisted with review of this text, I am solely responsible for any errors, and I hope that readers will notify me or the publisher of those that they find. Errata will be maintained at http://www.cambridge.org/kirby.

Brian J. Kirby
Ithaca, NY
May 2010

Cambridge University Press
978-0-521-11903-0 - Micro- and Nanoscale Fluid Mechanics: Transport in Microfluidic Devices
Brian J. Kirby
Frontmatter
More information

Nomenclature

Symbol	Meaning	Page of first use or definition
A	area	61
$\mathcal{A}$	Helmholtz free energy	324
α	coefficient	112
α	phase lag angle	69
α	rotation angle	158
α	thermal diffusivity	80
a	acceleration	255
a	particle radius	171
a_i	activity	413
β	compressibility	75
β	coefficient	236
b	slip length	xxxvi
$\vec{\boldsymbol{B}}$	applied magnetic field	391
$\mathcal{B}$	Brillouin function	104
c_i	species molar concentration	407
c_p	specific heat	80
c	passive scalar	80
C	capacitance	117
C	constant of integration	xliii
C_{h}	compliance	66
$\underset{\sim}{C}$	complex number	465
C_{D}	drag coefficient	188
Γ	2D vortex strength	163
Γ	circulation	xiv
Γ	surface chemical site density	229
Γ	magnitude of injected sample	90
γ	surface tension	xxi
γ_i	natural logarithm of species concentration	259
χ	electrokinetic coupling matrix	65
χ_{e}	electric susceptibility	100
χ_{m}	magnetic susceptibility	98
d	depth	140
d	diameter	xxiv
D	scalar diffusivity	80
D_i	species diffusivity	252
$\vec{\boldsymbol{D}}$	electric displacement	100

Symbol	Meaning	Page of first use or definition
Du	Dukhin number	263
δ	Dirac delta function	458
$\vec{\vec{\delta}}$	identity tensor	xvi
∇	del operator	426
e	eccentricity	188
e	fundamental charge	201
e_1	singlet potential	475
e_2	pair potential	476
e_{mf}	potential of mean force	227
$\vec{E}$	electric field	97
ε	electrical permittivity	98
$\underset{\sim}{\varepsilon}$	complex electrical permittivity for sinusoidal fields	113
ε_{S}	Stern layer permittivity	360
ε_0	electrical permittivity of free space	100
ε_{r}	relative permittivity, i.e., dielectric constant	101
ε'	reactive permittivity	115
ε''	dissipative permittivity	115
$\varepsilon_{\mathrm{LJ}}$	potential well depth	477
$\vec{\vec{\varepsilon}}$	strain rate tensor	x
$\frac{\partial \varepsilon}{\partial c}$	dielectric increment	413
F	Faraday constant	99
$\vec{F}$	force	108
$\vec{f}$	force per unit volume	vi
$\underset{\sim}{f}_{\mathrm{CM}}$	Clausius–Mossotti factor	393
f_{ad}	adjusted distribution function	480
f_{d}	distribution function	217
f_{dc}	direct correlation function	482
f_{tc}	total correlation function	482
f_{M}	Mayer f function	480
f_0	Henry's function	288
f	electrophoretic correction factor	287
ϕ	electric potential	97
φ	electric potential difference from bulk	133
φ_0	total potential drop across the double layer	133
ϕ_{v}	velocity potential	153
$\underset{\sim}{\phi}_{\mathrm{v}}$	complex velocity potential	158
φ	azimuthal coordinate	419
Φ	cross-correlation	189
ζ	zeta potential	139
G	Gibbs free energy	xxi
G	electrical conductance	117
G_{s}	excess surface conductance	262
$\vec{g}$	gravitational acceleration	vi
g_i	chemical potential	227
$\overline{g_i}$	electrochemical potential	227

Symbol	Meaning	Page of first use or definition
$\vec{\vec{G}}$	hydrodynamic interaction tensor	187
$\vec{\vec{G}}_0$	Oseen–Burgers tensor	187
H	capillary height	xxv
$\vec{H}$	induced magnetic field	98
h	height	xliii
η	dynamic viscosity	xviii
$\vec{i}$	current density	110
i_0	exchange current density	112
I	current	64
I	second moment of area	309
I_c	ionic strength	408
j	square root of minus 1	157
$\vec{j}$	scalar flux density	80
$\mathcal{J}$	Joukowski transform	171
k	spring constant	325
k	chemical reaction rate	409
k_{ve}	viscoelectric coefficient	235
k_B	Boltzmann constant	104
K_a	acid dissociation constant	409
K_{eq}	equilibrium constant	409
K_{sp}	solubility product	412
κ	2D doublet strength	165
κ	Debye screening parameter	288
Λ	molar conductivity	256
Λ	2D source strength	160
λ_B	Bjerrum length	478
λ_D	Debye length	202
λ_{HS}	hard-sphere packing length	213
λ_S	Stern layer thickness	360
ℓ_c	polymer contour length	301
ℓ_e	polymer end-to-end length	303
ℓ_K	polymer Kuhn length	312
ℓ_p	polymer persistence length	299
L	length	61
L	electrical inductance	117
L	depolarization factor	384
m	mass	184
$\vec{M}$	magnetization	98
μ	viscous mobility	252
μ_{DEP}	dielectrophoretic mobility	374
μ_{EK}	electrokinetic mobility	265
μ_{EO}	electroosmotic mobility	138
μ_{EP}	electrophoretic mobility	252
μ_{mag}	magnetic permeability	98
$\mu_{mag,0}$	magnetic permeability of free space	98
N_A	Avogadro's number	112

Symbol	Meaning	Page of first use or definition
N_{bp}	number of base pairs in DNA molecule	301
n	normal coordinate	106
p	pressure	vi
$\vec{\boldsymbol{p}}$	dipole moment	104
$\mathrm{p}K_{\mathrm{a}}$	negative logarithm of acid dissociation constant	410
pH	negative logarithm of molar proton concentration	410
pOH	negative logarithm of molar hydroxyl ion concentration	411
pzc	point of zero charge	230
$\mathcal{P}$	perimeter	63
$\mathcal{P}$	probability density function	313
Pe	mass transfer Peclet number	79
ϖ	dummy frequency integration variable	115
ψ	stream function	viii
ψ_{S}	Stokes stream function	ix
ψ_{e}	electric stream function	469
$\vec{\boldsymbol{P}}$	electric polarization	100
$\vec{\vec{\boldsymbol{P}}}$	pressure interaction tensor	187
Q	volumetric flow rate	60
q	electric charge	97
q''	electric areal charge density	359
ρ	fluid density	vi
ρ_{E}	net charge density	99
r	radial coordinate – spherical coordinates	418
r_{h}	hydraulic radius	63
Δr	radial distance – spherical coordinates	98
$\vec{\boldsymbol{\Delta r}}$	distance vector	98
$\imath$	radial coordinate – cylindrical coordinates	418
$\Delta \imath$	radial distance – cylindrical coordinates	157
Re	Reynolds number	442
R	universal gas constant	112
R	electrical resistance	117
R	radius of channel	xlv
R	radius of curvature	xxii
R	separation resolution	267
R_{h}	hydraulic resistance	61
$\langle r_{\mathrm{g}} \rangle$	radius of gyration	303
s	arc length	302
$\mathcal{S}$	entropy	324
$\mathcal{S}$	Schwarz–Christoffel transform	473
σ	conductivity	110
σ_{LJ}	Lennard–Jones “bond length”	477
$\underline{\sigma}$	complex electrical conductivity	114
σ_{s}	effective surface conductivity	210

Cambridge University Press
978-0-521-11903-0 - Micro- and Nanoscale Fluid Mechanics: Transport in Microfluidic Devices
Brian J. Kirby
Frontmatter
More information

Symbol	Meaning	Page of first use or definition
Sk	Stokes number	186
St	Strouhal number	442
t	time	vii
T	Kelvin temperature	xxi
$\vec{\boldsymbol{T}}$	torque	109
$\vec{\vec{\boldsymbol{T}}}$	Maxwell stress tensor	107
$\vec{\vec{\boldsymbol{\tau}}}$	stress tensor	xvi
τ	characteristic time	103
θ	polar coordinate – cylindrical coordinates	418
θ	contact angle	xxii
θ_0	corner angle	170
ϑ	colatitude coordinate – spherical coordinates	418
$\Delta\theta$	polar coordinate of distance vector	157
$\vec{\boldsymbol{u}}$	velocity vector	vii
$\underset{\sim}{u}$	complex velocity	159
u_{EK}	electrokinetic velocity	269
u_{EO}	electroosmotic velocity	140
u_{EP}	electrophoretic velocity	255
$u_{\mathscr{r}}$	radial velocity – cylindrical coordinates	ix
u_r	radial velocity – spherical coordinates	ix
u_θ	circumferential velocity – cylindrical coordinates	ix
u_ϑ	circumferential velocity – spherical coordinates	ix
$\mathcal{U}$	molecular internal energy	324
V	voltage	106
$\mathcal{V}$	volume	66
ω	angular frequency	xlvii
$\vec{\boldsymbol{\omega}}$	vorticity	xiii
$\vec{\vec{\boldsymbol{\omega}}}$	rotation rate tensor	xi
w	width	90
x	x coordinate	418
ξ	hard-sphere packing parameter	215
ξ	thermodynamic efficiency	143
y	y coordinate	418
Y	Young's modulus	309
z	z coordinate	418
z	valence magnitude for symmetric electrolytes	203
z_i	species valence	99
Z	partition function	326
$\underset{\sim}{Z}$	impedance	119
$\underset{\sim}{Z}_{\mathfrak{h}}$	hydraulic impedance	69

Subscript	Example	Meaning
0	p_0	phasor or sinusoid magnitude
0	w_0	value at reference state
∞	$c_{i,\infty}$	value in freestream or in bulk
bend	$\mathcal{U}_{\mathrm{bend}}$	bending
conv	$\vec{\boldsymbol{j}}_{\mathrm{conv},i}$	convective
diff	$\vec{\boldsymbol{j}}_{\mathrm{diff},i}$	diffusive
edl	q''_{edl}	electrical double layer
eff	ζ_{eff}	effective
ext	$\vec{\boldsymbol{E}}_{\mathrm{ext}}$	extrinsic
H	u_H	high
L	u_L	low
m	$\underset{\sim}{\varepsilon}_{\mathrm{m}}$	suspending medium
n	E_n	normal
p	ρ_{p}	particle
pre	$\vec{\vec{\boldsymbol{\tau}}}_{\mathrm{pre}}$	isotropic (pressure) components
str	I_{str}	streaming
t	u_{t}	tangential
visc	$\vec{\vec{\boldsymbol{\tau}}}_{\mathrm{visc}}$	deviatoric (viscous) components
w	ρ_{w}	water

Superscript, accent	Example(s)	Meaning
$\circ$	g_i°	value at reference condition
$'$	φ', y'	dummy integration variable
$'$	F', I'	per unit length
$''$	F'', q''	per unit area
$'$, $''$	f', f''	derivatives of functions
$'$	ε'	reactive component
$''$	ε''	dissipative component
$-$	$\overline{u}$	spatially averaged
$\sim$	$\underset{\sim}{Z}$	analytic representation of real parameters
$\rightarrow$	$\vec{\boldsymbol{u}}$, $\vec{\boldsymbol{T}}$	vector or pseudovector
$\rightrightarrows$	$\vec{\vec{\boldsymbol{\tau}}}$, $\vec{\vec{\boldsymbol{\varepsilon}}}$	rank 2 tensor
$\hat{}$	$\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{\vartheta}}$	unit vector
$\hat{}$	$\hat{e}_1$	molar value
$*$	d^*, p^*	nondimensionalized quantity
$\langle\rangle$	$\langle \ell_{\mathrm{e}} \rangle$, $\langle r_{\mathrm{g}} \rangle$	time- or ensemble-averaged property
Δ	Δp, Δx	difference in property