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Chapter 1
Coordinates, points and lines

This chapter uses coordinates to describe points and lines in two dimensions. When you have completed it, you should be able to

- find the distance between two points
- find the mid-point of a line segment, given the coordinates of its end points
- find the gradient of a line segment, given the coordinates of its end points
- find the equation of a line though a given point with a given gradient
- find the equation of the line joining two points
- recognise lines from different forms of their equations
- find the point of intersection of two lines
- tell from their gradients whether two lines are parallel or perpendicular.

1.1 The distance between two points

When you choose an origin, draw an x-axis to the right on the page and a y-axis up the page and choose scales along the axes, you are setting up a coordinate system. The coordinates of this system are called **cartesian coordinates** after the French mathematician René Descartes, who lived in the 17th century.

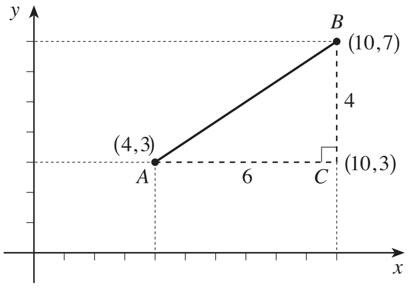

Fig. 1.1

In Fig. 1.1, two points A and B have cartesian coordinates (4,3) and (10,7). The part of the line AB which lies between A and B is called a **line segment**. The length of the line segment is the distance between the points.

A third point C has been added to Fig. 1.1 to form a right-angled triangle. You can see that C has the same x-coordinate as B and the same y-coordinate as A; that is, C has coordinates (10,3).

It is easy to see that AC has length $10 - 4 = 6$, and CB has length $7 - 3 = 4$. Using Pythagoras' theorem in triangle ABC shows that the length of the line segment AB is

$$\sqrt{(10-4)^2 + (7-3)^2} = \sqrt{6^2 + 4^2} = \sqrt{36+16} = \sqrt{52}.$$

You can use your calculator to give this as 7.21..., if you need to, but often it is better to leave the answer as $\sqrt{52}$.

The idea of coordinate geometry is to use algebra so that you can do calculations like this when A and B are any points, and not just the particular points in Fig. 1.1. It often helps to use a notation which shows at a glance which point a coordinate refers to. One way of doing this is with suffixes, calling the coordinates of the first point (x_1, y_1), and the coordinates of the second point (x_2, y_2). Thus, for example, x_1 stands for 'the x-coordinate of the first point'.

Fig. 1.2 shows this general triangle. You can see that C now has coordinates (x_2, y_1), and that $AC = x_2 - x_1$ and $CB = y_2 - y_1$. Pythagoras' theorem now gives

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

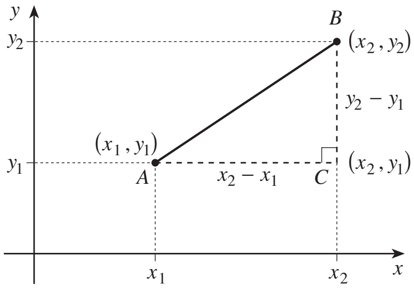

Fig. 1.2

An advantage of using algebra is that this formula works whatever the shape and position of the triangle. In Fig. 1.3, the coordinates of A are negative, and in Fig. 1.4 the line slopes downhill rather than uphill as you move from left to right. Use Figs. 1.3 and 1.4 to work out for yourself the length of AB in each case. You can then use the formula to check your answers.

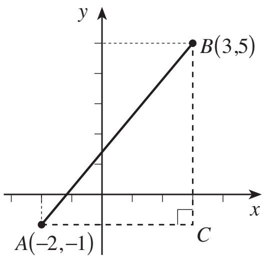

Fig. 1.3

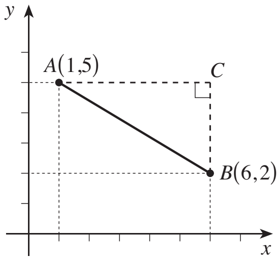

Fig. 1.4

In Fig. 1.3,

$$x_2 - x_1 = 3 - (-2) = 3 + 2 = 5 \quad \text{and} \quad y_2 - y_1 = 5 - (-1) = 5 + 1 = 6,$$

so $$AB = \sqrt{(3-(-2))^2 + (5-(-1))^2} = \sqrt{5^2 + 6^2} = \sqrt{25+36} = \sqrt{61}.$$

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And in Fig. 1.4,

$$x_2 - x_1 = 6 - 1 = 5 \quad \text{and} \quad y_2 - y_1 = 2 - 5 = -3,$$

so $$AB = \sqrt{(6-1)^2 + (2-5)^2} = \sqrt{5^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}.$$

Also, it doesn't matter which way round you label the points A and B. If you think of B as 'the first point' (x_1,y_1) and A as 'the second point' (x_2,y_2), the formula doesn't change. For Fig. 1.1, it would give

$$BA = \sqrt{(4-10)^2 + (3-7)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}, \text{ as before.}$$

The distance between the points (x_2,y_1) and (x_2,y_2) (or the length of the line segment joining them) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

1.2 The mid-point of a line segment

You can also use coordinates to find the mid-point of a line segment.

Fig. 1.5 shows the same line segment as in Fig. 1.1, but with the mid-point M added.

The line through M parallel to the y-axis meets AC at D. Then the lengths of the sides of the triangle ADM are half of those of triangle ACB, so that

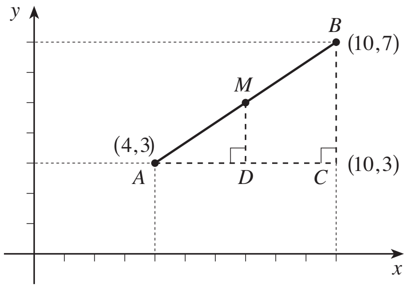

Fig. 1.5

$$AD = \tfrac{1}{2}AC = \tfrac{1}{2}(10-4) = \tfrac{1}{2}(6) = 3,$$
$$DM = \tfrac{1}{2}CB = \tfrac{1}{2}(7-3) = \tfrac{1}{2}(4) = 2.$$

The x-coordinate of M is the same as the x-coordinate of D, which is

$$4 + AD = 4 + \tfrac{1}{2}(10-4) = 4 + 3 = 7.$$

The y-coordinate of M is

$$3 + DM = 3 + \tfrac{1}{2}(7-3) = 3 + 2 = 5.$$

So the mid-point M has coordinates $(7,5)$.

In Fig. 1.6 points M and D have been added in the same way to Fig. 1.2. Exactly as before,

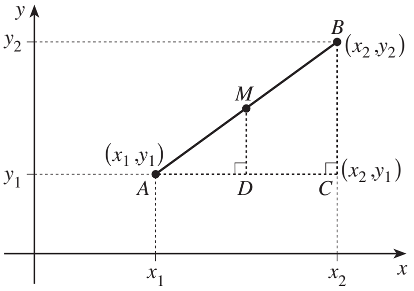

Fig. 1.6

$$AD = \tfrac{1}{2}AC = \tfrac{1}{2}(x_2 - x_1), \quad DM = \tfrac{1}{2}CB = \tfrac{1}{2}(y_2 - y_1).$$

So the x-coordinate of M is

$$x_1 + AD = x_1 + \tfrac{1}{2}(x_2 - x_1) = x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_1$$
$$= \tfrac{1}{2}x_1 + \tfrac{1}{2}x_2 = \tfrac{1}{2}(x_1 + x_2).$$

The y-coordinate of M is

$$y_1 + DM = y_1 + \tfrac{1}{2}(y_2 - y_1) = y_1 + \tfrac{1}{2}y_2 - \tfrac{1}{2}y_1$$
$$= \tfrac{1}{2}y_1 + \tfrac{1}{2}y_2 = \tfrac{1}{2}(y_1 + y_2).$$

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The mid-point of the line segment joining (x_1,y_1) and (x_2,y_2) has coordinates

$$\left(\tfrac{1}{2}(x_1+x_2), \tfrac{1}{2}(y_1+y_2)\right).$$

Now that you have an algebraic form for the coordinates of the mid-point M you can use it for any two points. For example, for Fig. 1.3 the mid-point of AB is

$$\left(\tfrac{1}{2}((-2)+3), \tfrac{1}{2}((-1)+5)\right) = \left(\tfrac{1}{2}(1), \tfrac{1}{2}(4)\right) = \left(\tfrac{1}{2}, 2\right).$$

And for Fig. 1.4 it is $\left(\tfrac{1}{2}(1+6)+3), \tfrac{1}{2}(5+2)\right) = \left(\tfrac{1}{2}(7), \tfrac{1}{2}(7)\right) = \left(3\tfrac{1}{2}, 3\tfrac{1}{2}\right)$.

Again, it doesn't matter which you call the first point and which the second. In Fig. 1.5, if you take (x_1,y_1) as $(10,7)$ and (x_2,y_2) as $(4,3)$, you find that the mid-point is $\left(\tfrac{1}{2}(10+4), \tfrac{1}{2}(7+3)\right) = (7,5)$, as before.

1.3 The gradient of a line segment

The gradient of a line is a measure of its steepness. The steeper the line, the larger the gradient.

Unlike the distance and the mid-point, the gradient is a property of the whole line, not just of a particular line segment. If you take any two points on the line and find the increases in the x- and y-coordinates as you go from one to the other, as in Fig. 1.7, then the value of the fraction

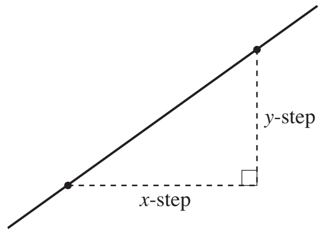

Fig. 1.7

$$\frac{y\text{-step}}{x\text{-step}}$$

is the same whichever points you choose. This is the **gradient** of the line.

In Fig. 1.2 the x-step and y-step are $x_2 - x_1$ and $y_2 - y_1$, so that:

The gradient of the line joining (x_1,y_1) to (x_2,y_2) is $\dfrac{y_2-y_1}{x_2-x_1}$.

This formula applies whether the coordinates are positive or negative. In Fig. 1.3, for example, the gradient of AB is $\dfrac{5-(-1)}{3-(-2)} = \dfrac{5+1}{3+2} = \dfrac{6}{5}$.

But notice that in Fig. 1.4 the gradient is $\dfrac{2-5}{6-1} = \dfrac{-3}{5} = -\dfrac{3}{5}$; the negative gradient tells you that the line slopes downhill as you move from left to right.

As with the other formulae, it doesn't matter which point has the suffix 1 and which has the suffix 2. In Fig. 1.1, you can calculate the gradient as either $\dfrac{7-3}{10-4} = \dfrac{4}{6} = \dfrac{2}{3}$, or $\dfrac{3-7}{4-10} = \dfrac{-4}{-6} = \dfrac{2}{3}$.

Two lines are **parallel** if they have the same gradient.

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EXAMPLE 1.3.1

The ends of a line segment are $(p-q, p+q)$ and $(p+q, p-q)$. Find the length of the line segment, its gradient and the coordinates of its mid-point.

For the length and gradient you have to calculate

$$x_2 - x_1 = (p+q)-(p-q) = p+q-p+q = 2q$$

and $$y_2 - y_1 = (p-q)-(p+q) = p-q-p-q = -2q.$$

The length is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2q)^2+(-2q)^2} = \sqrt{4q^2+4q^2} = \sqrt{8q^2}$.

The gradient is $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-2q}{2q} = -1$.

For the mid-point you have to calculate

$$x_1 + x_2 = (p-q)-(p+q) = p-q+p+q = 2p$$

and $$y_1 + y_2 = (p+q)+(p-q) = p+q+p-q = 2p.$$

The mid-point is $\left(\frac{1}{2}(x_1+x_2), \frac{1}{2}(y_1+y_2)\right) = \left(\frac{1}{2}(2p), \frac{1}{2}(2p)\right) = (p, p)$.

Try drawing your own figure to illustrate the results in this example.

EXAMPLE 1.3.2

Prove that the points $A(1,1)$, $B(5,3)$, $C(3,0)$ and $D(-1,-2)$ form a parallelogram.

You can approach this problem in a number of ways, but whichever method you use, it is worth drawing a sketch. This is shown in Fig. 1.8.

Method 1 (using distances) In this method, find the lengths of the opposite sides. If both pairs of opposite sides are equal, then $ABCD$ is a parallelogram.

$$AB = \sqrt{(5-1)^2+(3-1)^2} = \sqrt{20}.$$
$$DC = \sqrt{(3-(-1))^2+(0-(-2))^2} = \sqrt{20}.$$
$$CB = \sqrt{(5-3)^2+(3-0)^2} = \sqrt{13}.$$
$$DA = \sqrt{(1-(-1))^2+(1-(-2))^2} = \sqrt{13}.$$

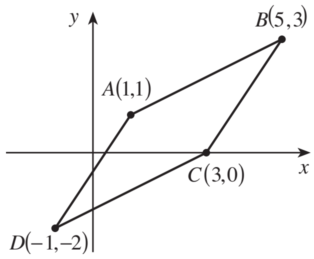

Fig. 1.8

Therefore $AB = DC$ and $CB = DA$, so $ABCD$ is a parallelogram.

Method 2 (using mid-points) In this method, begin by finding the mid-points of the diagonals AC and BD. If these points are the same, then the diagonals bisect each other, so the quadrilateral is a parallelogram.

The mid-point of AC is $\left(\frac{1}{2}(1+3), \frac{1}{2}(1+0)\right)$, which is $\left(2, \frac{1}{2}\right)$. The mid-point of BD is $\left(\frac{1}{2}(5+(-1)), \frac{1}{2}(3+(-2))\right)$, which is also $\left(2, \frac{1}{2}\right)$. So $ABCD$ is a parallelogram.

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Method 3 (using gradients) In this method, find the gradients of the opposite sides. If both pairs of opposite sides are parallel, then $ABCD$ is a parallelogram.

The gradients of AB and DC are $\frac{3-1}{5-1}=\frac{2}{4}=\frac{1}{2}$ and $\frac{0-(-2)}{3-(-1)}=\frac{2}{4}=\frac{1}{2}$ respectively, so AB is parallel to DC. The gradients of DA and CB are both $\frac{3}{2}$, so DA is parallel to CB. As the opposite sides are parallel, $ABCD$ is a parallelogram.

Exercise 1A

Do not use a calculator. Where appropriate, leave square roots in your answers.

1 Find the lengths of the line segments joining these pairs of points. In parts **e** and **h** assume that $a > 0$; in parts **j** and **i** assume that $p > q > 0$.

- **a** $(2, 5)$ and $(7, 17)$
- **b** $(-3, 2)$ and $(1, -1)$
- **c** $(4, -5)$ and $(-1, 0)$
- **d** $(-3, -3)$ and $(-7, 3)$
- **e** $(2a, a)$ and $(10a, -14a)$
- **f** $(a+1, 2a+3)$ and $(a-1, 2a-1)$
- **g** $(2, 9)$ and $(2, -14)$
- **h** $(12a, 5b)$ and $(3a, 5b)$
- **i** (p, q) and (q, p)
- **j** $(p+4q, p-q)$ and $(p-3q, p)$

2 Show that the points $(1, -2)$, $(6, -1)$, $(9, 3)$ and $(4, 2)$ are vertices of a parallelogram.

3 Show that the triangle formed by the points $(-3, -2)$, $(2, -7)$ and $(-2, 5)$ is isosceles.

4 Show that the points $(7, 12)$, $(-3, -12)$ and $(14, -5)$ lie on a circle with centre $(2, 0)$.

5 Find the coordinates of the mid-points of the line segments joining these pairs of points.

- **a** $(2, 11)$, $(6, 15)$
- **b** $(5, 7)$, $(-3, 9)$
- **c** $(-2, -3)$, $(1, -6)$
- **d** $(-3, 4)$, $(-8, 5)$
- **e** $(p+2, 3p-1)$, $(3p+4, p-5)$
- **f** $(p+3, q-7)$, $(p+5, 3-q)$
- **g** $(p+2q, 2p+13q)$, $(5p-2q, -2p-7q)$
- **h** $(a+3, b-5)$, $(a+3, b+7)$

6 A $(-2, 1)$ and B $(6, 5)$ are the opposite ends of the diameter of a circle. Find the coordinates of its centre.

7 $M(5, 7)$ is the mid-point of the line segment joining A $(3, 4)$ to B. Find the coordinates of B.

8 $A(1, -2)$, $B(6, -1)$, $C(9, 3)$ and $D(4, 2)$ are the vertices of a parallelogram. Verify that the mid-points of the diagonals AC and BD coincide.

9 Which one of the points $A(5, 2)$, $B(6, -3)$ and $C(4, 7)$ is the mid-point of the other two? Check your answer by calculating two distances.

10 Find the gradients of the lines joining the following pairs of points.

- **a** $(3, 8)$, $(5, 12)$
- **b** $(1, -3)$, $(-2, 6)$
- **c** $(-4, -3)$, $(0, -1)$
- **d** $(-5, -3)$, $(3, -9)$

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e $(p+3, p-3)$, $(2p+4, -p-5)$ **f** $(p+3, q-5)$, $(q-5, p+3)$

g $(p+q-1, q+p-3)$, $(p-q+1, q-p+3)$ **h** $(7, p)$, $(11, p)$

11 Find the gradients of the lines AB and BC where A is $(3, 4)$, B is $(7, 6)$ and C is $(-3, 1)$. What can you deduce about the points A, B and C?

12 The point $P(x, y)$ lies on the straight line joining $A(3, 0)$ and $B(5, 6)$. Find expressions for the gradients of AP and PB. Hence show that $y = 3x - 9$.

13 A line joining a vertex of a triangle to the mid-point of the opposite side is called a median. Find the length of the median AM in the triangle $A(-1, 1)$, $B(0, 3)$, $C(4, 7)$.

14 A triangle has vertices $A(-2, 1)$, $B(3, -4)$ and $C(5, 7)$.

a Find the coordinates of M, the mid-point of AB, and N, the mid-point of AC.

b Show that MN is parallel to BC.

15 The points $A(2, 1)$, $B(2, 7)$ and $C(-4, -1)$ form a triangle. M is the mid-point of AB and N is the mid-point of AC.

a Find the lengths of MN and BC. **b** Show that $BC = 2MN$.

16 The vertices of a quadrilateral $ABCD$ are $A(1, 1)$, $B(7, 3)$, $C(9, -7)$ and $D(-3, -3)$. The points P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.

a Find the gradient of each side of $PQRS$.

b What type of quadrilateral is $PQRS$?

17 The origin O and the points $P(4, 1)$, $Q(5, 5)$ and $R(1, 4)$ form a quadrilateral.

a Show that OR is parallel to PQ.

b Show that OP is parallel to RQ.

c Show that $OP = OR$.

d What shape is $OPQR$?

18 The origin O and the points $L(-2, 3)$, $M(4, 7)$ and $N(6, 4)$ form a quadrilateral.

a Show that $ON = LM$. **b** Show that ON is parallel to LM.

c Show that $OM = LN$. **d** What shape is $OLMN$?

19 The vertices of a quadrilateral $PQRS$ are $P(1, 2)$, $Q(7, 0)$, $R(6, -4)$ and $S(-3, -1)$.

a Find the gradient of each side of the quadrilateral.

b What type of quadrilateral is $PQRS$?

20 The vertices of a quadrilateral are $T(3, 2)$, $U(2, 5)$, $V(8, 7)$ and $W(6, 1)$. The mid-points of UV and VW are M and N respectively. Show that the triangle TMN is isosceles.

21 The vertices of a quadrilateral $DEFG$ are $D(3, -2)$, $E(0, -3)$, $F(-2, 3)$ and $G(4, 1)$.

a Find the length of each side of the quadrilateral.

b What type of quadrilateral is $DEFG$?

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22 The points $A(2, 1)$, $B(6, 10)$ and $C(10, 1)$ form an isosceles triangle with AB and BC of equal length. The point G is $(6, 4)$.

- **a** Write down the coordinates of M, the mid-point of AC.
- **b** Show that $BG = 2GM$ and that BGM is a straight line.
- **c** Write down the coordinates of N, the mid-point of BC.
- **d** Show that AGN is a straight line and that $AG = 2GN$.

1.4 What is meant by the equation of a straight line or of a curve?

How can you tell whether or not the points $(3, 7)$ and $(1, 5)$ lie on the curve $y = 3x^2 + 2$? The answer is to substitute the coordinates of the points into the equation and see whether they fit; that is, whether the equation is **satisfied** by the coordinates of the point.

For $(3, 7)$: the right side is $3 \times 3^2 + 2 = 29$ and the left side is 7, so the equation is not satisfied. The point $(3, 7)$ does not lie on the curve $y = 3x^2 + 2$.

For $(1, 5)$: the right side is $3 \times 1^2 + 2 = 5$ and the left side is 5, so the equation is satisfied. The point $(1, 5)$ lies on the curve $y = 3x^2 + 2$.

The equation of a line or curve is a rule for determining whether or not the point with coordinates (x, y) lies on the line or curve.

This is an important way of thinking about the equation of a line or curve.

1.5 The equation of a line

EXAMPLE 1.5.1

Find the equation of the line with gradient 2 which passes through the point $(2, 1)$.

Fig. 1.9 shows the line of gradient 2 through $A(2, 1)$, with another point $P(x, y)$ lying on it. P lies on the line if (and only if) the gradient of AP is 2.

The gradient of AP is $\frac{y-1}{x-2}$. Equating this to 2 gives $\frac{y-1}{x-2} = 2$, which is $y - 1 = 2x - 4$, or $y = 2x - 3$.

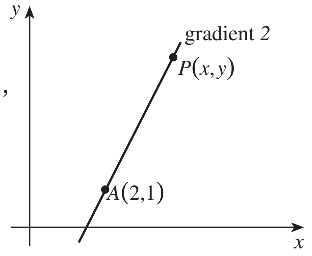

Fig. 1.9

In the general case, you need to find the equation of the line with gradient m through the point A with coordinates (x_1, y_1). Fig. 1.10 shows this line and another point P with coordinates (x, y) on it. The gradient of AP is $\frac{y-y_1}{x-x_1}$.

Equating to m gives $\frac{y-y_1}{x-x_1} = m$, or $y - y_1 = m(x - x_1)$.

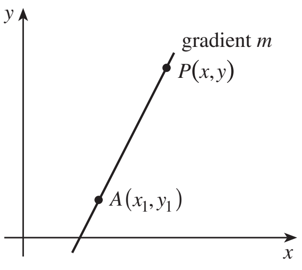

Fig. 1.10

The equation of the line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$.

Notice that the coordinates of $A(x_1, y_1)$ satisfy this equation.

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EXAMPLE 1.5.2

Find the equation of the line through the point $(-2, 3)$ with gradient -1.

Using the equation $y - y_1 = m(x - x_1)$ gives the equation $y - 3 = -1(x - (-2))$, which is $y - 3 = -x - 2$ or $y = -x + 1$. As a check, substitute the coordinates $(-2, 3)$ into both sides of the equation, to make sure that the given point does actually lie on the line.

EXAMPLE 1.5.3

Find the equation of the line joining the points $(3, 4)$ and $(-1, 2)$.

To find this equation, first find the gradient of the line joining $(3,4)$ to $(-1,2)$.

Then you can use the equation $y - y_1 = m(x - x_1)$.

The gradient of the line joining $(3, 4)$ to $(-1, 2)$ is $\frac{2-4}{(-1)-3} = \frac{-2}{-4} = \frac{1}{2}$.

The equation of the line through $(3,4)$ with gradient $\frac{1}{2}$ is $y - 4 = \frac{1}{2}(x - 3)$. After multiplying out and simplifying you get $2y - 8 = x - 3$, or $2y = x + 5$.

Check this equation mentally by substituting the coordinates of the other point.

1.6 Recognising the equation of a line

The answers to Examples 1.5.1–1.5.3 can all be written in the form $y = mx + c$, where m and c are numbers.

It is easy to show that any equation of this form is the equation of a straight line. If $y = mx + c$, then $y - c = m(x - 0)$, or

$$\frac{y-c}{x-0} = m \qquad \text{(except when } x = 0\text{)}.$$

This equation tells you that, for all points (x, y) whose coordinates satisfy the equation, the line joining $(0, c)$ to (x, y) has gradient m. That is, (x, y) lies on the line through $(0, c)$ with gradient m.

The point $(0, c)$ lies on the y-axis. The number c is called the ***y*-intercept** of the line.

To find the x-intercept, put $y = 0$ in the equation, which gives $x = -\frac{c}{m}$.

But notice that you can't do this division if $m = 0$. In that case the line is parallel to the x-axis, so there is no x-intercept.

When $m = 0$, all the points on the line have coordinates of the form (something, c). Thus the points $(1, 2)$, $(-1, 2)$, $(5, 2)$, ... all lie on the straight line $y = 2$, shown in Fig. 1.11. As a special case, the x-axis has equation $y = 0$.

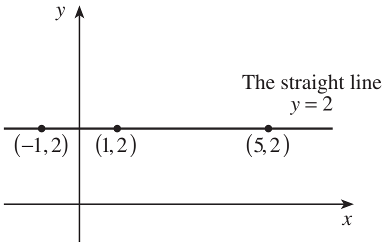

Fig. 1.11

Similarly, a straight line parallel to the y-axis has an equation of the form $x = k$. All points on it have coordinates (k, something). Thus the points $(3, 0)$, $(3, 2)$, $(3, 4)$, ... all lie on the line $x = 3$, shown in Fig. 1.12. The y-axis itself has equation $x = 0$.

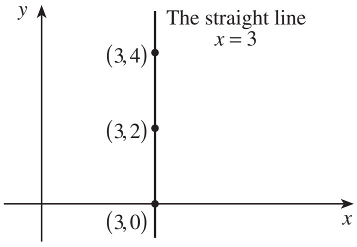

Fig. 1.12

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Hugh Neill , Douglas Quadling , Julian Gilbey
Excerpt
More Information

The line $x = k$ does not have a gradient; its gradient is undefined. Its equation cannot be written in the form $y = mx + c$.

1.7 The equation $ax + by + c = 0$

Suppose you have the equation $y = \frac{2}{3}x + \frac{4}{3}$. It is natural to multiply by 3 to get $3y = 2x + 4$, which can be rearranged to get $2x - 3y + 4 = 0$. This equation is in the form $ax + by + c = 0$ where a, b and c are constants.

Notice that the straight lines $y = mx + c$ and $ax + by + c = 0$ both contain the letter c, but it doesn't have the same meaning. For $y = mx + c$, c is the y-intercept, but there is no similar meaning for the c in $ax + by + c = 0$.

A simple way to find the gradient of $ax + by + c = 0$ is to rearrange it into the form $y = \ldots$. Here are some examples.

EXAMPLE 1.7.1

Find the gradient of the line $2x + 3y - 4 = 0$.

Write this equation in the form $y = \ldots$, and then use the fact that the straight line $y = mx + c$ has gradient m.

From $2x + 3y - 4 = 0$ you find that $3y = -2x + 4$ and $y = -\frac{2}{3}x + \frac{4}{3}$. Therefore, comparing this equation with $y = mx + c$, the gradient is $-\frac{2}{3}$.

EXAMPLE 1.7.2

One side of a parallelogram lies along the straight line with equation $3x - 4y - 7 = 0$. The point (2,3) is a vertex of the parallelogram. Find the equation of one other side.

The line $3x - 4y - 7 = 0$ is the same as $y = \frac{3}{4}x - \frac{7}{4}$, so its gradient is $\frac{3}{4}$. The line through $(2, 3)$ with gradient $\frac{3}{4}$ is $y - 3 = \frac{3}{4}(x - 2)$, or $3x - 4y + 6 = 0$.

1.8 The point of intersection of two lines

Suppose that you have two lines with equations $2x - y = 4$ and $3x + 2y = -1$. How do you find the coordinates of the point of intersection of these lines?

You want the point (x, y) which lies on both lines, so the coordinates (x, y) satisfy both equations. Therefore you need to solve the equations simultaneously.

From these two equations, you find $x = 1$, $y = -2$, so the point of intersection is $(1, -2)$.

This argument applies to straight lines with any equations provided they are not parallel. To find points of intersection, solve the equations simultaneously. The method can also be used to find the points of intersection of two curves.

