

Engineering Design Optimization

Based on course-tested material, this rigorous yet accessible graduate textbook covers both fundamental and advanced optimization theory and algorithms. It covers a wide range of numerical methods and topics, including both gradient-based and gradient-free algorithms, multidisciplinary design optimization, and uncertainty, with instruction on how to determine which algorithm should be used for a given application. It also provides an overview of models and how to prepare them for use with numerical optimization, including derivative computation. Over 400 high-quality visualizations and numerous examples facilitate understanding of the theory, and practical tips address common issues encountered in practical engineering design optimization and how to address them. Numerous end-of-chapter homework problems, progressing in difficulty, help put knowledge into practice.

Accompanied online by a solutions manual for instructors and source code for problems, this is ideal for a one- or two-semester graduate course on optimization in aerospace, civil, mechanical, electrical, and chemical engineering departments.

Joaquim R. R. A. Martins is a Professor of Aerospace Engineering at the University of Michigan. He is a fellow of the American Institute for Aeronautics and Astronautics, and the Royal Aeronautical Society.

Andrew Ning is an Associate Professor of Mechanical Engineering at Brigham Young University, and has previously worked at the National Renewable Energy Laboratory (NREL) as a Senior Engineer.

Engineering Design Optimization

Joaquim R. R. A. Martins
University of Michigan, Ann Arbor

Andrew Ning
Brigham Young University, Utah



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-83341-7 — Engineering Design Optimization
Joaquim R. R. A. Martins, Andrew Ning
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108833417
DOI: 10.1017/9781108980647

© Joaquim R. R. A. Martins and Andrew Ning 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2022

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Martins, Joaquim R. R. A., author. | Ning, S. Andrew (Simeon Andrew), author.

Title: Engineering design optimization / Joaquim R. R. A. Martins, Andrew Ning.

Description: Cambridge ; New York, NY : Cambridge University Press, 2021. |

Includes bibliographical references and index.

Identifiers: LCCN 2021024825 (print) | LCCN 2021024826 (ebook) | ISBN 9781108833417 (hardback) | ISBN 9781108833417 (ebook)

Subjects: LCSH: Engineering design – Mathematical models. | Mathematical optimization. | Multidisciplinary design optimization. | BISAC:

MATHEMATICS / Optimization

Classification: LCC TA174 .M354 2021 (print) | LCC TA174 (ebook) | DDC 620/.0042–dc23

LC record available at <https://lcn.loc.gov/2021024825>

LC ebook record available at <https://lcn.loc.gov/2021024826>

ISBN 978-1-108-83341-7 Hardback

Additional resources for this publication at www.cambridge.org/martins-ning

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

Contents v

Preface xi

Acknowledgements xiii

- 1 Introduction 1
 - 1.1 Design Optimization Process 2
 - 1.2 Optimization Problem Formulation 6
 - 1.3 Optimization Problem Classification 17
 - 1.4 Optimization Algorithms 21
 - 1.5 Selecting an Optimization Approach 26
 - 1.6 Notation 28
 - 1.7 Summary 29
 - Problems 30
- 2 A Short History of Optimization 33
 - 2.1 The First Problems: Optimizing Length and Area 33
 - 2.2 Optimization Revolution: Derivatives and Calculus 34
 - 2.3 The Birth of Optimization Algorithms 36
 - 2.4 The Last Decades 39
 - 2.5 Toward a Diverse Future 43
 - 2.6 Summary 45
- 3 Numerical Models and Solvers 47
 - 3.1 Model Development for Analysis versus Optimization 47
 - 3.2 Modeling Process and Types of Errors 48
 - 3.3 Numerical Models as Residual Equations 50
 - 3.4 Discretization of Differential Equations 52
 - 3.5 Numerical Errors 53
 - 3.6 Overview of Solvers 61
 - 3.7 Rate of Convergence 63
 - 3.8 Newton-Based Solvers 66
 - 3.9 Models and the Optimization Problem 70

3.10	Summary	73
	Problems	75
4	Unconstrained Gradient-Based Optimization	79
4.1	Fundamentals	80
4.2	Two Overall Approaches to Finding an Optimum	94
4.3	Line Search	96
4.4	Search Direction	110
4.5	Trust-Region Methods	139
4.6	Summary	147
	Problems	149
5	Constrained Gradient-Based Optimization	153
5.1	Constrained Problem Formulation	154
5.2	Understanding n -Dimensional Space	156
5.3	Optimality Conditions	158
5.4	Penalty Methods	175
5.5	Sequential Quadratic Programming	187
5.6	Interior-Point Methods	204
5.7	Constraint Aggregation	211
5.8	Summary	214
	Problems	215
6	Computing Derivatives	223
6.1	Derivatives, Gradients, and Jacobians	223
6.2	Overview of Methods for Computing Derivatives	225
6.3	Symbolic Differentiation	226
6.4	Finite Differences	227
6.5	Complex Step	232
6.6	Algorithmic Differentiation	237
6.7	Implicit Analytic Methods—Direct and Adjoint	252
6.8	Sparse Jacobians and Graph Coloring	262
6.9	Unified Derivatives Equation	265
6.10	Summary	275
	Problems	277
7	Gradient-Free Optimization	281
7.1	When to Use Gradient-Free Algorithms	281
7.2	Classification of Gradient-Free Algorithms	284
7.3	Nelder–Mead Algorithm	287
7.4	Generalized Pattern Search	292
7.5	DIRECT Algorithm	298
7.6	Genetic Algorithms	306

Contents

vii

- 7.7 Particle Swarm Optimization 316
- 7.8 Summary 321
 - Problems 323
- 8 Discrete Optimization 327
 - 8.1 Binary, Integer, and Discrete Variables 327
 - 8.2 Avoiding Discrete Variables 328
 - 8.3 Branch and Bound 330
 - 8.4 Greedy Algorithms 337
 - 8.5 Dynamic Programming 339
 - 8.6 Simulated Annealing 347
 - 8.7 Binary Genetic Algorithms 351
 - 8.8 Summary 351
 - Problems 352
- 9 Multiobjective Optimization 355
 - 9.1 Multiple Objectives 355
 - 9.2 Pareto Optimality 357
 - 9.3 Solution Methods 358
 - 9.4 Summary 369
 - Problems 370
- 10 Surrogate-Based Optimization 373
 - 10.1 When to Use a Surrogate Model 374
 - 10.2 Sampling 375
 - 10.3 Constructing a Surrogate 384
 - 10.4 Kriging 400
 - 10.5 Deep Neural Networks 408
 - 10.6 Optimization and Infill 414
 - 10.7 Summary 418
 - Problems 420
- 11 Convex Optimization 423
 - 11.1 Introduction 423
 - 11.2 Linear Programming 425
 - 11.3 Quadratic Programming 427
 - 11.4 Second-Order Cone Programming 429
 - 11.5 Disciplined Convex Optimization 430
 - 11.6 Geometric Programming 434
 - 11.7 Summary 437
 - Problems 438

- 12 Optimization Under Uncertainty 441
 - 12.1 Robust Design 442
 - 12.2 Reliable Design 447
 - 12.3 Forward Propagation 448
 - 12.4 Summary 469
 - Problems 471
- 13 Multidisciplinary Design Optimization 475
 - 13.1 The Need for MDO 475
 - 13.2 Coupled Models 478
 - 13.3 Coupled Derivatives Computation 501
 - 13.4 Monolithic MDO Architectures 510
 - 13.5 Distributed MDO Architectures 519
 - 13.6 Summary 533
 - Problems 535
- A Mathematics Background 539
 - A.1 Taylor Series Expansion 539
 - A.2 Chain Rule, Total Derivatives, and Differentials 541
 - A.3 Matrix Multiplication 544
 - A.4 Four Fundamental Subspaces in Linear Algebra 547
 - A.5 Vector and Matrix Norms 548
 - A.6 Matrix Types 550
 - A.7 Matrix Derivatives 552
 - A.8 Eigenvalues and Eigenvectors 553
 - A.9 Random Variables 554
- B Linear Solvers 559
 - B.1 Systems of Linear Equations 559
 - B.2 Conditioning 560
 - B.3 Direct Methods 560
 - B.4 Iterative Methods 562
- C Quasi-Newton Methods 571
 - C.1 Broyden's Method 571
 - C.2 Additional Quasi-Newton Approximations 572
 - C.3 Sherman–Morrison–Woodbury Formula 576
- D Test Problems 579
 - D.1 Unconstrained Problems 579
 - D.2 Constrained Problems 586

Contents

ix

Bibliography 591

Index 615

Preface

Despite its usefulness, design optimization remains underused in industry. One of the reasons for this is the shortage of design optimization courses in undergraduate and graduate curricula. This is changing; today, most top aerospace and mechanical engineering departments include at least one graduate-level course on numerical optimization. We have also seen design optimization increasingly used in an expanding number of industries.

The word *engineering* in the title reflects the types of problems and algorithms we focus on, even though the methods are applicable beyond engineering. In contrast to explicit analytic mathematical functions, most engineering problems are implemented in complex multidisciplinary codes that involve implicit functions. Such problems might require hierarchical solvers and coupled derivative computation. Furthermore, engineering problems often involve many design variables and constraints, requiring scalable methods.

The target audience for this book is advanced undergraduate and beginning graduate students in science and engineering. No previous exposure to optimization is assumed. Knowledge of linear algebra, multivariable calculus, and numerical methods is helpful. However, these subjects' core concepts are reviewed in an appendix and as needed in the text. The content of the book spans approximately two semester-length university courses. Our approach is to start from the most general case problem and then explain special cases. The first half of the book covers the fundamentals (along with an optional history chapter). In contrast, the second half, from Chapter 8 onward, covers more specialized or advanced topics.

Our philosophy in the exposition is to provide a detailed enough explanation and analysis of optimization algorithms so that readers can implement a basic working version. Although we do not encourage readers to use their implementations instead of existing software for solving optimization problems, implementing a method is crucial in understanding the method and its behavior.* A deeper knowledge of these methods is useful for developers, researchers, and those who want to use numerical optimization more effectively. The problems at

*In the words of Donald Knuth: *"The ultimate test of whether I understand something is if I can explain it to a computer. I can say something to you and you'll nod your head, but I'm not sure that I explained it well. But the computer doesn't nod its head. It repeats back exactly what I tell it. In most of life, you can bluff, but not with computers."*

the end of each chapter are designed to provide a gradual progression in difficulty and eventually require implementing the methods. Some of the problems are open-ended to encourage students to explore a given topic on their own. When discussing the various optimization techniques, we also explain how to avoid the potential pitfalls of using a particular method and how to employ it more effectively. Practical tips are included throughout the book to alert the reader to common issues encountered in engineering design optimization and how to address them.

We have created a repository with code, data, templates, and examples as a supplementary resource for this book: <https://github.com/mdobook/resources>. Some of the end-of-chapter exercises refer to code or data from this repository.

Go forth and optimize!

Acknowledgments

Our workflow was tremendously enhanced by the support of Edmund Lee and Aaron Lu, who took our sketches and plots and translated them to high-quality, consistently formatted figures. The layout of this book was greatly improved based in part on a template provided by Max Opgenoord. We are indebted to many students and colleagues who provided feedback and insightful questions on our concepts, examples, lectures, and manuscript drafts. At the risk of leaving out some contributors, we wish to express particular gratitude to the following individuals who helped create examples, problems, solutions, or content that was incorporated in the book: Tal Dohn, Xiaosong Du, Sicheng He, Jason Hicken, Donald Jones, Shugo Kaneko, Taylor McDonnell, Judd Mehr, Santiago Padrón, Sabet Seraj, P. J. Stanley, and Anil Yildirim. Additionally, the following individuals provided helpful suggestions and corrections to the manuscript: Eytan Adler, Josh Anibal, Eliot Aretskin-Hariton, Alexander Coppeans, Alec Gallimore, Philip Gill, Justin Gray, Christina Harvey, John Hwang, Kevin Jacobsen, Kai James, Eirikur Jonsson, Matthew Kramer, Alexander Kleb, Michael Kokkolaras, Yingqian Liao, Sandy Mader, Marco Mangano, Giuliana Mannarino, Yara Martins, Johannes Norheim, Bernardo Pacini, Malhar Prajapati, Michael Saunders, Nikhil Shetty, Tamás Terlaky, and Elizabeth Wong. We are grateful to peer reviewers who provided enthusiastic encouragement and helpful suggestions and wish to thank our editors at Cambridge University Press, who quickly and competently offered corrections. Finally, we express our deepest gratitude to our families for their loving support.

Joaquim Martins and Andrew Ning