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THE ANGULAR MOMENTUM OF LIGHT

Recent developments in the angular momentum of light present fresh challenges to long-established concepts and pave the way for new and wide-ranging applications. The scope for structured light such as optical vortices, in particular, now extends from microfluidics to quantum information.

This comprehensive edited collection deals with light carrying spin and orbital angular momentum, covering both fundamental and applied aspects. Written by internationally leading specialists, the chapters have been compiled to reflect the latest scientific progress and to address the multitude of theoretical, experimental and technical issues associated with this vibrant and exciting field.

The volume is an authoritative reference for academic researchers and graduate students engaged in theoretical or experimental study of optical angular momentum and its applications. It will also benefit professionals in physics, optics and optical engineering, chemistry and biology.

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Preface

For any sceptic of the continued capacity of science to uncover new truth, to pave the way for previously unimagined applications, there is hardly a better corrective than to invite reflection on recent discoveries in the science of light. It may be unscientific to say that light is unfathomable, but it certainly is a characteristic of the subject that there is always more to be learned, just when the utmost depths seem within grasp. There is no better illustration than the specific subject of the volume before you.

It has long been known that light conveys energy, and the associated linear momentum has also been understood since the days of Maxwell and Bartoli.¹ With angular momentum the history is more recent, and the property a little less straightforward. What we quickly learned is that light has a propensity to convey angular momentum, depending on its state. The pioneering work in which Beth² established a link with circular polarisation is nonetheless already three-quarters of a century old. Once the quantum theory of light was developed, many would have surmised that the science was complete, the concept of angular momentum so beautifully related to the unit spin of the photon—the hallmark of a boson. But what has been discovered in the past quarter century has shown that the spin angular momentum is only half the story – and the other half has no ending yet in sight.

Recent developments in the angular momentum of light are leading to new and wide-ranging applications, even as the subject presents fresh challenges to long established and cherished concepts. The present volume of contributed chapters has been compiled to reflect how far the science has now progressed, and to address the multitude of theoretical, experimental and technical issues associated with this vibrant and exciting field. The chapters are as follows.

In Chapter 1, Götte and Barnett formally introduce orbital angular momentum, the topic that has opened the door to a broader understanding of angular momentum in light. The twisted wave-front structures that deliver such features have a characteristic form whose vortex properties are then discussed by Molina-Terriza in Chapter 2. Chapter 3 by Galvez describes the detailed relationships between polarization and wave-front morphology, revealing the connectivity between spin and orbital angular momentum. Desyatnikov

¹ J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 2 (Macmillan, London, 1873); A. Bartoli, Il calorico raggiante e il secondo principio di termodinamica, *Il Nuovo Cimento* **15**, 196–202 (1884).

² R. A. Beth, Mechanical detection and measurement of the angular momentum of light, *Phys. Rev.* **50**, 115–25 (1936).

and Kivshar address the additional complexities that arise when twisted light propagates through an optically nonlinear medium, in Chapter 4. In Chapter 5, Nienhuis provides a study of the connections between ray optics, wave optics and quantum mechanics, associated with optical angular momentum. In a second contribution by Götte and Barnett, Chapter 6 continues the theme with an analysis of the interplay between discrete and quantum mechanically uncertain angular properties. Chapter 7 by Bialynicki-Birula and Bialynicka-Birula then provides the theory of rotational frequency shift, a dynamically controllable feature that affords another tier of opportunities for application.

In Chapter 8, Bliokh *et al.* give a detailed account of the interactions between the two forms of optical angular momentum, spin and orbital. Theory continues with Chapter 9 by Andrews and Babiker, detailing the quantum electrodynamical basis for the interactions of optical vortices, lending insights into the connected issues of chirality and propagated angular momentum. In a second contribution by Bialynicki-Birula *et al.*, Chapter 10 focuses attention on the trapping of charged particles by Bessel beams, a theme continued in a second contribution by Babiker *et al.*, addressing charged and uncharged atoms in Laguerre–Gaussian light, in Chapter 11. As an introduction to specifically quantum optical applications, Romero *et al.* describe in Chapter 12 the practical aspects of producing and deploying light with orbital angular momentum. In Chapter 13, also from Padgett’s group, the means of experimentally determining and separating beams with such angular momentum are described. More experimental detail is provided in Chapter 14 by Daria *et al.*, concerning the efficient generation and deployment of twisted optical beams. In Chapter 15, Firstenberg *et al.* address the issues of coherent diffusion, and finally in Chapter 16 van Exter *et al.* return to a distinctly quantum mechanical topic, developing and exploiting the orbital angular momentum of light to achieve azimuthal entanglement.

For the editors it has been a particular pleasure to undertake the responsibility of collating contributions from an outstanding set of authors, who include many of the key players in recent developments. On behalf of all our readers, we express our indebtedness to them for bringing a new level of clarity and accessibility to this fascinating subject.