A probabilistic approach for financial IoT data

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Abstract. The extraction of information from the Internet of Things (IoT) plays a fundamental role in many research fields. In this work we focus our attention on financial data, used to describe self-financing portfolios in a complete market. Here, the absence of the arbitrage principle, the existence and the uniqueness of no arbitrage price are valid. With these hypotheses we can resort to the *Black-Scholes model* in order to determine the expression of no arbitrage price. In this model, frictional costs are avoided. Moreover, selling and buying of every amount of the assets and short sellings are allowed. In other words, traders can sell amount of assets even if they do not own them. Finally, this model is composed by a *risk-free* and a *Geometric Brownian motion risk* assets.

1 Introduction

The Internet of Things (IoT) [3] denotes a system based on the link between the real world and Internet and characterized by big data flows, as occurs for systems managing financial data. The number of internet services, as home banking and on-line trading, which let people access Internet databases and do financial transactions, is increasing more and more. This implies the necessity to have very fast and efficient methods able to solve problems by available informations in real time. As a case study, we propose the problem of *pricing of derivatives.* A derivative is a kind of contract whose value (represented by a function F named *Payoff function*) depends on another entity, called *underlying.* Derivatives are used especially in risk management and speculation field. Common examples of derivatives are *options.* More in detail, an option gives the right to buy (*Call option*) or to sell (*Put option*) its underlying in future dates by paying a *strike price.* In the following section we briefly describe the analytical model and apply a numerical approach to an example.

2 Analytical and Numerical Model

A portfolio (or strategy) is an integrable stochastic process rappresenting the shares of the assets and identified by its value function. In particular, *self-financing portfolios*, are strategies where a change of its value depends only on a change in the values of assets. The following equation, known as *Black-Scholes equation* [1], characterizes the self-financing portfolios f depending both on present time and the risk asset:

$$\frac{\sigma^2 s^2}{2}\partial_{ss} f(t;s) + rs\partial_s f(t;s) + \partial_t f(t;s) = rf(t;s) \quad \forall (t;s) \in [0;T[\times \mathbb{R}^+$$

where $r \in \mathbb{R}^+$ is the free risk interest rate. An arbitrage strategy is a self-financing portfolio ensuring a future positive also with a null initial value. More in detail, in a Black-Scholes market the no arbitrage price P_0 of a derivative $F(S)$ assumes the expression $P_0 = e^{-rT}E[F(S_T)]$, where S is the solution of the stochastic differential equation $dS_t = \mu S_t dt + \sigma S_t dW_t$ with $\mu, \sigma \in \mathbb{R}^+$ and W_t is a Brownian motion. Now we focus on a statistical approach (i.e., Monte Carlo method [2]) to evaluate the price formula. The first step consists in estimating σ. We extract a sample of past values of the underlying from a sample database and calculate the corresponding values of the normal process $X = (X_t)_{t \in [0;T]} = \log\left(\frac{S_t}{S_0}\right)$. Here, we have $Var(X) = T\sigma^2$, where $Var(X)$ can be approximated by the sample variance. The second step consists in determining the simulations of the underlying $\tilde{S}_k = S_0 exp\left(\sigma\sqrt{T}\tilde{Z}_k + \left(r - \frac{\sigma^2}{2}\right)T\right)$, where $\tilde{Z}_k$ are normal standard casual numbers. The last step consists in finding the value of P_0 by the *Law of big numbers*: $P_0 \approx \frac{e^{-rT}}{n}\sum_{k=1}^{n} F(\tilde{S}_k)$. We apply the previous numerical results to determine an approximation of a Call with a generic underlying S and $S_0 = 100,\ K = 100,\ T = 1,\ r = 0,1$. Table 1 contains some historical values of S in the first row and in the second simulations of underlying.

Table 1. Extraction of historical data and Simulations of underlying.

His. Val.	101,8	109,7	119,3	109,2	113,4	108,1	107,9	109,9	105,9	115,0
Sim. Val.	110,4	110,6	110,2	110,3	110,6	110,0	110,1	110,56	110,5	110,3

From the first row we obtain $\sigma \approx 0,002$. Since the derivative is an option, by using the values of the second row, the expression of the price becomes:

$$P_0 = \frac{e^{-0,1}}{10}\sum_{\tilde{S}_n > K}(\tilde{S}_n - 100) \approx 9,4$$

References

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