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On the Progression of Knowledge in Multiagent Systems

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Abstract

In a seminal paper, Lin and Reiter introduced the progression of basic action theories in the situation calculus. In this paper, we study the progression of knowledge in multiagent settings, where after actions, an agent updates her beliefs but also updates what she believes other agents know given what has occurred. By appealing to the notion of only knowing, we are able to avoid limitations of earlier work on multiagent progression, and obtain a new general account: we show that after an action, knowledge bases are updated in a Lin and Reiter fashion at every nesting of modalities. Consequently, recent results on the first-order definability of progression carry over to a multiagent setting without too much effort.

Introduction

Long-lived agents, such as autonomous robots, have to operate purposefully after thousands of actions. As argued in (Reiter 2001), there is only one computational methodology that addresses this problem at large: the idea of *progressing* the current world state, investigated in a general way for the language of the situation calculus in (Lin and Reiter 1997). Indeed, applying repeated regression over so many actions would simply not be feasible. STRIPS technology is a simple form of progression, Lin and Reiter (LR) observe, but in an open-world setting with non-trivial action types, appealing to second-order logic is necessary (Vassos and Levesque 2008). Nonetheless, LR identified two simple cases where progression is first-order definable, and other companion results have been identified since (Liu and Lakemeyer 2009).

However, LR did not consider knowledge. The progression of epistemic theories is very easily motivated in a world where the agent can both *act* and *sense*, and the agent would deliberate on her actions based on what she knows and does not know. An equally compelling case for the progression of knowledge is in multiagent settings, such as the ones robots would inhabit. Here, it is reasonable that after actions, for computational reasons among others, an agent updates her beliefs but also updates her beliefs about what others know

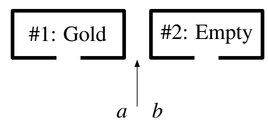

Figure 1: Agents a and b look for a room with gold.

given that these actions have occurred.[1] Imagine, for example, a simple domain, involving agents a and b looking for gold (Figure 1). When a observes b moving to Room 2 and looking through its window, her beliefs would be as follows:

- she still does not know where the gold is;
- a knows that b knows whether Room 2 has the gold, but cannot say what b now knows.

In general, then, progression should say precisely how a's knowledge and a's beliefs about b's knowledge would be updated after any sequence of physical and sensing actions.[2]

The progression of knowledge in the single agent case is considered in (Liu and Wen 2011), but under syntactic restrictions to the initial theory, such as limitations on quantifying-in, and that knowledge does not appear negatively. The progression of knowledge is also considered in (Lakemeyer and Levesque 2009). The results here are general. By appealing to Levesque's (1990) logic of *only knowing*, they obtain a simple specification: given an agent who only knows an action theory, the result of doing an action is that the agent only knows the LR progression of this theory.

The progression of knowledge in the multiagent case is also considered in (Liu and Wen 2011). In addition to the above limitations, formulas such as $\boldsymbol{K}_i p$, where p is objective, can be progressed, but not $\boldsymbol{K}_i \boldsymbol{K}_j \boldsymbol{K}_i p$. That is, among other things, they cannot deal with an agent's beliefs about what others believe about her. In this work, we substantially improve and extend these results. We provide a new general account of the progression of knowledge in multiagent systems that is essentially equivalent to the results in (Lakemeyer and Levesque 2009) when only a single agent is concerned. As a simple consequence of this work, earlier advances on tractable progression apply easily to our results.

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[1]We use the terms "knowledge" and "belief" interchangeably, and never require that knowledge needs to be true in the real world.

[2]For simplicity, executed actions are assumed to be publicly observable, even if the information that agents receive may be private and unobservable.

Logical Foundations

The language is a second-order modal dialect with equality, and can be thought of as a situation-suppressed epistemic version of Reiter's (2001) situation calculus, as defined in (Lakemeyer and Levesque 2009). We will not go over the details here, except to note:

- the symbols of the language consist of first-order variables, second-order predicate variables, rigid functions, fluent predicate symbols, as well as these connectives and other symbols: $=, \wedge, \neg, \forall, \Box$, epistemic modalities, parentheses, period, comma;
- the language includes a countably infinite supply of *rigid terms*, which we denote R, that are taken to be isomorphic to the *domain of discourse*;
- *multiagent knowledge* and *only knowing*: the language includes modalities $\boldsymbol{K}_i$ and $\boldsymbol{O}_i$ to reason about what i knows as well as what i only knows;
- *sensing*: every action is assumed to have a binary sensing result. After an action, the agent comes to know that the action was executable (via a special symbol *Poss*), and whether the corresponding sensing outcome holds (via a special symbol SF_i). For simplicity, we keep sensing actions separate from physical ones which only affect the world but do not provide any knowledge to the agent.

A semantics is provided for well-formed formulas in this language by means of *possible worlds*. Formally, let $\mathcal{Z}$ be the set of all finite sequences of elements of R including $\langle\rangle$, the empty sequence. $\mathcal{Z}$ should be understood as the set of all finite sequences of actions. Then a *world* $w \in \mathcal{W}$ is any function from G (the set of ground atoms) and $\mathcal{Z}$ to $\{0, 1\}$.

To interpret epistemic operators, and only knowing in particular, we appeal to the notion of k-structures (Belle and Lakemeyer 2010). For ease of exposition, assume that there are only 2 agents: a (Alice) and b (Bob); a generalization to more agents is straightforward. Then,

Definition 1: A k-structure ($k \geq 1$), say e^k, for an agent is defined inductively as:

– $e^1 \subseteq \mathcal{W} \times \{\{\}\}$,

– $e^k \subseteq \mathcal{W} \times \mathbb{E}^{k-1}$, where $\mathbb{E}^m$ is the set of all m-structures.

Intuitively, a k-structure for a, denoted e^k_a, determines what a believes about the world, but also at each world, what a believes b to know, to depth $k-1$. Such an epistemic state captures the initial beliefs of an agent, and of course, as the agent acts and senses in the world, some possibilities would be discarded. To enable this, we define $w' \simeq^i_z w$ (read: "i considers w' to agree with w on the sensing results throughout action sequence z") inductively by the following:

- $w' \simeq_{\langle\rangle} w$ for all w';
- $w' \simeq^i_{z\cdot t} w$ iff $w' \simeq^i_z w$, $w'[Poss(t), z] = 1$ and $w'[SF_i(t), z] = w[SF_i(t), z]$.

Putting all these together, we now turn to the semantic rules for sentences of the logic. These will be defined wrt a world w, an action sequence z, a variable map μ to interpret quantification over second-order variables (see (Lakemeyer and Levesque 2009)) a structure of some depth for a, say e^k_a, and a structure of some depth for b, say e^l_b.[3] The rules for the formulas in the language are given inductively, as usual. The cases for formulas not mentioning epistemic operators is as discussed in (Lakemeyer and Levesque 2009). For epistemic operators, we use the following definition:

Definition 2: Let w be a world, e^k_i a k-structure and z any sequence of actions. Then:

- w_z is a world such that $w_z[p, z'] = w[p, z\cdot z']$ for all ground atoms p and action sequences z';
- $(e^k_i)^w_z$ is defined inductively:
  - $(e^1_i)^w_z = \{(w'_z, \{\}) \mid (w', \{\}) \in e^1_i \text{ and } w' \simeq^i_z w\}$;
  - $(e^k_i)^w_z = \left\{(w'_z, (e^{k-1}_j)^{w'}_z) \mid (w', e^{k-1}_j) \in e^k_i \text{ and } w' \simeq^i_z w\right\}$.

Here, w_z is exactly like w after z. Thus, it is intuitive to think of w_z as the *progression* of w wrt z, and analogously, $(e^k_i)^w_z$ is the progression of e^k_i wrt z given the (real) world w. Indeed, when we progress the structures in e^k_i we are insisting that only those worlds which are compatible with w are considered.[4] Then, the rules for epistemic operators are as follows:

- $e^k_a, e^l_b, w, z, \mu \models \boldsymbol{K}_a\alpha$ iff for all w', for all e^{k-1} for b, if $(w', e^{k-1}_b) \in (e^k_a)^w_z$ then $(e^k_a)^w_z, e^{k-1}_b, w', \langle\rangle, \mu \models \alpha$;
- $e^k_a, e^l_b, w, z, \mu \models \boldsymbol{O}_a\alpha$ iff for all w', for all e^{k-1} for b, $(w', e^{k-1}_b) \in (e^k_a)^w_z$ iff $(e^k_a)^w_z, e^{k-1}_b, w', \langle\rangle, \mu \models \alpha$.

The semantics for $\boldsymbol{K}_b$ and $\boldsymbol{O}_b$ is defined analogously.

Roughly speaking, then, i is said to know α after z at w iff all the structures in i's progressed epistemic state wrt (w, z) are those where α holds initially. Analogously, α is said to be only known after z at w if structures where α holds are precisely the ones in the progressed epistemic state.[5]

We say a sentence α is *satisfiable* if there is a model of "appropriate" depth where α is true (Belle and Lakemeyer 2010). We say α is *valid*, written $\models \alpha$, with the understanding that α is true wrt all models of the appropriate depth.[6]

Finally, the usual properties on introspection can be shown to hold for all action sequences. That is, both $\Box(\boldsymbol{K}_i\alpha \supset \boldsymbol{K}_i\boldsymbol{K}_i\alpha)$ and $\Box(\neg\boldsymbol{K}_i\alpha \supset \boldsymbol{K}_i\neg\boldsymbol{K}_i\alpha)$ are valid. Of course, from only knowing (Levesque 1990), we also get knowledge, that is, $\Box(\boldsymbol{O}_i\alpha \supset \boldsymbol{K}_i\alpha)$ is valid.

[3]The idea is that only formulas with some number of epistemic operators are interpreted wrt structures of a corresponding depth; see (Belle and Lakemeyer 2010) on how to make this precise.

[4]We remark that when constructing $(e^k_i)^w_z$, we use worlds w' that are compatible with w wrt i's sensing results, but then we progress the structures e^{k-1}_j that i considers possible for j at w' as if w' is the real world to obtain $(e^{k-1}_j)^{w'}_z$. The reason is that i simply does not know which world is the real one, and so at each of her possible worlds w', she would "progress" her version of j's epistemic state using w' instead of w.

[5]The difference between the rules for $\boldsymbol{K}_i\alpha$ and $\boldsymbol{O}_i\alpha$ is the "if" in the second line of the former *vs.* "iff" in the latter.

[6]In particular, it is possible to show that if a sentence is satisfiable wrt every model of the appropriate depth, it is also satisfiable wrt every model of higher depths (Belle and Lakemeyer 2010).

Basic Action Theories

We now consider the equivalent of Reiter's *action theories*, the components of which we illustrate using an example:[7]

Example 3: We develop a simple basic action theory for the gold domain in Figure 1, where the gold is to be found in either $r1$ (Room 1) or $r2$ (Room 2). We imagine a single physical action $mv(x, y)$ which gets the agent x to Room y, and a single sensing action $see(x, y)$ which allows the agent x to see through Room y's window and the action reports whether the gold is present. We also use a fluent $Gold(x)$ to say that Room x has the gold, and a fluent $At(x, y)$ to indicate that the agent x is at Room y. More precisely, the basic action theory we are imagining has the following components:

1. $\Box Poss(o) \equiv \exists x, y.\ o = see(x, y) \wedge At(x, y)\ \vee$ $o = mv(x, y) \wedge true.$
2. $\Box[o]Gold(x) \equiv Gold(x).$
3. $\Box[o]At(x, y) \equiv o = mv(x, y)\ \vee$ $At(x, y) \wedge \neg\exists u(o = mv(x, u)).$
4. $\Box SF_a(o) \equiv \exists x, y.\ o = see(x, y) \wedge x = a \wedge Gold(y)\ \vee$ $o = see(x, y) \wedge x \neq a \wedge true.$
5. $\Box SF_b(o) \equiv \exists x, y.\ o = see(x, y) \wedge x = b \wedge Gold(y)\ \vee$ $o = see(x, y) \wedge x \neq b \wedge true.$

Item 1 is a precondition axiom, items 2 and 3 are successor state axioms, and the remaining are sensing axioms. These latter axioms say this: if x is at y then its sensor would return *true* if y has the gold, and false otherwise. We assume that actions are always public in this work, and so we also need to describe what i learns when he senses j looking through the window. It is this definition that eventually leads to information asymmetry; here, we want that i does not obtain new knowledge on j's sensing. For example, SF_a will simply be *true* when $see(b, r1)$ is performed.

Finally, we want to also say that the gold is in Room 1, and so for an initial theory, we will have $\text{KB}_0 = Gold(r1)$. Moreover, to say that the agents do not know where the gold is, let KB denote the conjunction of: $Gold(r1) \vee Gold(r2)$, $Gold(r1) \equiv \neg Gold(r2)$, and $\forall x, y \neg At(x, y)$. Then, lumping items 1-5 from the example as $\Box\beta$, the following sentence:

$$\text{KB}_0 \wedge \boldsymbol{O}_a(\text{KB} \wedge \Box\beta \wedge \boldsymbol{O}_b(\text{KB} \wedge \Box\beta)) \qquad (1)$$

is a domain where Room 1 has the gold, a does not know where the gold is, and believes the same of b. In contrast, (1) $\wedge\ \boldsymbol{O}_b(\text{KB}_0 \wedge \Box\beta)$ is a domain where, in fact, b does know where the gold is located and so, a has false beliefs about b.

Let us conclude the section with some entailments which will guide us when accounting for progression.

Proposition 4: *The following are entailed by* (1)*:*

1. $[mv(b, r2)][see(b, r2)]\neg\exists x \boldsymbol{K}_a(Gold(x));$
2. $[mv(b, r2)][see(b, r2)]\boldsymbol{K}_a\exists x \boldsymbol{K}_b(Gold(x));$
3. $[mv(b, r2)][see(b, r2)][mv(a, r1)][see(a, r1)]$ $(\boldsymbol{K}_a Gold(r1) \wedge \boldsymbol{K}_a\boldsymbol{K}_b Gold(r1)).$

[7] See (Lakemeyer and Levesque 2009) for the general definition. Most significantly, since situation terms do not appear in the language, Reiter's (2001) second-order induction axiom, among other things, will not be needed, but the remaining parts of an action theory have an equivalent formulation.

Progression

In this section, we turn to the main results of this paper. The question we would like to ask is this: suppose agents are given basic action theories as their initial knowledge bases, how are we to characterize the knowledge bases after an action is performed? Only knowing will give us the answer.

What we desire for a progression theorem are transformations obeying the following properties:

1. a general characterization of the new knowledge bases regardless of the syntactic restrictions to action types;
2. progression must be allowed to iterate.

We present results supporting this desiderata for a fragment of the language referred to as *progressable sentences*:

Definition 5: A OCNF formula is any conjunction of formulas of the form $\phi \vee \boldsymbol{O}_i\psi$, where ψ is any formula and ϕ is any fluent formula.

Definition 6: We define *progressable sentences* of depth $k \geq 1$ inductively as follows:

- *i-progressable* and *j-progressable* sentences of depth 1 are basic action theories;
- a *i-progressable* sentence of depth $k > 1$ is the conjunction of any basic action theory Σ and a OCNF formula $\bigwedge(\phi \vee \boldsymbol{O}_j\psi)$, where each ψ is a j-progressable sentence of depth $< k$;
- a *j-progressable* sentence of depth $k > 1$ is the conjunction of any basic action theory Σ and a OCNF formula $\bigwedge(\phi \vee \boldsymbol{O}_i\psi)$, where each ψ is a i-progressable sentence of depth $< k$.

If α is a i-progressable sentence (of any depth) then $\boldsymbol{O}_i\alpha$ is referred to a *progressable sentence.* If α is a j-progressable sentence (of any depth) then $\boldsymbol{O}_j\alpha$ is referred to as a *progressable sentence.*

Roughly, progressable sentences are those where i's beliefs about the world are precisely determined by an action theory, and i has beliefs about possible knowledge bases that j only knows.[8] Clearly, (1) would be classified as progressable, as would a sentence of the form $\boldsymbol{O}_a(\phi \wedge \Box\beta \wedge (\varphi \supset \boldsymbol{O}_b\psi \wedge \varphi' \supset \boldsymbol{O}_b\psi'))$ where, by requirement, ψ and ψ' are b-progressable, and thus, perhaps action theories themselves.

Progression for Physical Actions In the following we assume π to refer to the RHS of the precondition axioms, and γ_F to refer to the RHS of successor state axioms. Also, let $\alpha^{\vec{F}}_{\vec{P}}$ denote the formula α with every occurrence of fluent predicate symbols F_i replaced by second-order variables P_i of the same arity. The first main result is a characterization, in general terms, of what is known after a physical action.[9]

[8] Note that, then, the single agent case is essentially identical to (Lakemeyer and Levesque 2009).

[9] To avoid notational clutter, we make two simplifying assumptions: (a) we state theorems for progressable sentences of depth 2, and (b) we assume that the dynamic components $\Box\beta$ are the same at all depths. Generalizing these is straightforward but tedious.

Theorem 7: *Suppose $\Upsilon = \boldsymbol{O}_i(\phi \wedge \Box\beta \wedge \bigwedge(\varphi_u \vee \boldsymbol{O}_j(\psi_u \wedge \Box\beta)))$, where $u \in \{1, \ldots, n\}$, is a progressable theory, where ϕ, φ_u and ψ_u are fluent sentences and $\Box\beta$ are the dynamic components. Suppose t is a physical action. Then:*

$$\models \Upsilon \supset [t]\boldsymbol{O}_i(\mathcal{P}_\phi \wedge \Box\beta \wedge \bigwedge(\varphi_u \vee \boldsymbol{O}_j(\mathcal{P}_{\psi_u} \wedge \Box\beta)))$$

where for a fluent formula α:

$$\mathcal{P}_\alpha = \exists\vec{P}.\, [(\alpha \wedge \pi_t^o)^{\vec{F}}_{\vec{P}} \wedge \bigwedge \forall\vec{x}.\, F(\vec{x}) \equiv \gamma_{F\,t}^{\;o\vec{F}}{}_{\vec{P}}].$$

This theorem says that, at all depths, if all that i knows is a basic action theory, then after an action, he knows another basic action theory where the knowledge base is the *progression* of the previous initial knowledge base. The resulting theory is still *progressable*, and so progression can iterate.

On the correctness of belief expansion, we have:

Theorem 8: *Suppose Υ is as above. Then*

$$\Upsilon \models [t]\boldsymbol{K}_i\alpha \text{ iff } \Upsilon' \models \boldsymbol{K}_i\alpha$$

where Υ' is the new progressed theory from Theorem 7.

This result says that whatever is believed by the progressed theory is equivalently believed after t from the initial knowledge base. That is, the two theories agree on the future.

Example 9: Consider the effect of $mv(b, r2)$ on the progressable theory (1). From Theorem 7, we have:

$$\models (1) \supset [mv(b, r2)]\boldsymbol{O}_a(\mathcal{P}_{\text{KB}} \wedge \Box\beta \wedge \boldsymbol{O}_b(\mathcal{P}_{\text{KB}} \wedge \Box\beta))$$

where $\mathcal{P}_{\text{KB}}$ can be shown to be equivalent to $\{Gold(r1) \vee Gold(r2), Gold(r1) \equiv \neg Gold(r2), At(b, r2), \forall x, y.\ x \neq b \supset \neg At(x, y)\}$. Then, readers may note that the following sentences are entailed by $\boldsymbol{O}_a(\mathcal{P}_{\text{KB}} \wedge \Box\beta \wedge \boldsymbol{O}_b(\mathcal{P}_{\text{KB}} \wedge \Box\beta))$: $\boldsymbol{K}_a At(b, r2)$, $\boldsymbol{K}_a\boldsymbol{K}_b At(b, r2)$ and $\boldsymbol{K}_a\boldsymbol{K}_b\neg At(a, r2)$.

Finally, we remark that $\mathcal{P}_\phi$ for any initial theory ϕ is essentially the second-order account of progression given by LR; see (Lakemeyer and Levesque 2009) for discussions.

From this general definition, standard progression results on first-order definability (Liu and Lakemeyer 2009) apply in a straightforward manner. For example, we have:

Theorem 10: *Suppose Υ is as above, where ϕ and ψ_u are first-order formulas. If Υ is local-effect, then $\mathcal{P}_\phi$ and $\mathcal{P}_{\psi_u}$ from Theorem 7 are definable as first-order formulas.*

See (Liu and Lakemeyer 2009) for a definition of local-effect action theories; analogously, first-order progression for other classes of actions carry over without too much effort. See a long version of this paper for details.

Progression for Sensing Actions Letting θ_i refer to the RHS of SF_i in a basic action theory, we have the second main result of this paper:

Theorem 11: *Suppose $\Upsilon = \boldsymbol{O}_i(\phi \wedge \Box\beta \wedge \bigwedge(\varphi_u \vee \boldsymbol{O}_j(\psi_u \wedge \Box\beta)))$, where $u \in \{1, \ldots, n\}$, is a progressable theory as before. Suppose t is a sensing action. Then,*

$$\begin{aligned} \models \Upsilon \supset & \\ & (SF_i(t) \supset [t]\boldsymbol{O}_i(\phi \wedge \pi_t^o \wedge \theta_{i_t}^o \wedge \Box\beta \wedge \bigwedge(\varphi_u \vee \Psi_u))) \wedge \\ & (\neg SF_i(t) \supset [t]\boldsymbol{O}_i(\phi \wedge \pi_t^o \wedge \neg\theta_{i_t}^o \wedge \Box\beta \wedge \bigwedge(\varphi_u \vee \Psi_u))) \end{aligned}$$

where Ψ_u is the following sentence:

$$\begin{aligned} & \theta_{j_t}^o \supset \boldsymbol{O}_j(\psi_u \wedge \pi_t^o \wedge \theta_{j_t}^o \wedge \Box\beta) \wedge \\ & \neg\theta_{j_t}^o \supset \boldsymbol{O}_j(\psi_u \wedge \pi_t^o \wedge \neg\theta_{j_t}^o \wedge \Box\beta). \end{aligned}$$

This theorem says that after a sensing action, what i learns for herself is contingent on the outcome of $SF_i(t)$. But as far as i's beliefs about j goes, she is uncertain about the outcome of sensing for j, and so her beliefs about j would differ at one place: $\theta_{j_t}^o$ *vs.* $\neg\theta_{j_t}^o$. Let us illustrate this using an example:

Example 12: Consider the progression of the theory from Example 9 wrt the sensing action $see(b, r2)$, denoted t. Let ζ denote $\mathcal{P}_{\text{KB}}$ from that example, and so for an initial theory, we have $\Upsilon = Gold(r1) \wedge \boldsymbol{O}_a(\zeta \wedge \Box\beta \wedge \boldsymbol{O}_b(\zeta \wedge \Box\beta))$. After t, from Theorem 11 we have $\Upsilon \models SF_a(t) \supset [t]\boldsymbol{O}_a(\zeta^* \wedge \Box\beta\ \wedge \theta_{b_t}^o \supset \boldsymbol{O}_b(\zeta' \wedge \Box\beta)\ \wedge \neg\theta_{b_t}^o \supset \boldsymbol{O}_b(\zeta'' \wedge \Box\beta))$ because $SF_a(t)$ is *true*, and where $\zeta^* = \zeta \wedge \pi_t^o \wedge \theta_{a_t}^o$, which is equivalent to ζ; $\zeta' = \zeta \wedge \pi_t^o \wedge \theta_{b_t}^o$, where $\theta_{b_t}^o$ is $\exists x, y.\ (t = see(x, y) \wedge x = b \wedge Gold(y) \vee (t = see(x, y) \wedge x \neq b))$, which is equivalent to $Gold(r2)$; and $\zeta'' = \zeta \wedge \pi_t^o \wedge \neg\theta_{b_t}^o$. Thus, ζ' is equivalent to $\zeta \wedge Gold(r2)$ and ζ'' is equivalent to $\zeta \wedge \neg Gold(r2)$. Thus, a now knows that b knows whether Room 2 has the gold. Formally, the progressed theory can be shown to entail $\boldsymbol{K}_a(\boldsymbol{K}_b Gold(r2) \vee \boldsymbol{K}_b\neg Gold(r2))$, $\boldsymbol{K}_a\exists x\boldsymbol{K}_b Gold(x)$, and $\neg\exists x\boldsymbol{K}_a\boldsymbol{K}_b Gold(x)$.

Example 13: As a final example on progression, consider the sentence $\text{KB}_0 \wedge \boldsymbol{O}_a(\text{KB} \wedge \Box\beta \wedge \boldsymbol{O}_b(\text{KB} \wedge \Box\beta \wedge \boldsymbol{O}_a(\text{KB} \wedge \Box\beta)))$. Its progression wrt $mv(b, r2) \cdot see(b, r2)$ simplifies to:

$$\begin{aligned} & \boldsymbol{O}_a(\zeta \wedge \Box\beta \wedge \\ & \quad (Gold(r2) \supset \boldsymbol{O}_b(\zeta \wedge Gold(r2) \wedge \Box\beta \wedge \boldsymbol{O}_a(\zeta \wedge \Box\beta))) \wedge \\ & \quad (\neg Gold(r2) \supset \boldsymbol{O}_b(\zeta \wedge \neg Gold(r2) \wedge \Box\beta \wedge \boldsymbol{O}_a(\zeta \wedge \Box\beta)))) \end{aligned}$$

where ζ is from the previous example. The new knowledge base entails $\boldsymbol{K}_a\boldsymbol{K}_b\boldsymbol{K}_a At(b, r2)$ and $\boldsymbol{K}_a\boldsymbol{K}_b\neg\exists x\boldsymbol{K}_a Gold(x)$.

Conclusions

We obtained new *general* results on the progression of knowledge in multiagent systems that substantially extended previous work: we showed that given any action theory, knowledge bases at all depths can be progressed in a LR fashion after actions. For the future, we would like to consider *private* actions, and go beyond OCNF formulas.

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