Development of a Finite Element Method Mode Solver Application for Optical Waveguides

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ABSTRACT

Finite Element Method constitutes one of the most powerful numerical methods when there is need for approximate solutions of a problem. In this work an application with a user-friendly graphical interface, developed in Matlab, is presented which facilitates the calculation and graphical presentation of the propagation constants and modes of various types of optical waveguides using FEM. To check the validity of our simulations the results have been compared with those presented in the literature.

General Terms

Computing methodologies, Modeling and simulation.

Keywords

Finite Element Method, Variational Method, Propagation Modes.

1. INTRODUCTION

Optical Waveguides have been used for decades giving the ability to transfer the information through high speed networks and devices. In order to achieve better performance or adaptation to specific problems, waveguides with different characteristics and materials were introduced. The need to calculate some important characteristics, such as the number of modes and their corresponding propagation constants of the waveguides was necessary and both analytical and numerical methods have been proposed [2-3].

One of the most powerful analytical methods is the Finite Element Method (FEM) [4]. The main advantage of the method is the fact that it can be applied to any waveguide geometry. This flexibility gives us the ability to use FEM to any waveguide, without changing the course of the analysis, but just using the corresponding functions for every type of waveguide.

The main notion behind FEM is that the region is divided into many small elements and then analytical functions are applied to every element. The results are summed up to global matrixes and an eigenvalue matrix equation is produced. The final step is to solve the eigenvalue matrix equation and retrieve the propagation modes and the corresponding propagation constants [5]. Thus, instead of trying to solve a problem with infinite number of unknowns, we transformed it to a problem with finite unknowns for which a solution can be reached [4].

In this work we present a FEM mode solver for optical waveguides which is easy to configure through its Graphical User Interface (GUI). The application gives the ability to analyze rectangular 3D (ridge or rib) waveguides, find the propagation constants of the supported modes and plot the results.

In section 2 the selected FEM and the developed Matlab [8] application will be described. The simulation results are presented in section 3. Finally in section 4 we summarize our results.

2. THE APPLICATION

2.1 The selected FEM

In the Finite Element Method there are two main directions which can be followed: The Scalar FEM – SC FEM and the Fully Vectorial FEM – V FEM [5]. Both of them have their merits and their weaknesses. However, the former has been chosen in our implementation, because there are no spurious problems and the produced eigenvalue matrixes are sparse and symmetrical, a characteristic which makes the calculations more efficient and consequently the application faster. Furthermore, regarding the calculation of the characteristics of the elements, for simplicity reasons we used the Variational Approach compared to the Weighted Residual Approach [6]. More specifically, the methodology described by Katsunari Okamoto [1] has been followed as the theoretical basis for the application. In this methodology the Scalar Wave Equation (1) is solved using the Scalar FEM in combination with the Variational Approach.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + [k^2 n^2(x,y) - \beta^2]\phi(x,y) = 0 \tag{1}$$

Before a detailed description of the application, it is necessary to provide the steps which are followed and also provide the necessary definitions. The steps can be summarized as follows:

- The meshing of the area is performed in many small elements. An **element** consists of **nodes**. An element can be either a triangle with tree nodes (first order element) or six nodes (second order element), or a rectangle with four nodes (first order element) or eight nodes (second order element) and similarly any other shape which follows the above pattern (Figure 1). The meshing can be performed by a plethora of algorithms, such as Delaunay [7] or any other which meets our needs.

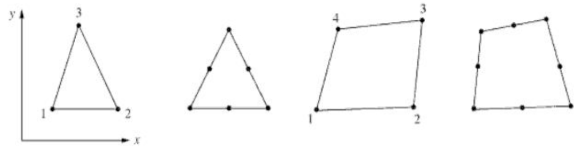

Figure 1. Different element shapes

- The **interpolation functions** are chosen and applied to the nodes of the elements. Usually, polynomial functions are used because they can be easily differentiated. Also, the degree of the polynomial is correlated with the order of the element used.
- The parameters of the elements are calculated and expressed in a matrix form. For the calculation of the parameters we used the Variational approach.
- The matrixes of the elements are combined in order to generate an eigenvalue matrix equation which describes the global system.
- The necessary boundary conditions are introduced in the system. The boundary conditions which are used are the Neumann and Dirichlet conditions [5].
- Finally, the eigenvalue matrix system is solved using a suitable method according to the problem at hand. The most common algorithms which can be used are the Jacobi's and Householder's methods [1]. By solving the equation, the propagation constants of the modes and the corresponding field distributions are produced.

2.2 The implementation

Crucial to our implementation was the need for accuracy in the results and at the same time efficiency and fast mathematical calculations. The complexity of the system is in a direct correlation with the number of the elements which are used for the meshing of the area. The higher the number of elements, the higher the complexity of the system which is going to be solved and consequently the slower the calculation of the results [10]. Matlab has been chosen as the development platform. The application consists of the following:

Waveguide creating engine

This module is creating the waveguide and its related parameters. One could choose between two different 3D waveguide types: Ridge (or Rib) and/or Rectangular. Also, the user has to setup the initial parameters of the waveguide which describe its characteristics, the optical pulse and the area of analysis. The parameters are summarized in Table 1.

Table 1. Waveguide parameters

Parameter	Description
n_c	Refractive index of the core
n_s	Substrate refractive index
n_a	Refractive index of air
Wavelength	The wavelength (λ) of the optical pulse
Core with	The width of the core at the central area of the waveguide
Core height	The maximum height of the core
Profile height	The height of the core at the side of the central core of the waveguide (*for the ridge waveguides only*)
X axis analysis length	The length (in μm) of the analysis area along the x axis
Area space north	The free space at the top part of the analysis area
Area space south	The free space at the bottom part of the analysis area
Elements at core x axis	The number of elements along the x axis at the central core of the waveguide
Elements at profile y axis	The number of elements along the y axis at the side core of the waveguide.
Percentage change x axis	The percentage (0.00 to 1.00) change of the side of the elements at the x axis.
Percentage change y axis	The percentage (0.00 to 1.00) change of the side of the elements at the y axis.

Meshing engine

This module is responsible for meshing the examining area with elements and calculating the appropriate properties for the meshed area. A variety of algorithms could be used for this purpose depending on the application. In our case a simple custom meshing algorithm has been used. According to the given parameters, points are created in the analysis area and every point is connected to its neighboring point in order to produce first order triangle elements. Figure 2 shows the representation of the nodes.

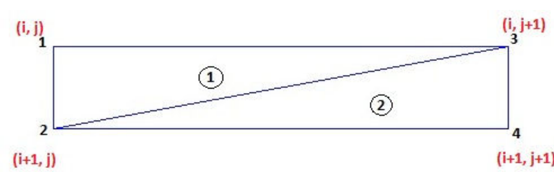

Figure 2. Example of node connection

The connection is performed starting from the upper left corner of the area to the bottom right corner moving vertically at each pass. Every node is examined from the top to bottom direction and two triangles are created. When we examine the node i, then the triangles (i,j) – (i+1,j) – (i, j+1) and (i+1,j) – (i+1, j+1) – (i, j+1) are created. Anticlockwise numbering is used for the nodes.

The meshing engine produces two arrays. The first array holds the node data while the second array the element data, where the node values and the refractive index of every element are stored. A similar approach has been used at [9]. Figure 3 shows an example mesh of a ridge waveguide. The outline of the waveguide is marked with a thicker stroke.

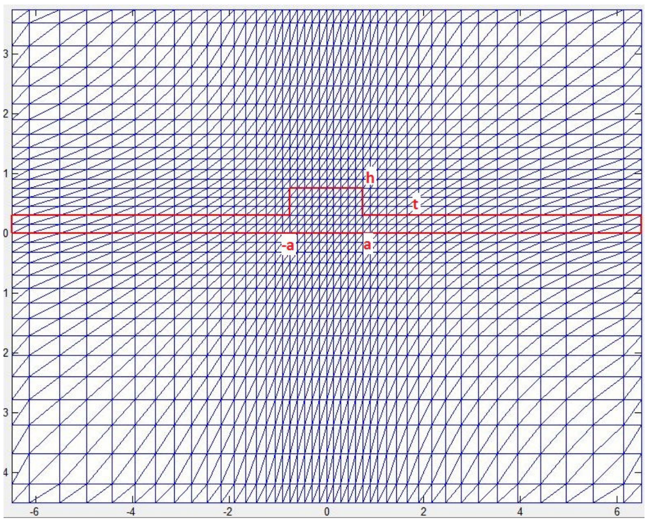

Figure 3. Area meshing

The meshing of the area is performed with different densities for the core and the cladding. The higher the accuracy we want to achieve the higher the number of elements that have to be used. However, as it has already been mentioned at the preceding section, the complexity of the calculations is increased when more elements are used. Although we want more accuracy and better results, it is not necessary to perform high meshing at all parts of the examining area, but mainly at the core of the waveguide. For that reason, the parameters «*percentage change x axis*» and «*percentage change y axis*» were introduced. The former indicates the step of the element side change along the x axis. This step is used for the elements that their nodes have $x > a$ and $x < -a$. The latter indicates the step of the element side change along the y axis. This step is used for the elements that their nodes have $y > h$ and $y < 0$. By applying this technique the meshing of the area has higher density at the core of the waveguide and lower density at the cladding. This is also shown at Figure 3, where the meshing is gradually decreased for the areas that are away from the waveguide core. In that way, the computations are kept in a balance and both accuracy and low cost performance are achieved. Furthermore, presuming that the waveguides are symmetrical, there is no need for meshing all the area, but only half of it and then mirror the mesh a process that speeds up the meshing time.

FEM solving engine

The solving engine is responsible for the application of FEM functions, the creation of the eigenvalue matrix equation and the solution of it. After the meshing has been performed, then the analytical functions are applied to each element. The results are added to global matrixes and the eigenvalue matrix equation [10] is created (2).

$$\left([A] - \lambda^2 [B]\right)\{\phi\} = 0 \qquad (2)$$

The global matrixes A and B in equation (2) are created from the analytical function at the elements. The λ is the unknown eigenvalue which we derive from the solution of the system, in our case the propagation constants, and ϕ is the distribution of the field for the corresponding eigenvalue. The number of propagation constants shows the number of supported modes for the waveguide. The solutions of the eigenvalue equation are not all acceptable. We accept only those which meet the necessary criteria. More specifically, the n_{eff} (the effective index) has to satisfy the following inequality:

$$n_s < n_{eff} < n_c \qquad (3)$$

where n_{eff} is defined as:

$$n_{eff} = \sqrt{b(n_c^2 - n_s^2) + n_s^2} \qquad (4)$$

where b is the propagation constant which has been calculated from the solution of equation (2). After deriving the accepted propagation constants, then the corresponding field values are retrieved.

Graphical User Interface (GUI)

The solutions of the system and the graphical field distribution are presented to the GUI. The main purpose of the GUI design was to achieve an intuitive environment where a non-expert user could employ the application by providing only the necessary parameters and without any knowledge for command line configuration and this is the main advantage of our implementation. Anyone, without having to deal with a complex configuration, can use the application and generate results fast providing an ideal tool for students who want a free and optical-oriented FEM mode solver in order to experiment with optical waveguide propagation characteristics. Figure 4 presents a screenshot of the GUI with a meshed Ridge waveguide. Moreover, the user can select from the options menu to show the number label of the elements and their corresponding node number labels. Finally, using any of the solutions generated one can produce the corresponding contour diagram, Figure 5.

File import/export handler

The import/export handler is responsible for the file manipulation of the application. The user has the ability to store the setup of the waveguide and the results to a file. Also, the user can import the exported file to any other application without having the need to make the computations from scratch. The results and the setup are saved in a Matlab .mat format.

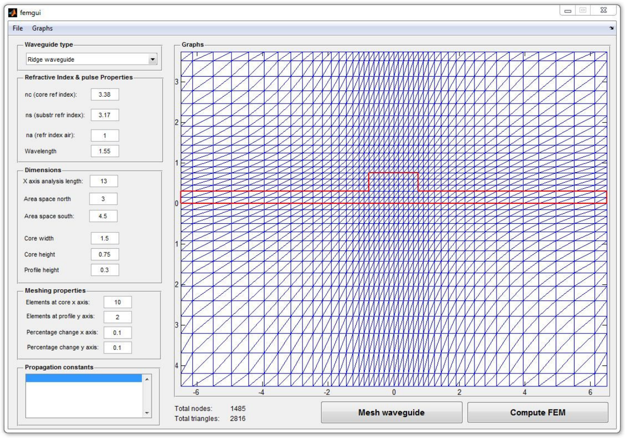

Figure 4. FEM mode solver GUI

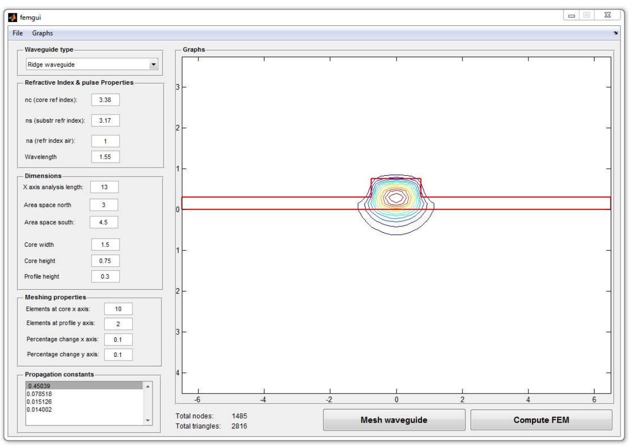

Figure 5. Contour diagram for selected mode

3. SIMULATION RESULTS AND CONCLUSION

3.1 Simulation results

In order to verify that the proposed implementation produces results that are comparable with those presented in the literature, a series of simulations have been run.

Simulation No1

For the first simulation the setup was selected from Katsunari Okamoto [1] and some slight modifications have been made in order to be adapted to our implementation. The setup is summarized in Table 2 and 3.

Table 2. Ridge waveguide parameters

Parameter	Value
n_c	3.38
n_s	3.17
n_a	1
Core width	1.5μm
Core height	0.75μm
Core profile height	0.3μm

Table 3. Area parameters

Parameter	Value
Area width	13μm
Area north space	3μm
Area south space	4.5μm
Elements at core x axis	10
Elements at core y axis	2
Percentage change x axis	0.1
Percentage change y axis	0.1

The wavelength used is λ = 1.55μm. The meshing generated 1485 nodes and 2816 elements. The results are presented at table 4.

Table 4. Results of simulation No1

Propagation constants	N_{effs}
0.45039	3.2663
0.078518	3.187
0.015126	3.1733
0.014002	3.1733

The results in table 4 indicate that the described waveguide can support 4 different modes. However, examining more carefully the results, we could eliminate the last three values as unacceptable. Plotting some of the results in contour diagrams (Figures 6, and 7) it is apparent that for the mode with b = 0.0152126 the propagation takes place mostly at the cladding while for the mode with b = 0.45039, the propagation is confined in the core. As a result, we conclude that the aforementioned modes (with b = 0.078518, 0.015126, 0.014002) can not be propagated. The reason for these false results can be attributed to the solving method used for the eigenvalue matrix equation or the errors produced from the rounding calculations. Our results are in good agreement with the results presented in [1].

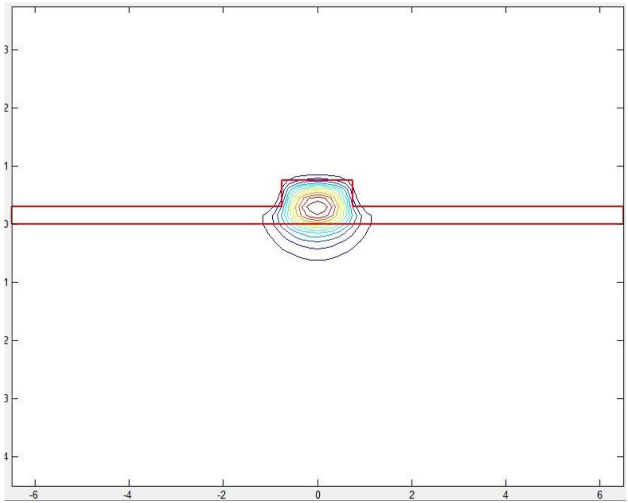

Figure 6. Contour diagram for mode with b = 0.45039

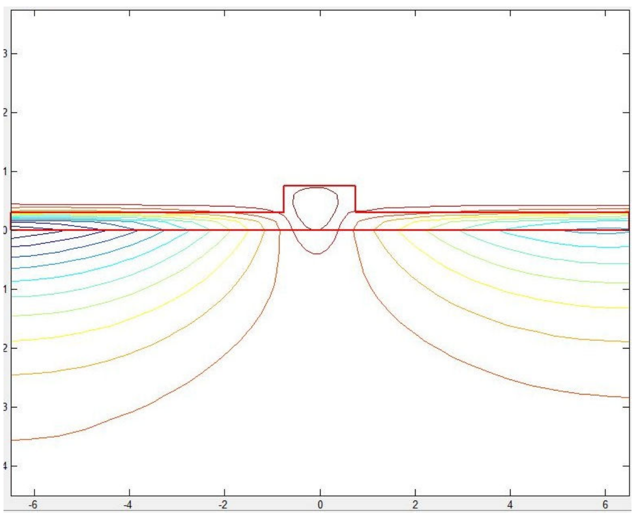

Figure 7. Contour diagram for mode with b = 0.015126

Simulation No2

For the second simulation a Ridge waveguide has been used and its parameters are presented in Table 5 and 6.

Table 5. Ridge waveguide parameters

Parameter	Value
n_c	3.48
n_s	1.48
n_a	1
Core width	1μm
Core height	1μm
Core profile height	0.6μm

Table 6. Area parameters

Parameter	Value
Area width	15μm
Area north space	3μm
Area south space	2μm
Elements at core x axis	10
Elements at core y axis	4
Percentage change x axis	0.1
Percentage change y axis	0.1

The simulation wavelength is $\lambda = 1.55\mu m$. The meshing generated 1595 nodes and 3024 elements. The results are presented in Table 7.

Table 7. Results of simulation No2

Propagation constants	Neffs
0.87043	3.2902
0.8547	3.2663
0.85461	3.2662
0.84208	3.2624

Similarly to the previous example, plotting the corresponding contour diagrams, it can be seen that the mode with b = 0.87043 is the only mode which can propagate at the core of the waveguide, Figure 8.

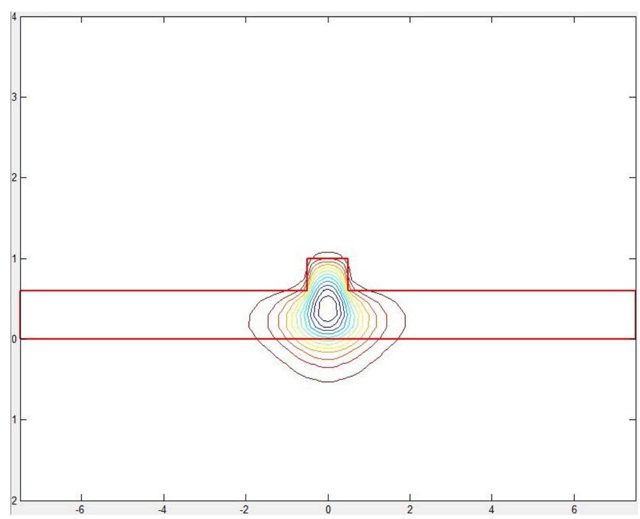

Figure 8. Contour diagram for mode with b = 0.87043

Although the accuracy of the results is of vital importance, another factor which has to be taken into consideration is the performance of the application. For that reason a benchmark test has been conducted. The computer used for the simulations had an Intel Core i5 M450, 2.4GHz CPU and 4.00 GB RAM with the Matlab version 7.12.0 (R2011a) - 64bit.

To run the benchmark five different waveguide setups have been selected. All of them had the same basic settings. The only different parameter is the number of elements, i.e., the different percentage changes of the elements at the axis, and consequently the total number of nodes at the examining area. The setups are presented in the Table 8.

Table 8. Waveguide setups

Waveguide setup	Number of elements	Number of nodes
Setup 1	1120	609
Setup 2	1320	713
Setup 3	1700	910
Setup 4	2816	1485
Setup 5	4032	2109

The results of the benchmark are displayed in the Figure 9. It can be easily observed that as we increase the number of the elements more time is needed for the calculations. However, the most costly operation in the calculation process is the solution of the eigenvalue matrix equation. From the diagram it is obvious that the time needed for that operation increases exponentially. The calculation time difference between the Setup 3 and Setup 4 can be marked as the most significant. The Setup 4 has 1116 more elements compared to the Setup 3 and the calculation time is 90.45 seconds more. A possible improvement in the calculation process could be achieved by analyzing only half of the examining area. Our presumption is based on the fact that the Ridge waveguides are symmetrical along the y axis [11].

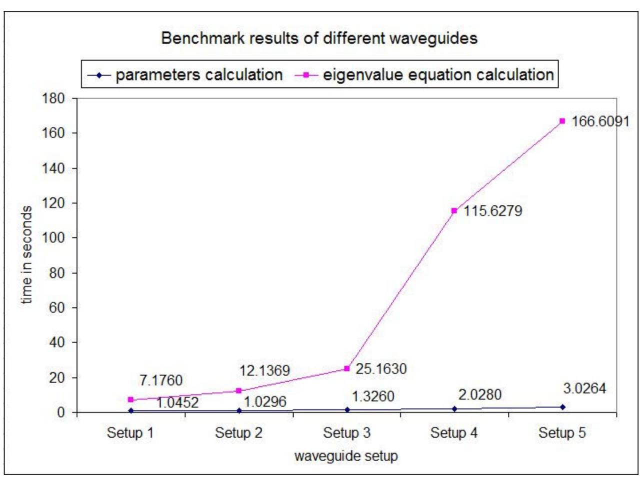

Figure 9. Benchmark results

Furthermore, a better eigenvalue equation solving algorithm could be used, which would handle the sparsity of the matrixes in a more efficient way. Examining the curve of the calculation parameters, it can be concluded that this process is fast and is insignificantly influenced by the number of elements in comparison with the solution process of the eigenvalue matrix equation. In addition, the meshing times for the corresponding Setups are presented in Figure 10 showing an insignificant increase with the increase in the elements number.

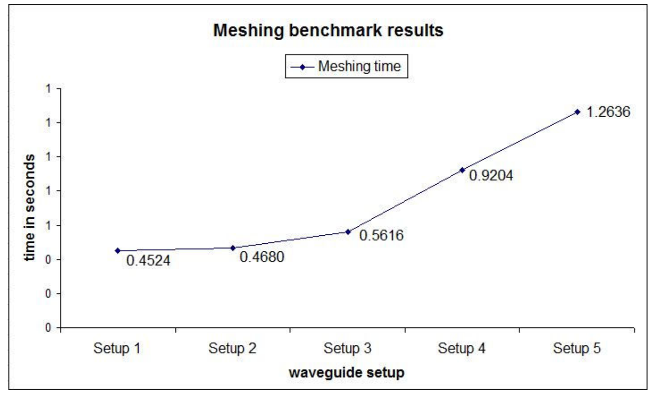

Figure 10. Meshing benchmark results

4. CONCLUSIONS & FUTURE WORK

The presented implementation constitutes an accurate and easy to use application. With a complete GUI, the experimentation and the simulation of a Ridge (or Rib) or Rectangular waveguide can be performed without having to deal with complex configurations. Our application can be used by students or researchers investigating the characteristics of a waveguide before fabrication. The accuracy of our application was successfully proven comparing our results with those in the literature.

Some issues that we have already scheduled to deal with are the introduction of a more sophisticated meshing engine and the performance improvement of the algorithm for the solution of the eigenvalue matrix equation. Finally, the construction of a web platform and a complete infrastructure where the application would be available publicly through a web interface are also at our plans.

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