NEUTRINO PHYSICS

Thoroughly revised and updated, this new edition presents a coherent and comprehensive overview of modern neutrino physics.

The book covers all the major areas of current interest, with chapters discussing the intrinsic properties of neutrinos, the theory of the interaction of neutrinos with matter, experimental investigations of the weak interaction in neutrino processes, the theory and supporting experiment for the basic properties of the interaction of neutrinos with fermions, and on astrophysics and cosmology. Several chapters have been completely rewritten.

This edition presents new data on solar neutrinos and an update of the results of searches for double beta decay. It also contains a new chapter on direct measurements of the neutrino mass, with high precision data from experiments at Fermilab and CERN, and at the Kamiokande Laboratory in Japan.

An essential reference text for particle physicists, nuclear physicists and astrophysicists.

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NEUTRINO PHYSICS

Second edition

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Preface to the second edition

Looking today, roughly ten years after this book was written, at the status of Neutrino Physics, it is amazing how much progress has been achieved. Some of it is truly fundamental, the discovery of neutrino flavor oscillation. Of course, according to our rules, it has still to be confirmed. It provides a first outlook into physics beyond the Standard Model. The Standard Model itself has now reached a status of maturity, after confirmation by measurements with a precision which seemed unthinkable a decade ago. It is now generally believed that its underlying symmetry pattern cannot be accidental and that it will be incorporated into a future Grand Unified Theory.

The structure of this book has met, I am told, with general approval. It has therefore not been modified. Most chapters have been updated, some had to be completely rewritten. Because of the original approach that the book is written by scientists who have themselves made important contributions, some new authors appear.

Looking at the open questions, details of the neutrino mixing and neutrino masses, and of CP violation in neutrino reactions, the particle-antiparticle properties of neutrinos, the problem of the detection of relic neutrinos, their density in space and their contribution to the energy density of the universe, their flavor composition and neutrino-antineutrino asymmetry, and the electromagnetic properties of neutrinos, I feel assured that Neutrino Physics will continue to develop and to become an even more central part of elementary particle physics.

I am grateful to the authors of the first and the second edition and to many colleagues who have again helped me to clarify the topics covered in this book.

It is a special pleasure to thank my wife Krisztina who has always given me her full support; I am gratefully dedicating this book to her.

Klaus Winter
Geneva, 3 November 1998

1

History

1.1 On the earlier and more recent history of the neutrino

WOLFGANG PAULI, 1957*

1 Problems concerning the interpretation of the continuous energy spectrum of beta rays

The continuous energy spectrum of beta rays discovered by J. Chadwick in 1914 [CHA 14] immediately posed difficult problems with respect to its theoretical interpretation. Was it directly due to the primary electrons emitted from the radioactive nucleus or was it to be attributed to secondary processes? The first hypothesis, which proved to be the correct one, was advocated by C. D. Ellis [ELL 22a], the second one by L. Meitner [MEI 22]. Meitner appealed to the fact that nuclei possess discrete energy states, as was known from alpha and gamma rays. She focused attention on the discrete energies of electrons, which had also been observed for many beta-radioactive nuclei. Ellis interpreted them as electrons being ejected from the outer shells by inner conversion of monochromatic nuclear gamma rays and assigned them to the observed X-ray lines. According to Meitner's theory, however, at least one of the electrons of discrete energy should be a genuine primary electron from the nucleus, which, in a secondary process, could then emit from the outer shells more electrons with smaller energies.[1] However, this postulated primary electron of discrete energy was never detected. Moreover, there are beta-radioactive nuclei, like RaE, that do not emit gamma rays and for which the electrons with discrete energies are missing altogether. In the polemic that arose between Ellis and Meitner, Ellis summarized [ELL 22b] his point of view in the following way:

> The theory of Miss Meitner is a very interesting attempt to provide a simple explanation of β-decay. The experimental facts, however, do not fit the framework of this theory and there is every indication that the simple analogy between α- and β-decay cannot be maintained. The β-decay is a considerably more complicated process and the general suggestions I made in this context appear to me to require the least constraint.

---

* Translation by Gabriele Zacek (CERN, Geneva), of "Zur älteren und neueren Geschichte des Neutrinos," published in Wolfgang Pauli, *Physik und Erkenntnistheorie*, pp. 156–80; Friedr, Vieweg, & Sohn, Braunschweig/Wiesbaden, 1984.

[1] In a later work [MEI 25] Meitner has proven experimentally that the γ-rays, contrary to an earlier opinion of Ellis, were emitted by the nucleus, which is generated *after* the emission of the α- or β-particle.

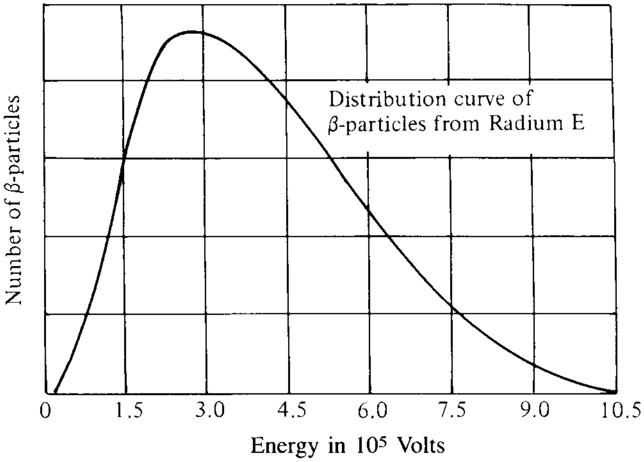

Fig. 1 Continuous beta spectrum of RaE.

This statement obviously did not bring researchers any closer to an answer to the question of how to interpret the continuous beta spectrum, and opinion remained divided on whether the spectrum was of primary origin (Ellis) or whether an initially discrete energy did broaden into a continuum by subsequent secondary processes (Meitner). This dispute finally came to an end in an experiment: the *measurement of the absolute heat in the absorption of beta electrons*. It was known from counting experiments that *one* electron is emitted from the nucleus per decay. In subsequent secondary processes, the heat measured in the calorimeter per decay should correspond to the upper limit of the beta spectrum; in the primary process, however, it should correspond to its mean energy. Ellis and W. A. Wooster [ELL 27] performed the measurement on RaE. The result for each decay, converted to Volts, was a heat of

$$344\,000 \text{ Volts} \pm 10\%$$

which corresponded well to the mean energy of the beta spectrum (Fig. 1). The upper boundary of the beta spectrum, however, would correspond to about 1 million Volts, which was completely excluded by the experiments. Ellis stressed that his experiment still left open the possibility of restoring the energy balance by a continuous gamma spectrum that would not have been absorbed in the calorimeter and would have escaped observation.

Meitner was not yet convinced by this experiment and immediately decided to repeat it with an improved apparatus. W. Orthmann, a collaborator of Nernst, designed a special differential calorimeter for this purpose. This calorimeter made it possible to repeat the heat measurement of the beta electrons from RaE with increased precision. The outcome,

$$337\,000 \text{ Volts} \pm 6\%$$

confirmed the result from Ellis and Wooster.

Moreover, in special experiments using ionization tubes, Meitner [MEI 30] proved that the continuous gamma spectrum postulated by Ellis was not present. Following these experimental results, there remained only two theoretical possibilities for the *interpretation of the continuous beta spectrum*:

1. The conservation of energy holds only statistically in this particular interaction, which gives rise to beta radioactivity.
2. The conservation of energy holds strictly in each primary process; however, an additional, very penetrating radiation is emitted together with the electrons, which consists of *new*, *neutral particles*.

The first possibility was supported by Bohr, the second one by myself. Before treating the history of these further questions, which was finally settled in favor of the second possibility, we must explain how our ideas about nuclear structure developed.

2 *Neutrino and nuclear structure*

Following Rutherford's first experiments on artificially induced transformations of nuclei, it was generally accepted that nuclei consist of protons and electrons. Rutherford himself discussed nuclear structure in this way in his famous Bakerian Lecture [RUT 20]. Among other things, the lecture presented the hypothesis of the existence of a nucleus with charge 0 and its eventual properties. Soon it became known (compare, e.g., [CLA 21]) that Rutherford had proposed the name *neutron* for these new hypothetical particles. He thought of them as a combination of protons and electrons of nuclear dimensions. Consequently, he urged his laboratory to perform experiments looking for these neutrons in hydrogen discharges, which of course had to remain fruitless.

The idea that the nuclei were made up of protons and electrons was eventually dismissed, albeit reluctantly. The decisive blow came from the quantum and wave mechanics theory advanced in 1927. According to this theory, there are two sorts of particles, the antisymmetric fermions and the symmetric bosons. Composite particles are fermions or bosons with the number of their constitutive fermions odd or even. An equivalent argument also holds for the spin, with fermions always possessing half a unit and bosons always an entire unit of spin. Since it was soon found that electrons and protons are fermions, the idea that they alone were the building blocks of all nuclei led to the conclusion that the parity of the charge number should determine the symmetry character of the nuclei. This conclusion was not confirmed by experience. The first counterexample was the "nitrogen anomaly," as we called it then. Using the band spectra, R. Kronig [KRO 28] and W. Heitler and G. Herzberg [HEI 29] showed that nitrogen with a charge number 7 and mass number 14 has spin 1 and Bose statistics. Similar cases followed, such as Li 6 (charge 3, mass 6) and the deuteron (charge 1, mass 2); both also had spin 1 and Bose statistics. Thus it was shown that the symmetry character of the nuclei was determined by the parity of the mass number and not by the parity of the charge number.

Using the idea of a new particle, I tried to combine this problem of the spin and statistics of nuclei with the problem of the continuous beta spectrum, without abandoning the conservation of energy. In December 1930, when the heavy neutron had not yet been discovered experimentally, I sent a letter on this topic to a meeting of physicists in Tübingen, where Geiger and Meitner in particular were present.[2]

> Public letter to the group of the Radioactives at the district society meeting in Tübingen:
>
> Physikalisches Institut Zürich, 4. Dec. 1930
> der Eidg. Technischen Hochschule Gloriastr.
>
> Zürich
>
> Dear Radioactive Ladies and Gentlemen,
>
> As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the "exchange theorem"[3] of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $\frac{1}{2}$ and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. – The continuous β-spectrum would then become understandable by the assumption that in β-decay, a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant. Now the question that has to be dealt with is which forces act on the neutrons? The most likely model for the neutron seems to me, because of wave mechanical reasons (the details are known by the bearer of these lines), that the neutron at rest is a magnetic dipole of a certain moment μ. The experiments seem to require that the effect of the ionization of such a neutron cannot be larger than that of a γ-ray and then μ should not be larger than $e * 10^{-3}$ cm.

---

[2] I am indebted to Mrs. Meitner for keeping a copy of this letter and for leaving it to me.

[3] This reads: exclusion principle (Fermi statistics) and half-integer spin for an odd number of particles; Bose statistics and integer spin for an even number of particles.

> For the moment, however, I do not dare to publish anything on this idea and I put to you, dear Radioactives, the question of what the situation would be if one such neutron were detected experimentally, if it would have a penetrating power similar to, or about 10 times larger than, a γ-ray.
>
> I admit that on a first look my way out might seem to be unlikely, since one would certainly have seen the neutrons by now if they existed. But nothing ventured nothing gained, and the seriousness of the matter with the continuous β-spectrum is illustrated by a quotation of my honored predecessor in office, Mr. Debey, who recently told me in Brussels: "Oh, it is best not to think about it, like the new taxes." Therefore one should earnestly discuss each way of salvation. – So, dear Radioactives, examine and judge it. – Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December. – With my best regards to you, and also to Mr. Back, your humble servant,
>
> W. Pauli

You see how modest the numbers were that I still had in mind at that time. To tell the truth, the penetration power of these particles, which today are called neutrinos, is about 100 light-years of Pb instead of 10 cm; compared with the gamma rays the factor is 10^{16} to 10^{17} instead of 10, the rest mass and the magnetic moment theoretically are 0, and the experimental upper limits are 0.002 electron masses and 10^{-9} Bohr magnetons [COW 57a].

I soon received a reply to my letter from Geiger, who had discussed my question with the others in Tübingen, especially with Meitner. Unfortunately, I do not have this reply any more. I recall, however, that his answer was positive and encouraging: From the experimental point of view, my new particles would indeed be possible.

Because of the empirical nuclear masses, I had quickly abandoned the idea that the neutral particles emitted in beta decay were at the same time constituents of the nuclei.

In a talk I gave on the occasion of a meeting of the American Physical Society in Pasadena in June 1931, I reported for the first time on my idea of new, very penetrating neutral particles in beta decay. I no longer believed that they made up the building blocks of the nucleus and hence did not call them neutrons any more. In fact, I used no special name for them. The matter still seemed to me to be quite uncertain, however, and I did not have my talk printed. In the same year, 1931, I traveled from America to Rome, where a large international congress on nuclear physics was to take place in October. There I met Fermi, who immediately expressed a lively interest in my idea and a very positive attitude toward my new neutral particles, as well as Bohr, who on the contrary advocated his idea of the statistical conservation of energy in beta decay. A little later he published this idea in his Faraday lecture [BOH 32]. To give you an impression of his ideas at that time, I quote the following section ([BOH 32], p. 383).

> At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β-ray disintegrations, and are even led to complications and difficulties in trying to do so. Of course, a radical departure from this principle would imply strange consequences, in case such a process could be reversed. Indeed, if, in a collision process, an electron could attach itself to a nucleus with loss of its mechanical individuality, and subsequently be recreated as a β-ray, we should find that the energy of this β-ray would generally differ from that of the original electron. Still, just as the account of those aspects of atomic constitution essential for the explanation of the ordinary physical and chemical properties of matter implies a renunciation of the classical idea of causality, the features of atomic stability, still deeper-lying, responsible for the existence and the properties of atomic nuclei, may force us to renounce the very idea of energy balance. I shall not enter further into such speculations and their possible bearing on the much debated question of the source of stellar energy. I have touched upon them here mainly to emphasize that in atomic theory, notwithstanding all the recent progress, we must still he prepared for new surprises.

Concerning the more general possibility of surprises in those interactions that we today call "weak," Bohr should maintain his point in another respect. However, his idea that there was only a statistical conservation of energy in these interactions seemed unacceptable to both Fermi and me. We had many private discussions on this topic in Rome in 1931, and I saw no theoretical reason to consider the law of the conservation of energy as less certain than, for example, the law of the conservation of electric charge. From an empirical point of view, it seemed to me decisive, whether the beta spectra of electrons showed a sharp upper limit or whether they showed a Poisson distribution dropping off toward infinity. In the first case, in my opinion, my idea of new particles would be established.[4] At that time the question was not yet decided experimentally, but Ellis, who was also present in Rome, already had plans to take this experimental problem up once more.

In the following year, Chadwick discovered the long-searched-for neutron with charge number 0 and mass number 1 through the bombardment of lighter nuclei with alpha particles. My new particle emitted in beta decay was thereupon called *neutrino* by Fermi in talks in Rome, to distinguish it from the heavier neutron,[5] and this Italian name was soon commonly adopted. Then the new idea about nuclear structure rapidly took shape, with the nuclei consisting of protons and neutrons, which we today call "nucleons," and which are both fermions with spin $\frac{1}{2}$. Various authors came to this idea independently; in Italy it was advocated by Majorana, who was supported by Fermi.

---

[4] For the theoretical interpretation of the upper limit of the spectrum, see also Ellis and Mott [ELL 33].

[5] I owe this information to Mr. E. Amaldi.

Thus at the Solvay meeting on atomic nuclei in Brussels in October 1933, where Joliot and Chadwick, among others, reported on their experimental discovery of positron decay and of the neutron and Heisenberg reported on the structure of the nucleus, a general clarification took place. Also, Fermi and Bohr were again present. It was now evident that, on the basis of this conception of nuclear structure, the neutrinos, as they were now called, had to be fermions in order to conserve statistics in beta decay. Furthermore, Ellis reported on new experiments carried out by his student W. J. Henderson [HEN 34], which established the sharp upper limit of the beta spectrum and consolidated its interpretation.

In view of the new circumstances, my earlier precaution of delaying publication now seemed to me unnecessary.

Following Heisenberg's lecture, I communicated my ideas on the neutrino (as it now was called) in the discussion, which also was printed in the report of the conference [PAU 34] and is reproduced here:

> The difficulty connected with the existence of the continuous spectra of beta rays arises, as one knows, from the fact that the mean lifetimes of the nuclei that emit these rays and also of the resulting daughter nuclei, have well determined values. Thus one necessarily concludes that the state, as well as the energy and the mass of the nucleus, which is left over after the expulsion of the β-particles, are also well determined. I do not want to elaborate on the efforts one could use to avoid this conclusion, but I think in accordance with Mr. Bohr, that one will always encounter unsurmountable difficulties in the explanation of the experimental facts.
>
> In the context of these ideas, two interpretations of the experiments are suggested. The one that is defended by Mr. Bohr admits that the laws of energy and momentum conservation are violated if one deals with a nuclear process where light particles play an essential role. This hypothesis seems to me unsatisfactory, not even plausible. First, the electric charge is conserved in the process and I do not see why the conservation of charge should be more fundamental than the conservation of energy and momentum. Furthermore, it is precisely the kinematic relations that govern various properties of the β-spectra (the existence of an upper limit and the connection to the γ-spectra, Heisenberg's criterion of stability). If the conservation laws should not hold, one would obviously have to conclude from these relations that β-decay is always accompanied by a loss and never by a gain in energy; this conclusion implies an irreversibility of this process with respect to time, which seems to me not to be acceptable at all.
>
> In June 1931 on the occasion of a conference in Pasadena I proposed the following interpretation: The conservation laws remain valid, since the emission of the β-particles is accompanied by a very penetrating radiation of neutral particles, which has not been observed up to now. The sum of the energies of the β-particle and the neutral particle (or the neutral particles, since one does not know whether there is only one or

> whether there are several), which are emitted by the nucleus in a single process equals the energy which corresponds to the upper limit of the β-spectra. It goes without saying that we admit for all elementary processes not only the conservation of energy but also the conservation of momentum, of angular momentum and of the type of statistics.
>
> As for the properties of these neutral particles, the atomic weights of the radioactive elements in particular teach us that their mass cannot exceed the mass of the electron by a lot. To distinguish them from the heavy neutrons Mr. Fermi has suggested the name "neutrino." It is possible that the rest mass of the neutrinos equals zero, so that they have to propagate, like the photons, with the speed of light. In any case their penetrating power exceeds many times that of photons of the same energy. It seems to me admissible that the neutrinos have spin $\frac{1}{2}$ and that they obey Fermi statistics, even though experience does not provide us with any direct proof of this hypothesis. We do not know anything about the interaction of the neutrinos with other matter particles and with photons: The hypothesis that they possess a magnetic moment, as I have proposed earlier (Dirac's theory foresees the possibility of the existence of neutral magnetic particles), does not seem to me established at all.
>
> In connection with these ideas, the experimental study of the momentum balance in β-decays is a problem of utmost importance; one can predict that the difficulties will be great because of the smallness of the recoil energy of the nucleus.

The difficulty with recoil measurements referred to above was not overcome until quite recently.

Subsequently, Chadwick reported on the first unsuccessful efforts to experimentally detect an absorption of neutrinos, which yielded an upper limit on the magnetic moment of the neutrino of 0.001 magnetons. Bohr's opposition had weakened considerably since his Faraday lecture. Having become very cautious about claiming the invalidity of the conservation of energy, he restricted himself to his much more general statement that nobody knew which surprises still were in store for us in this field. By the way, only as late as 1936 [BOH 36] he accepted entirely the validity of the conservation of energy in beta decay and the neutrino, even though Fermi's theory had already been successfully developed by then.

3 Formulation of a theory of beta decay

Soon afterward, stimulated by the discussions at the Solvay conference, Fermi developed his theory of beta decay [FER 33, 34]. Part of Fermi's conclusions concerning the shape of the beta spectrum and the inference about the rest mass of the neutrino were drawn at the same time and independently by F. Perrin [PER 33], who was also present at the Solvay conference. For this, a complete theory of the interaction is not necessary if one restricts oneself to the so-called allowed transitions, where the nonrelativistic approximation for the nucleons in the nucleus is sufficient. Apart from corrections, which only become important for larger nuclear charges due to the Coulomb interaction between the nucleus and the electron, the shape of the beta spectrum for these transitions is entirely determined by the statistical weight factor $\rho(E_e)$ of the density of states in phase space. This factor, depending very sensitively on the value of the rest mass m_ν of the neutrinos, is given by

$$\rho(E_e)\,dE_e = p_e^2\,dp_e\,p_\nu^2\,\frac{dp_\nu}{dE_\nu} = p_e E_e p_\nu E_\nu\,dE_e. \tag{1}$$

Here, the natural units $\hbar = c = 1$ are adopted, the indices e, ν refer to electron and neutrino, respectively, and the energy E is related to the momentum through the relation $E^2 = p^2 + m^2$, such that $dE/dp = p/E$.

If ΔE is the energy difference of the nucleus in the initial and final state of the decay, the law of energy conservation requires

$$E_\nu = \Delta E - E_e. \tag{2}$$

Since m_ν is the minimum energy of the neutrino, the upper limit E_0 of the electron energy of the spectrum is

$$E_0 = \Delta E - m_\nu. \tag{3}$$

Thus,

$$E_\nu = E_0 - E_e + m_\nu \tag{4}$$

and

$$\rho(E_e)\,dE_e = p_e E_e (E_0 - E_e + m_\nu)\sqrt{(E_0 - E_e)(E_0 - E_e + 2m_\nu)}\,dE_e. \tag{5}$$

In the case $m_\nu \neq 0$, the behavior of (5) in the vicinity of the upper limit E_0, namely, for $E_0 - E_e \ll m_\nu$, is completely different from the behavior for $m_\nu = 0$; that is,

$$\rho(E_e)\,dE_e = p_e E_e (E_0 - E_e)^2\,dE_e, \quad \text{for } m_\nu = 0. \tag{6}$$

In comparison with the empirical shape of the spectrum, Fermi and Perrin had already inferred $m_\nu = 0$ in 1933.

In accordance with the same principles, the most precise estimate of the upper limit on the rest mass of the neutrino m_ν is derived from the precise measurements of the beta spectra of tritium (H_3) by L. M. Langer and R. J. D. Moffat [LAN 52].[6] The result is found in the discussions of L. Friedman and Smith [FRI 58a], J. J. Sakurai

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[6] Besides the statistical factor of ρ an additional correction had to be taken into account here for $m_\nu \neq 0$, which was noted for the first time by J. R. Pruett [PRU 48]. The correction depends on a factor, which in general can lie between -1 and $+1$. For the general expression of this factor, see E. P. Enz [ENZ 57]. For the type of interaction assumed today, however, this factor is equal to zero.

[SAK 58a], and L. Friedman [FRI 58b]

$$m_\nu < 250\,\mathrm{eV} = 0.002 m_e.$$

Thus in what follows we always assume $m_\nu = 0$.

The Kurie plot of allowed transitions shows that (besides a factor $F(Z, E_e)$, i.e., the Coulomb correction) the statistical density $\rho(E_e)$ alone determines the shape of the beta spectrum. The experimental technique had to be refined before this result could be established.[7] In the Kurie plot,

$$\sqrt{N(E_e)/F(Z, E_e) p_e E_e} = K(x) \tag{7}$$

is plotted as a function of

$$x = (E_e - m_e)/(\Delta E - m_e) \tag{8}$$

where $N(E_e)\,dE_e$ is the number of electrons emitted per second and integrated over all directions.

For $m_\nu = 0$ the theory yields

$$K(x) = 1 - x. \tag{9}$$

Figure 2 shows a typical example of the linear character of the Kurie plot. On the basis of Fermi's theory of beta decay from 1933 and its generalizations, further conclusions can be drawn from the empirical result that, for allowed transitions, already the statistical weight factor alone determines the shape of the beta spectrum. Fermi had devoted all of his attention to the formalism of quantum electrodynamics developed by Heisenberg and myself, where the fields are represented as sums of space-time-dependent creation and absorption operators, and soon had reformulated them more elegantly in his own contributions. Immediately after the congress in Brussels, he began to develop a theory of beta decay as an example of an application of these field quantization methods in as close connection to quantum electrodynamics as possible. For the energy of the interaction per cm^3, he thus made the ansatz of a sum of products of the components of *four* different spinor fields (corresponding to two nucleons and two leptons, respectively) at the same space-time point. It is possible that this *local character of the Fermi interaction* will have to be refined later, but in any case it has proved to be an extremely good approximation. The entire expression describing the density of the interaction energy has to be a relativistic invariant, which, moreover, strictly obeys the law of conservation of electric charge. There are five typical possibilities, depending on whether the scalar products used are of two scalars (S), two pseudoscalars (P), two

---

[7] An example for a forbidden transition is the beta decay of RaE, which has played such an important role in the history of the interpretation of the continuous electron spectrum. The shape of the RaE spectrum is determined not only by the factor ρ, the density of states, which even today still makes an interesting object for study [BUH 58a,b].

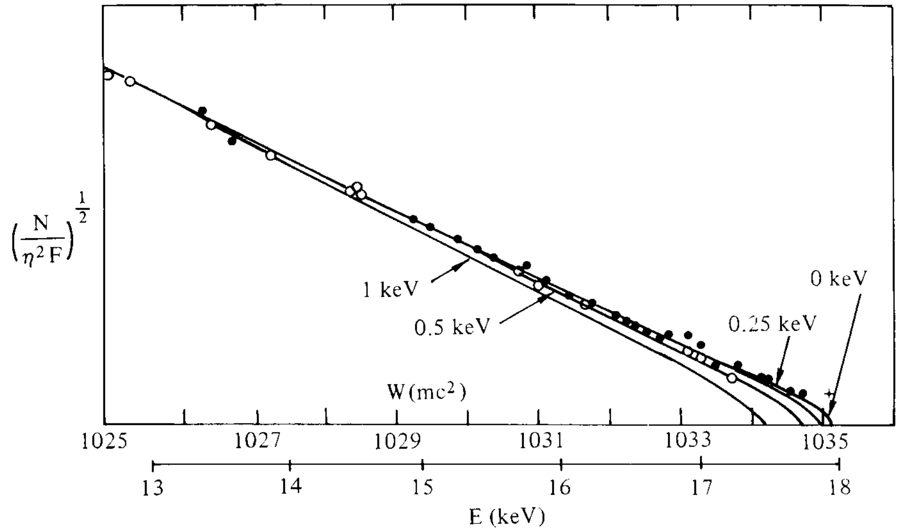

Fig. 2 Kurie plot of the tritium spectrum.

vectors (V), two pseudo- or axialvectors (A), or two antisymmetric tensors (T). By analogy with quantum electrodynamics, Fermi chose the V type in particular.

Initially, each of these types seemed to result in only *one* constant. However, this is based on special assumptions. One of them, as illustrated in the next paragraph, is the conservation of a leptonic charge, which up to now has withstood all the tests. The other one is the assumption of an invariance under spatial reflection and unchanged electrical charge ("parity"). In the last paragraph we will see that, surprisingly, this assumption did *not* prove to be correct. Thus, in the case of the "Fermi interaction," the most general expression that corresponds to the five types contains 10 arbitrary constants. However, in nature one special case is realized (see Section 5), so that finally only *one* quotient of coupling constants still remained undefined.

For the following discussion, we note, first, that in the nonrelativistic approximation the pseudoscalar type P makes no contribution to nucleons. To obtain information about the type P, it is necessary to consider "forbidden" transitions, for which this nonrelativistic approximation vanishes, while here we confine ourselves to "allowed" beta decay transitions in the nonrelativistic approximation and consequently omit the case P.

According to the selection rules for the angular momentum J of the nuclei, these transitions divide into two classes:

$$S, V \quad \begin{array}{l} \Delta J = 0 \\ (0 \to 0 \text{ allowed}) \end{array} \quad \text{Fermi (F)} \tag{10a}$$

$$T, A \quad \begin{array}{l} \Delta J = 0, \pm 1 \\ (0 \to 0 \text{ forbidden}) \end{array} \quad \text{Gamow–Teller (GT).} \tag{10b}$$

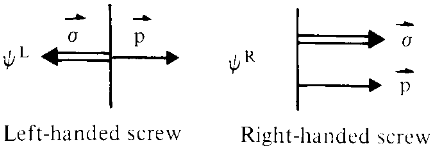

Fig. 3 Relative direction of spin σ and momentum p, for states characterized by ψ^L and ψ^R of a free Dirac particle with zero rest mass.

There are both pure Fermi and pure Gamow–Teller transitions, while in the general case both matrix elements differ from zero.

Fierz [FIE 37] was the first to draw the important conclusion that in the general case an additional factor of $(1 \pm bm_e/E_e)$ arises in the expression for the energy distribution of the beta spectrum, and, moreover, that this is only the case where S, V or T, A are mixed. The linearity of the Kurie plot showed, however, that to a good approximation these "Fierzterms" should be zero. This leads to the conclusion *that cases S and V and cases T and A cannot both be present at the same time.*[8]

B. Stech and J. H. D. Jensen [STE 55] have related this result to a formal transformation property of the density of the interaction energy, which proved to be successful and suitable for generalization when parity violation was later discovered. To illustrate this, we have to introduce the 4 × 4 matrix denoted by γ_5. This matrix has two eigenvalues +1 and two eigenvalues −1, such that

$$(1+\gamma_5)/2 \equiv a^L, \quad (1-\gamma_5)/2 \equiv a^R \tag{11}$$

are projection operators. The letters L and R refer to left and right and justify themselves by the fact that the corresponding spinor components

$$\psi^L = a^L\psi; \quad \psi^R = a^R\psi \tag{12}$$

refer to states with spin σ and momentum p (i.e., direction of motion) either antiparallel or parallel (Fig. 3).

These states are identical to the stationary states of a free particle only in the case of a particle with rest mass 0, like the neutrino, while for the electron the mass term in the Dirac equation couples the L and R components. However, for electron energies that are large compared to their rest mass[9] one can still talk more or less about L and R states in the case of a free particle.

The original "Stech–Jensen transformation" now corresponds to the fact that one has to multiply the L-component of the electron and of the neutrino at the same time by +1 and the R-component by −1, which according to (11), (12) is equivalent to

$$\psi' = \gamma_5\psi. \tag{13}$$

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[8] In this form the conclusion is correct only if invariance with respect to time reversal holds, which seems to be fulfilled in nature.

[9] We always use the natural units $\hbar = c = 1$.