Rational Elimination of DL-Lite TBox Axioms

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Abstract. An essential task in managing description logic (DL) ontologies is the elimination of problematic axioms. Such elimination is formalised as the operation of contraction in belief change. In this paper, we investigate contraction over DL-Lite$_\mathcal{R}$ TBoxes. In belief change, a well known approach for defining contraction is via epistemic entrenchments which are preference orderings over formulas. The approach however is not applicable in DL-Lite$_\mathcal{R}$ as classic belief change assumes an underlying logic that is different from DL-Lite$_\mathcal{R}$. Thus we reformulate the epistemic entrenchment approach to make it applicable to DL-Lite$_\mathcal{R}$. We then provide instantiation for the reformulated approach.

1 Introduction

Description logic (DL) [2] ontologies are subject to frequent changes. For instance, outdated or incorrect axioms have to be eliminated from the ontologies and newly formed axioms have to be incorporated into the ontologies. Therefore a mandatory task for managing DL ontologies is to deal with such changes. In the field of belief change, extensive work has been done on formalising various kinds of changes over knowledge bases. In particular, the elimination of old knowledge is called *contraction* and incorporation of new knowledge is called *revision*. To handle changes over DL ontologies it makes sense to take advantage of the many existing techniques in belief change. Consequently, many have investigated contraction and revision under DLs [7,8,18,23,22,19,20].

The dominant approach in belief change is the so called AGM framework [1,9] which assumes an underlying logic that includes propositional logic. In the framework, the knowledge base to which changes are made is called a *belief set* which is a logically closed set of formulas. An AGM contraction function $\dot{-}$ takes as input a belief set K and a formula ϕ and returns another belief set $K \dot{-} \phi$ such that ϕ is not entailed. The *epistemic entrenchment contraction* function [9,10] is one such contraction function. The basic idea for epistemic entrenchment contraction is to rank all the formulas by their informational value. The higher up in the ranking the more informational value a formula holds. The formulas returned by the contraction function are then determined by comparing rankings of the related formulas with the intuition that if some formulas have to be removed then remove the ones with the least informational value whenever possible.

In this paper, we will define and instantiate contraction functions under DLs. We focus on DL-Lite$_\mathcal{R}$ which is the main language of the DL-Lite family [5]. DL-Lite underlies the OWL 2 QL profile of OWL 2 and gains its popularity through efficient query answering. The contraction function to be defined is for logically closed DL-Lite$_\mathcal{R}$

TBoxes. Inspired by epistemic entrenchment contraction function, we first rank all the DL-Lite$_\mathcal{R}$ TBox axioms such that the higher up in the ranking the more preferred an axiom is. As in epistemic entrenchment contraction function, we then need a mechanism for comparing rankings of related axioms so as to determining the outcomes of the contraction function. However, due to the assumption of AGM framework on a logic that subsumes propositional logic, the mechanism used in epistemic entrenchment contraction function are not applicable under DL-Lite$_\mathcal{R}$. An obvious obstacle is that DL-Lite$_\mathcal{R}$ does not allow disjunction of axioms whereas the comparison of rankings between disjunction of formulas with the formula to be contracted is essential to the mechanism for epistemic entrenchment contraction.

Our solution to the inapplicability which is also our main technical contribution is to reformulate the mechanism so as to make it independent of the underlying logic. That means although we use it for DL-Lite$_\mathcal{R}$ it has the potential to be applied for DLs in general. We first propose the notion of *critical axioms*. Roughly speaking, the critical axioms of ψ with respect to ϕ are those axioms that are critical in deciding whether to retain or to remove ψ when contracting by ϕ. The decision is then made by comparing the critical axioms with ϕ. The rule is that if any of the critical axioms are strictly more preferred than ϕ then ψ will be retained otherwise it is removed. We identify a set of rationality postulates satisfied by the contraction functions defined with the reformulated mechanism. Among other properties, it is clear from the postulates, functions thus defined are always successful in accomplishing the required elimination of axioms. An algorithm that instantiates the functions is provided at the end.

2 DL-Lite

In this section we introduce the family of DL-Lite languages. The core of the family is DL-Lite$_{core}$ which has the following syntax:

$$B \rightarrow A \mid \exists R \qquad C \rightarrow B \mid \neg B \qquad R \rightarrow P \mid P^- \qquad E \rightarrow R \mid \neg R$$

where A denotes an *atomic concept*, P an *atomic role*, P^- the *inverse* of the atomic role P. B denotes a *basic concept* which can be either an atomic concept or an unqualified existential quantification on basic role. C denotes a *general concept* which can be either a basic concept or its negation. E denotes a *general role* which can be either an atomic role or its negation. We also include $\bot$ denoting the empty set and $\top$ denoting the whole domain. We use $\mathcal{B}$ to represent the universal set of basic concepts and $\mathcal{R}$ as the universal set of atomic roles and their inverses. For an inverse role $R = P^-$, we write R^- meaning P for the convenience of presentation. In this paper, we assume $\mathcal{B}$ and $\mathcal{R}$ are finite.

A DL-Lite$_{core}$ knowledge base consists of a TBox and an ABox. A TBox is a finite set of *concept inclusion axioms* of the form $B \sqsubseteq C$. That is only basic concepts can appear on the left-hand side of a concept inclusion. An ABox is a finite set of *assertions* of the form $A(a)$ or $P(a,b)$.

There are two major extensions of DL-Lite$_{core}$, namely DL-Lite$_\mathcal{R}$ and DL-Lite$_\mathcal{F}$. DL-Lite$_\mathcal{R}$ extends DL-Lite$_{core}$ with *role inclusion axioms* of the form $R \sqsubseteq E$. That is only basic roles can appear on the left-hand side of a role inclusion. DL-Lite$_\mathcal{F}$ extends

DL-Lite$_{core}$ with assertions of the form (funct R) which specifies functionality on basic roles.

The semantics of a DL-Lite language is given in terms of interpretations. An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty domain $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ that assigns to each atomic concept A a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$, and to each atomic role P a binary relation $P^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$, and to each individual name a an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$. The interpretation function is extended to general concept, general roles, and special symbols as follows: $\bot^{\mathcal{I}} = \emptyset$, $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$, $(P^-)^{\mathcal{I}} = \{(o_2, o_1) \mid (o_1, o_2) \in P^{\mathcal{I}}\}$, $(\exists R)^{\mathcal{I}} = \{o \mid \exists o'.(o, o') \in R^{\mathcal{I}}\}$, $(\neg B)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus B^{\mathcal{I}}$, and $(\neg R)^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus R^{\mathcal{I}}$. An interpretation $\mathcal{I}$ satisfies a concept inclusion $B \sqsubseteq C$ if $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$, a role inclusion $R \sqsubseteq E$ if $R^{\mathcal{I}} \subseteq E^{\mathcal{I}}$, a concept assertion $A(a)$ if $a^{\mathcal{I}} \in A^{\mathcal{I}}$, a role assertion $P(a, b)$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}}$, and a functionality assertion (funct R) if $(o, o_1) \in R^{\mathcal{I}}$ and $(o, o_1) \in R^{\mathcal{I}}$ implies $o_1 = o_2$. $\mathcal{I}$ satisfies a TBox $\mathcal{T}$ (or ABox $\mathcal{A}$) if $\mathcal{I}$ satisfies each axiom in $\mathcal{T}$ (resp., each assertion in $\mathcal{A}$). $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ (or a TBox axiom ϕ) denoted as $\mathcal{I} \models \mathcal{T}$ (resp., $\mathcal{I} \models \phi$) if it satisfies $\mathcal{T}$ (resp., ϕ). A TBox or an axiom is *consistent* if it has at least one model. A TBox $\mathcal{T}$ logically implies an axiom ϕ, written $\mathcal{T} \vdash \phi$, if all models of $\mathcal{T}$ are also models of ϕ. Two TBox axioms ϕ and ψ are logically equivalent, written $\phi \equiv \psi$, if they have identical set of models. We use $\vdash \phi$ to denote that ϕ is a tautology such as $A \sqsubseteq A$. $\top$ standing alone also denotes a tautology. We use $\{\top \sqsubseteq \bot\}$ to denote the (unique) inconsistent TBox.

The closure of a TBox $\mathcal{T}$, denoted as $cl(\mathcal{T})$, is the set of all TBox axioms ϕ such that $\mathcal{T} \models \phi$. The closure of a DL-Lite TBox is finite, in fact, the size of the closure of a TBox $\mathcal{T}$ is quadratic with respect to the size of $\mathcal{T}$ [17]. In the upcoming sections all TBoxes are assumed to be closed.

In general, TBox axioms are logically connected meaning that they imply and are implied by other axioms. One exception is the functionality assertions. For DL-Lite$_{\mathcal{F}}$, if ABox is not considered and the TBox is coherent then a functionality assertion do not imply and is not implied by any other axioms, thus its removal and addition is nothing but set operations. For this reason, we only consider DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$ TBoxes. Since DL-Lite$_{core}$ is a subset of DL-Lite$_{\mathcal{R}}$, contraction functions for DL-Lite$_{core}$ can be obtained identically as for DL-Lite$_{\mathcal{R}}$ only more simpler. We present the results for DL-Lite$_{\mathcal{R}}$ only. In the upcoming sections all TBoxes are assumed to be DL-Lite$_{\mathcal{R}}$ ones which are denoted by lower case Greek letters ($\phi, \psi, \ldots$). Also we denote the universal set of TBox axioms for DL-Lite$_{\mathcal{R}}$ as $\mathcal{L}_{\mathcal{R}}$. Given a set of TBox axioms $\phi_1, \ldots, \phi_n$, their conjunction is denoted as $\phi_1 \wedge \cdots \wedge \phi_n$. As expected, an interpretation $\mathcal{I}$ is a model of $\phi_1 \wedge \cdots \wedge \phi_n$ if it satisfies all the conjuncts. We assume $\mathcal{L}_{\mathcal{R}}$ contains any conjunction of TBox axioms.

3 Entrenchment-Based Contraction for DL-Lite$_{\mathcal{R}}$

In this section, we deal with the problem of eliminating axioms from DL-Lite$_{\mathcal{R}}$ TBoxes. Our strategy is to define a contraction function $\dot{-}$ that takes as input a logically closed TBox $\mathcal{T}$ and an axiom ϕ and returns as output a TBox $\mathcal{T} \dot{-} \phi$ such that ϕ is not entailed.

In order to eliminate an axiom ϕ from a TBox $\mathcal{T}$, we need to remove at least one axiom from each subset S of $\mathcal{T}$ such that $S \vdash \phi$. For example, if $\mathcal{T}$ consists of $A \sqsubseteq$

$B, B \sqsubseteq C$, and $A \sqsubseteq C$ and we want to remove $A \sqsubseteq C$ then since the set $\{A \sqsubseteq B, B \sqsubseteq C\}$ implies $A \sqsubseteq C$, either $A \sqsubseteq B$ or $B \sqsubseteq C$ must be removed from $\mathcal{T}$. Obviously, we need some preference information over the axioms to guide us in deciding which one of $A \sqsubseteq B$ and $B \sqsubseteq C$ to remove.

Thus we start with specifying a preference ordering for all the TBox axioms. Following the AGM tradition we call the ordering an *epistemic entrenchment*. Formally, an epistemic entrenchment is a binary relation over $\mathcal{L}_\mathcal{R}$ and is associated with a TBox. Given an epistemic entrenchment $\leq$, we say an axiom ϕ is equally or more entrenched than an axiom ψ iff $\psi \leq \phi$. The strict relation $\psi < \phi$ is defined as $\psi \leq \phi$ and $\phi \not\leq \psi$ and the equivalent relation $\psi \simeq \phi$ is defined as $\psi \leq \phi$ and $\phi \leq \psi$. We say ϕ is strictly more entrenched than ψ iff $\psi < \phi$, and ϕ is equally entrenched to ψ iff $\psi \simeq \phi$. The basic idea is that the more entrenched an axiom the more important or more preferred it is thus in deciding which axioms to remove it is intuitive to remove the less entrenched ones whenever possible.

Not all preference ordering are appropriate in this context. We require an epistemic entrenchment $\leq$ to satisfy the following conditions:

$(DE1)$ If $\phi \leq \psi$ and $\psi \leq \delta$ then $\phi \leq \delta$ (Transitivity)
$(DE2)$ If $\phi \vdash \psi$ then $\phi \leq \psi$ (Dominance)
$(DE3)$ $\phi \leq \phi \wedge \psi$ or $\psi \leq \phi \wedge \psi$ (Conjunctiveness)
$(DE4)$ If $\mathcal{T}$ is consistent then $\phi \notin \mathcal{T}$ iff $\phi \leq \psi$ for every ψ (Minimality)
$(DE5)$ If $\phi \leq \psi$ for every ϕ then $\vdash \psi$ (Maximality)

Thus an epistemic entrenchment is transitive $(DE1)$; logically weaker axioms are equally or more entrenched than logically stronger ones $(DE2)$; conjunctions of axioms are equally or more entrenched than one of their conjuncts $(DE3)$; axioms not in the associated TBox are least entrenched $(DE4)$; and tautologies are most entrenched $(DE5)$. $(DE1)$–$(DE5)$ are obtained by recasting conditions $(EE1)$–$(EE5)$ of [10] to DL-Lite$_\mathcal{R}$ thus they carry the same intuitions as $(EE1)$–$(EE5)$ which are widely accepted in belief revision community. Among other properties, it can be derived from $(DE1)$–$(DE5)$ that an epistemic entrenchment is connected, a conjunction of axioms is equally entrenched to its least entrenched conjunct, and logically equivalent axioms are equally entrenched.

Lemma 1 *Let $\leq$ be an entrenchment.*

1. *Either $\phi \leq \psi$ or $\psi \leq \phi$ (Connectivity)*
2. *If $\psi \leq \phi$ then $\phi \wedge \psi \simeq \psi$*
3. *If $\phi \equiv \psi$ then $\phi \simeq \psi$*

for all $\phi, \psi \in \mathcal{L}_\mathcal{R}$.

Proof. 1. For any $\phi, \psi \in \mathcal{L}_\mathcal{R}$ we have by Conjunctiveness that either $\phi \leq \phi \wedge \psi$ or $\psi \leq \phi \wedge \psi$. If $\phi \leq \phi \wedge \psi$ we get from Dominance $\phi \wedge \psi \leq \psi$ and from Transitivity $\phi \leq \psi$. If $\psi \leq \phi \wedge \psi$ we get from Dominance $\phi \wedge \psi \leq \phi$ and from Transitivity $\psi \leq \phi$.

2. Suppose $\psi \leq \phi$. We have by Conjunctiveness that either $\phi \leq \phi \wedge \psi$ or $\psi \leq \phi \wedge \psi$. If $\phi \leq \phi \wedge \psi$ then we have by Transitivity that $\psi \leq \phi \wedge \psi$. Thus in either case we have $\psi \leq \phi \wedge \psi$. Since we also have $\phi \wedge \psi \leq \psi$ follows from Dominance, it must be that $\phi \wedge \psi \simeq \psi$.

3. Let $\phi \equiv \psi$. Then it follows from Dominance that $\phi \leq \psi$ and $\psi \leq \phi$ which give us $\phi \simeq \psi$.

$\square$

As a direct consequence of Lemma 1 (i.e. Part 2) the ranking of a conjunction in an epistemic entrenchment is uniquely determined by its conjuncts. This means we only need to specify preferences between all the single axioms.

Having an epistemic entrenchment associated with each TBox is only the first step. It remains to devise mechanism to make use of the preference information in deciding the axioms to remove and to retain during a contraction. Although AGM epistemic entrenchment contraction provides such a mechanism, the contraction is defined while assuming an underlying logic that subsumes propositional logic. As most DLs including DL-Lite$_R$ do not subsume propositional logic, the direct transition of the mechanism to contraction under DLs is not possible. An obvious obstacle in such transition is that the mechanism refers to disjunctions of formulas and DLs do not allow disjunctions of axioms. We deal with this matter by reformulating the mechanism such that logic dependent terms such as disjunctions do not play a part.

Let's consider the contraction of an axiom ϕ from a TBox $\mathcal{T}$ with an associated epistemic entrenchment $\leq$. Since the purpose is to eliminate ϕ, we have to make sure the axioms retained do not entail ϕ. For each axiom ψ of $\mathcal{T}$ if it has nothing to do with the entailment of ϕ that is there is no set of axioms S such that $S \nvdash \phi$ and $\{\psi\} \cup S \vdash \phi$ then we can be sure it is safe to retain ψ. In fact ψ must be retained if we respect the *principle of minimal change* [1,11] which requires to retain as much as possible axioms of $\mathcal{T}$. If ψ does play a part in the entailment of ϕ that is there is an S such that $S \nvdash \phi$ and $\{\psi\} \cup S \vdash \phi$ then it is safe to retain ψ only if for each such set S at least one axiom of it is not retained. Thus we have to decide whether to favour ψ or axioms of S. We propose the following notion of *critical axioms* that will aid such decisions. Basically, the critical axioms of ψ with respect to ϕ are those axioms whose rankings in $\leq$ are critical in deciding whether to retain ψ in the contraction of ϕ.

Definition 1 *Let ϕ, ψ be DL-Lite$_\mathcal{R}$ TBox axioms. The set of critical axioms of ψ with respect to ϕ, denoted as $\mathcal{C}^\phi(\psi)$, is defined as follows:*
If there is no set of axioms S such that $\{\psi\} \cup S \vdash \phi$ and $S \nvdash \phi$ then $\mathcal{C}^\phi(\psi) = \{\top\}$. Otherwise, $\sigma \in \mathcal{C}^\phi(\psi)$ iff

1. *For each set of axioms S such that $\{\psi\} \cup S \vdash \phi$ and $S \nvdash \phi$ we have $\{\sigma\} \cup S \vdash \phi$,*
2. *There is no axiom γ which satisfies condition 1 and is such that $\gamma \nvdash \sigma$ and $\sigma \vdash \gamma$.*

If ψ has nothing to do with the entailment of ϕ, then intuitively no axiom is critical to ψ in contracting ϕ, however, for technical reason we take that the only critical axiom in this case is the tautology. Besides the limiting cases, the critical axioms are logically the weakest axioms that can replace ψ in entailing ϕ with the help of other axioms (i.e., the set of axioms S).

The following properties for critical axioms follow immediately from Definition 1.

Lemma 2 *Let ϕ, ψ be DL-Lite$_\mathcal{R}$ TBox axioms. Then*

1. *If $\psi \vdash \phi$ then $\mathcal{C}^\phi(\psi) = \{\phi\}$*

2. *If $\phi \equiv \psi$ then $\mathcal{C}^\psi(\delta) = \mathcal{C}^\phi(\delta)$ and $\mathcal{C}^\delta(\psi) = \mathcal{C}^\delta(\phi)$ for any $\delta \in \mathcal{L}_\mathcal{R}$*

3. *If $\sigma \in \mathcal{C}^\phi(\psi)$ then $\mathcal{C}^\phi(\sigma) \subseteq \mathcal{C}^\phi(\psi)$*

Proof. 1. Since $\psi \vdash \phi$ we have $\emptyset \nvdash \phi$ and $\{\psi\} \cup \emptyset \vdash \phi$. Moreover, ϕ is the weakest axiom such that $\{\phi\} \cup \emptyset \vdash \phi$ thus it is the only axiom qualified as an element of $\mathcal{C}^\phi(\psi)$.

2. Since the definition of critical axioms only concerns the logical contents, logical equivalent axioms always produce the identical set of critical axioms.

3. Suppose $\sigma \in \mathcal{C}^\phi(\psi)$ then for all S such that $\{\psi\} \cup S \vdash \phi$ and $S \nvdash \phi$, $\{\sigma\} \cup S \vdash \phi$. Then by the definition of critical axioms, for any $\sigma' \in \mathcal{C}^\phi(\sigma)$ we have $\{\sigma'\} \cup S \vdash \phi$ and σ' is logically the weakest axiom satisfying the condition. Clearly, σ' fulfils all criteria for being an element of $\mathcal{C}^\phi(\psi)$.

$\square$

Now let's demonstrate why the critical set of axioms are so critical in deciding the axioms to retain in a contraction. Recall that the retainment of ψ in contracting ϕ has a lot to do with the set S such that $S \nvdash \phi$ and $\{\psi\} \cup S \vdash \phi$. We begin with a lemma showing how rankings of critical axioms of ψ with respect to ϕ affect the rankings of axioms in S.

Lemma 3 *Let ϕ, ψ be DL-Lite$_\mathcal{R}$ TBox axioms and $\leq$ an epistemic entrenchment. If $\sigma \in \mathcal{C}^\phi(\psi)$ and $\phi < \sigma$ then for any set of axioms S such that $\{\psi\} \cup S \vdash \phi$ and $S \nvdash \phi$ there is $\gamma \in S$ such that $\gamma \leq \phi$.*

Proof. Suppose $\sigma \in \mathcal{C}^\phi(\psi)$ and $\phi < \sigma$. The definition of critical axioms implies that for any set of axioms S such that $\{\psi\} \cup S \vdash \phi$ and $S \nvdash \phi$, $\{\sigma\} \cup S \vdash \phi$. Suppose without loss of generality that $S = \{\gamma_1, \ldots, \gamma_n\}$, thus $\sigma \wedge \gamma_1 \wedge \cdots \wedge \gamma_n \vdash \phi$. It then follows from Dominance that $\sigma \wedge \gamma_1 \wedge \cdots \wedge \gamma_n \leq \phi$. Assumes $\phi < \gamma_i$ for $1 \leq i \leq n$. Then it follows from Conjunctiveness that $\phi < \sigma \wedge \gamma_1 \wedge \cdots \wedge \gamma_n$ which implies by Transitivity $\phi < \phi$ a contradiction! Thus there is $\gamma \in S$ such that $\gamma \leq \phi$.

$\square$

According to Lemma 3, if any critical axiom σ of ψ with respect to ϕ is strictly more entrenched than ϕ then for any set of axioms S which does not entail ϕ but does so when united with ψ, there is at least one axiom of S that is not strictly more entrenched than ϕ. This means ψ together with all axioms strictly more entrenched than ϕ do not entail ϕ. It will be clear that in contracting ϕ if we only retain such ψ's then it is guaranteed ϕ will no longer be entailed. This leads to the following definition of *entrenchment-based contraction* function.

Definition 2 *Let $\mathcal{T}$ be a DL-Lite$_\mathcal{R}$ TBox with an associated epistemic entrenchment $\leq$. A function $\dot{-}$ is an entrenchment-based contraction function for $\mathcal{T}$ iff*

$$\mathcal{T} \dot{-} \phi = \begin{cases} \mathcal{T} \cap \{\psi \mid \text{there is } \sigma \in \mathcal{C}^\phi(\psi) \text{ such that } \phi < \sigma\} & \text{if } \phi \in \mathcal{T} \text{ and } \nvdash \phi \\ \mathcal{T} & \text{otherwise} \end{cases}$$

for all DL-Lite$_\mathcal{R}$ TBox axioms ϕ.

By Definition 2, in the contraction of $\mathcal{T}$ by ϕ, if ψ is in $\mathcal{T}$, then the existence of a critical axiom of ψ (w.r.t. ϕ) being strictly more entrenched than ϕ is a sufficient condition for retaining ψ. In the two limiting cases that ϕ is a tautology or it is not in $\mathcal{T}$, the original TBox $\mathcal{T}$ is returned meaning that all axioms are retained. For convenience we refer to ϕ as the contracting axiom, $\mathcal{T}$ the original TBox, and $\mathcal{T} \dot{-} \phi$ the resulting TBox and we say the function $\dot{-}$ is determined by the epistemic entrenchment $\leq$. To show that entrenchment-based contraction behaves properly and most importantly is always successful in eliminating the contracting axioms we provide the following representation theorem.

Theorem 1 *Let $\mathcal{T}$ be a DL-Lite$_\mathcal{R}$ TBox and $\dot{-}$ an entrenchment-based contraction function for $\mathcal{T}$ then it satisfies the following postulates:*

$(T \dot{-} 1)$ $\mathcal{T} \dot{-} \phi = cl(\mathcal{T} \dot{-} \phi)$
$(T \dot{-} 2)$ $\mathcal{T} \dot{-} \phi \subseteq \mathcal{T}$
$(T \dot{-} 3)$ *If $\phi \notin \mathcal{T}$, then $\mathcal{T} \dot{-} \phi = \mathcal{T}$*
$(T \dot{-} 4)$ *If $\nvdash \phi$, then $\phi \notin \mathcal{T} \dot{-} \phi$*
$(T \dot{-} cr)$ *If $\psi \in \mathcal{T}$ and there is $\sigma \in \mathcal{C}^\phi(\psi)$ such that $\sigma \in \mathcal{T} \dot{-} \phi$ then $\psi \in \mathcal{T} \dot{-} \phi$*
$(T \dot{-} 6)$ *If $\phi \equiv \psi$, then $\mathcal{T} \dot{-} \phi = \mathcal{T} \dot{-} \psi$*

Proof. Suppose $\dot{-}$ is an entrenchment-based contraction function for the DL-Lite$_\mathcal{R}$ TBox $\mathcal{T}$ with an associated epistemic entrenchment $\leq$. We need to show $\dot{-}$ satisfies $(T \dot{-} 1)$–$(T \dot{-} 4)$, $(T \dot{-} cr)$, and $(T \dot{-} 6)$. Satisfaction of $(T \dot{-} 2)$ and $(T \dot{-} 3)$ follows immediately from Definition 2.

$(T \dot{-} 1)$: If $\vdash \phi$ or $\phi \notin \mathcal{T}$ then it follows from Definition 2 that $\mathcal{T} \dot{-} \phi = \mathcal{T}$. Since $\mathcal{T} = cl(\mathcal{T})$, we have $\mathcal{T} \dot{-} \phi = cl(\mathcal{T} \dot{-} \phi)$. For the principal case, let $\nvdash \phi$ and $\phi \in \mathcal{T}$. Suppose $\psi \in cl(\mathcal{T} \dot{-} \phi)$, we need to show $\psi \in \mathcal{T} \dot{-} \phi$. According to Definition 2, it suffices to show $\psi \in \mathcal{T}$ and there is $\sigma \in \mathcal{C}^\phi(\psi)$ such that $\phi < \sigma$. There are two cases:

Case 1, $\nvdash \psi$: Since $\psi \in cl(\mathcal{T} \dot{-} \phi)$, by the compactness of DL-Lite$_\mathcal{R}$, there is a finite subset $\{\delta_1, \ldots, \delta_n\}$ of $\mathcal{T} \dot{-} \phi$ such that $\{\delta_1, \ldots, \delta_n\} \vdash \psi$. Then by Definition 2, we have $\{\delta_1, \ldots, \delta_n\} \subseteq \mathcal{T}$ and there is $\sigma_i \in \mathcal{C}^\phi(\delta_i)$ such that $\phi < \sigma_i$ for $1 \leq i \leq n$. Then since $\mathcal{T} = cl(\mathcal{T}), \{\delta_1, \ldots, \delta_n\} \subseteq \mathcal{T}$, and $\{\delta_1, \ldots, \delta_n\} \vdash \psi$ we have $\psi \in \mathcal{T}$. If there is no S such that $S \cup \{\psi\} \vdash \phi$ and $S \nvdash \phi$ then by Definition 1, $\mathcal{C}^\phi(\psi) = \{\top\}$. It then follows from Maximality and Connectivity (part 1 of Lemma 1) that $\phi \leq \top$ and we are done. Now suppose there is S such that $S \cup \{\psi\} \vdash \phi$ and $S \nvdash \phi$. Since $\{\delta_1, \ldots, \delta_n\} \vdash \psi$, we have $\{\delta_1, \ldots, \delta_n\} \cup S \vdash \phi$. Now let's show $\{\sigma_1, \ldots, \sigma_n\} \cup S \vdash \phi$. If $\{\delta_2, \ldots, \delta_n\} \cup S \vdash \phi$ then by the monotonicity of DL-Lite$_\mathcal{R}$ we have $\{\delta_1\} \cup \{\delta_2, \ldots, \delta_n\} \cup S \vdash \phi$ and if $\{\delta_2, \ldots, \delta_n\} \cup S \nvdash \phi$ then it follows from Definition 1 and $\sigma_1 \in \mathcal{C}^\phi(\delta_1)$ that $\{\delta_1\} \cup \{\delta_2, \ldots, \delta_n\} \cup S \vdash \phi$. Thus in either case we have $\{\delta_1\} \cup \{\delta_2, \ldots, \delta_n\} \cup S \vdash \phi$. Similarly we can replace δ_i by σ_i for $2 \leq i \leq n$ to get $\{\sigma_1, \ldots, \sigma_n\} \cup S \vdash \phi$ which implies $\{\sigma_1 \wedge \cdots \wedge \sigma_n\} \cup S \vdash \phi$. Since by Definition 1 any critical axiom $\sigma \in \mathcal{C}^\phi(\psi)$ is logically the weakest one such that $\{\sigma\} \cup S \vdash \phi$, there must be a σ such that $\sigma_1 \wedge \cdots \wedge \sigma_n \vdash \sigma$ which implies by Dominance $\sigma_1 \wedge \cdots \wedge \sigma_n \leq \sigma$. Since $\phi < \sigma_i$ for $1 \leq i \leq n$, we have by Lemma 1 (part 2) and Transitivity that $\phi < \sigma_1 \wedge \cdots \wedge \sigma_n$. Again by Transitivity we have $\phi < \sigma$ and we are done.

Case 2, $\vdash \psi$: Since $\mathcal{T} = cl(\mathcal{T})$ and $\vdash \psi$, we have $\psi \in \mathcal{T}$. Since $\vdash \psi$, there is no S such that $S \cup \{\psi\} \vdash \phi$ and $S \nvdash \phi$. Thus by Definition 1, $\mathcal{C}^\phi(\psi) = \{\top\}$. Since $\nvdash \phi$ we have by Maximality and Connectivity that $\phi < \top$.

$(T\dot{-}4)$: Suppose $\not\vdash \phi$. By Lemma 2 (part 1) we have $\mathcal{C}^\phi(\phi) = \{\phi\}$. It follows from $\phi \not< \phi$, $\not\vdash \phi$, and Definition 2 that $\phi \notin \mathcal{T}\dot{-}\phi$.

$(T\dot{-}6)$: Suppose $\phi \equiv \psi$. Then we have by Lemma 2 (part 2) $\mathcal{C}^\phi(\delta) = \mathcal{C}^\psi(\delta)$ for any δ and we have by Lemma 1 (part 3) $\phi \simeq \psi$. Then according to Definition 2, $\mathcal{T}\dot{-}\phi$ and $\mathcal{T}\dot{-}\psi$ can not be different.

$(T\dot{-}cr)$: Suppose $\psi \in \mathcal{T}$ and $\sigma \in \mathcal{C}^\phi(\psi)$ is such that $\sigma \in \mathcal{T}\dot{-}\phi$. If $\phi \notin \mathcal{T}$ or $\vdash \phi$ then $\psi \in \mathcal{T}\dot{-}\phi$ follows immediately from Definition 2. So suppose $\phi \in \mathcal{T}$ and $\not\vdash \phi$. By Definition 2, $\sigma \in \mathcal{T}\dot{-}\phi$ implies that $\sigma \in \mathcal{T}$ and there is $\delta \in \mathcal{C}^\phi(\sigma)$ such that $\phi < \delta$. Since $\sigma \in \mathcal{C}^\phi(\psi)$ we have by Lemma 2 (part 3) that $\mathcal{C}^\phi(\sigma) \subseteq \mathcal{C}^\phi(\psi)$. Thus $\delta \in \mathcal{C}^\phi(\psi)$. It then follows from $\phi < \delta$ and Definition 2 that $\psi \in \mathcal{T}\dot{-}\phi$.

$\square$

According to Theorem 1, an entrenchment-based contraction function produces a logically closed TBox $(T\dot{-}1)$ which does not entail the contracting axiom unless it is a tautology $(T\dot{-}4)$. The produced TBox is not larger than the original one $(T\dot{-}2)$. If the contracting axiom is not entailed, then nothing has to be done $(T\dot{-}3)$. The contraction function is syntax-insensitive $(T\dot{-}6)$. $(T\dot{-}1)$–$(T\dot{-}4)$ and $(T\dot{-}6)$ all have their origins in the AGM framework. Unlike them, $(T\dot{-}cr)$ has no counterpart in the AGM framework. It captures the sufficient condition for retaining an axiom in our reformulated mechanism.

$$\left\{ \boxed{\frac{A \sqsubseteq C}{C \sqsubseteq D}} \right\} < \left\{ \begin{matrix} B \sqsubseteq D \\ B \sqsubseteq A \end{matrix} \right\} < \left\{ \boxed{\begin{matrix} B \sqsubseteq C \\ A \sqsubseteq D \end{matrix}} \right\}$$

To illustrate entrenchment-based contraction, suppose a TBox $\mathcal{T}$ and its associated epistemic entrenchment $\leq$ are as shown above. Notice that conjunction of axioms are not shown as their rankings in the epistemic entrenchment uniquely are determined by the least entrenched conjunct. In eliminating $B \sqsubseteq D$ through the entrenchment-based contraction determined by $\leq$, only the boxed axioms are retained. Let's examine the fate of $A \sqsubseteq C$ in this elimination. According to [3,4], a set of axioms S must contain $B \sqsubseteq A$ and $C \sqsubseteq D$ for $S \not\vdash B \sqsubseteq D$ and $\{A \sqsubseteq C\} \cup S \vdash B \sqsubseteq D$. Thus we have $\mathcal{C}^{B \sqsubseteq D}(A \sqsubseteq C) = \{B \sqsubseteq C, A \sqsubseteq D, A \sqsubseteq \neg B, B \sqsubseteq D, A \sqsubseteq C\}$. Since both $B \sqsubseteq C$ and $A \sqsubseteq D$ are strictly more entrenched than $B \sqsubseteq D$, $A \sqsubseteq C$ is retained. It is easy to see the retained axioms do not entail $B \sqsubseteq D$. Moreover, since $C \sqsubseteq D$ together with $B \sqsubseteq C$ entails $B \sqsubseteq D$ and also $B \sqsubseteq A$ together with $A \sqsubseteq D$, the removal of $C \sqsubseteq D$ and $B \sqsubseteq A$ are necessary.

Notice that although we allow conjunction of axioms, we do not have to check for their retainments. It follows from $(\mathcal{T}\dot{-}1)$ that $\phi \wedge \psi \in \mathcal{T}\dot{-}\delta$ if and only if $\phi \in \mathcal{T}\dot{-}\delta$ and $\psi \in \mathcal{T}\dot{-}\delta$. Thus it is sufficient to consider single axioms only. This significantly simplifies the computation of the contraction result. We provide the CONT algorithm for obtaining the contraction outcomes of entrenchment-based contraction functions. CONT takes as input a logically closed TBox $\mathcal{T}$, the epistemic entrenchment $\leq$ for $\mathcal{T}$, and the TBox axiom ϕ to be contracted. CONT starts by initiating the resulting TBox $\mathcal{T}_\phi^-$ to empty. Then it checks for each TBox axioms of $\mathcal{T}$ (line 2) weather any of its critical axioms (line 3) is strictly more entrenched than ϕ (line 4). If it is the case then ψ is added to the resulting TBox $\mathcal{T}_\phi^-$ (line 5). Finally, the resulting TBox $\mathcal{T}_\phi^-$ is returned. It is easy to see that CONT instantiates entrenchment-based contraction function.

Algorithm 1: CONT

    Input: TBox $\mathcal{T}$, epistemic entrenchment $\leq$ for $\mathcal{T}$, and TBox axiom ϕ
    Output: TBox $\mathcal{T}_\phi^-$
1 $\mathcal{T}_\phi^- := \emptyset$;
2   **foreach** *TBox axiom ψ of $\mathcal{T}$* **do**
3   **foreach** $\sigma \in \mathcal{C}^\phi(\psi)$ **do**
4   **if** $\phi < \sigma$ **then**
5 $\mathcal{T}_\phi^- := \mathcal{T}_\phi^- \cup \{\psi\}$;
6   **break**;

7   **return** $\mathcal{T}_\phi^-$;

Proposition 1 *Let $\mathcal{T}$ be a DL-Lite$_\mathcal{R}$ TBox with an associated epistemic entrenchment $\leq$. A function $\dot{-}$ is an entrenchment-based contraction function for $\mathcal{T}$ and is determined by $\leq$ iff*

$$\mathcal{T} \dot{-} \phi = CONT(\mathcal{T}, \leq, \phi)$$

for all DL-Lite$_\mathcal{R}$ TBox axioms ϕ.

4   Related Work

In dealing with changes over DL ontologies, many [7,8,18,23,22,19,20,24] have taken the same strategy as ours by considering it as a belief change problem. Instead of focusing on contraction, [18,23] defined specific revision operators for incorporating new axioms. [22,19,20] studied both contraction and revision but over TBoxes and knowledge bases that are not necessarily closed. This means only the axioms explicitly presented in the TBox or knowledge base are considered. The implicit axioms which logically follow from the explicit ones but are not presented are discarded during the operation. Thus the logical contents are not maximally preserved during the operation as we did by considering logically closed TBoxes. [24] works with logically closed TBoxes and provides a model-theoretic approach for both contraction and revision under DL-Lite$_{core}$. Axiom negation is not supported by most DLs but is required in defining some belief change operations. [8] proposed several notions of negated axioms for DLs. They also explored the notions of inconsistent and incoherent TBoxes.

    In a more general setting, [7,21] identified properties of a monotonic logic under which a contraction function can be defined that satisfies the postulates of Recovery [1] and Relevance [14] respectively. By their results, it is possible to define DL-Lite contraction functions that satisfy Relevance.

    [13] studied operations that contract and revise at the same time. A constraint which states the set of axioms to be incorporated and those to be eliminated is first specified. Then the operation maps a knowledge base to another that satisfies the constraint. The operation reduces to a revision and contraction after making empty the eliminating set and the incorporating set respectively.

[12,6,15,16] also dealt with changes over DL-Lite ontologies. Instead of considering it as a belief change problem, they focus on issues with expressibility of the outcomes for model-based change operations.

5 Conclusion

We looked into methods of eliminating problematic axioms of DL-Lite$_\mathcal{R}$ TBoxes. The entrenchment-based contraction is thus defined by reformulating the AGM epistemic entrenchment contraction. The reformulation on the one hand solves the expressibility issues and on the other hand yields a proper contraction function.

There are many aspects of this work that are worth investigating further. We are in the process of devising an algorithm for computing the critical axioms which will guarantee CONT runs in polynomial time. The definition of critical axioms and also that of entrenchment-based contraction although assuming DL-Lite$_\mathcal{R}$ are in fact independent of the underlying logic. Their generalisation to DLs in general is on the way. The main difficulty in the generalisation is that unlike DL-Lite$_\mathcal{R}$ most DLs are not finite under logical closure.

References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. The Journal of Symbolic Logic 50(2), 510–530 (1985)
2. Baader, F., Calvanese, D., McGuinness, D., Nardi, D., Patel-Schneider, P. (eds.): The Description Logic Handbook. CUP, Cambridge, UK (2003)
3. Borgida, A., Calvanese, D., Rodriguez-Muro, M.: Explanation in dl-lite. In: Proceedings of the 21st Int. Workshop on Description Logics (DL-2008) (2008)
4. Borgida, A., Calvanese, D., Rodriguez-Muro, M.: Explanation in the dl-lite family of description logics. In: Proceedings of the 7th Int. Conf. on Ontologies, DataBases, and Applications of Semantics (ODBASE-2008). pp. 1440–1457 (2008)
5. Calvanese, D., Giacomo, G.D., Lembo, D., Lenzerini, M., Rosati, R.: Tractable reasoning and efficient query answering in description logics: The DL-Lite family. Journal of Automatic Reasoning 39(3), 385–429 (2007)
6. Calvanese, D., Kharlamov, E., Nutt, W., Zheleznyakov, D.: Evolution of dl-lite knowledge bases. In: Proceedings of the 9th International Semantic Web Conference on The Semantic Web (ISWC-2010). pp. 112–128. ISWC'10 (2010)
7. Flouris, G., Plexousakis, D., Antoniou, G.: Generalizing the AGM postulates: preliminary results and applications. In: Proceedings of the 10th International Workshop on Non-Monotonic Reasoning (NMR-2004). pp. 171–179 (2004)
8. Flouris, G., Huang, Z., Pan, J.Z., Plexousakis, D., Wache, H.: Inconsistencies, negations and changes in ontologies. In: Proceedings of the 21st National Concference on Artificial Intelligence (AAAI-2006) (2006)
9. Gärdenfors, P.: Knowledge in Flux: Modelling the Dynamics of Epistemic States. MIT Press (1988)
10. Gärdenfors, P., Makinson, D.: Revisions of knowledge systems using epistemic entrenchment. In: Proceedings of the 2nd conference on Theoretical Aspects of Reasoning about Knowledge (TARK-1988). pp. 83–95 (1988)

11. Gärdenfors, P., Rott, H.: Belief revision. In: Handbook of Logic in AI and LP, vol. 4. OUP (1995)
12. Giacomo, G.D., Lenzerini, M., Poggi, A., Rosati, R.: On instance-level update and erasure in description logic ontologies. Journal of Logic Computation 19(5), 745–770 (2009)
13. Grau, B.C., Ruiz, E.J., Kharlamov, E., Zhelenyakov, D.: Ontology evolution under semantic constraints. In: Proceedings of the 13th International Conference on Principles of Knowledge Representation and Reasoning (KR-2012). pp. 137–147 (2012)
14. Hansson, S.O.: Belief Contraction Without Recovery. Studia Logica 50(2), 251–260 (1991)
15. Kharlamov, E., Zheleznyakov, D.: Capturing instance level ontology evolution for dl-lite. In: Proceedings of the 10th International Conference on The Semantic Web (ISWC-2011). pp. 321–337 (2011)
16. Kharlamov, E., Zheleznyakov, D., Calvanese, D.: Capturing model-based ontology evolution at the instance level: The case of dl-lite. Journal of Computer and System Sciences 79(6), 835–872 (2013)
17. Pan, J.Z., Thomas, E.: Approximating owl-dl ontologies. In: Proceedings of the 22nd National Conference on Artificial Intelligence (AAAI-2007). pp. 1434–1439 (2007)
18. Qi, G., Du, J.: Model-based revision operators for terminologies in description logics. In: Proc. IJCAI-2009. pp. 891–897 (2009)
19. Qi, G., Haase, P., Huang, Z., Ji, Q., Pan, J.Z., Vlker, J.: A kernel revision operator for terminologies algorithms and evaluation. In: Proceedings of the 7th International Semantic Web Conference, (ISWC-2008). pp. 419–434 (2008)
20. Ribeiro, M.M., Wassermann, R.: Base revision for ontology debugging. Journal of Logic Computation 19(5), 721–743 (2009)
21. Ribeiro, M.M., Wassermann, R., Flouris, G., Antoniou, G.: Minimal change: Relevance and recovery revisited. Artificial Intelligence 201, 59–80 (2013)
22. Ribeiro, M.M., Wassermann, R.: First step towards revising ontologies. In: Proc. WONTO-2006 (2006)
23. Wang, Z., Wang, K., Topor, R.: A new approach to knowledge base revision in dl-lite. In: Proc. AAAI-2010 (2010)
24. Zhuang, Z., Wang, Z., Wang, K., Qi, G.: Contraction and revision over dl-lite tboxes. In: Proceedings of the 28th AAAI Conference (AAAI-2014) (2014)