Temporal Description Logics over Finite Traces

Alessandro Artale, Andrea Mazzullo, Ana Ozaki

KRDB Research Centre, Free University of Bozen-Bolzano, Italy
surname@inf.unibz.it

Abstract. Linear temporal logic interpreted over finite traces (LTL_f) has been used as a formalism for temporal specification in automated planning, process modelling and (runtime) verification. In this paper, we lift some results from the propositional case to a decidable fragment of first-order temporal logic obtained by combining the description logic $\mathcal{ALC}$ with LTL_f, denoted as $T^f_{\mathcal{U}}\mathcal{ALC}$. We show that the satisfiability problem in $T^f_{\mathcal{U}}\mathcal{ALC}$ is ExpSpace-complete, as already known for the infinite trace case, while it decreases to NExpTime when we consider finite traces with a fixed number of instants, and even to ExpTime when additionally restricting to globally interpreted TBoxes. Moreover, we investigate two model-theoretical notions for $T^f_{\mathcal{U}}\mathcal{ALC}$: we show that, differently from the infinite trace case, it enjoys the *bounded model property*; we also discuss the notion of *insensitivity to infiniteness*, providing characterisations and sufficient conditions to preserve it.

1 Introduction

Since the introduction of *linear temporal logic* (LTL) by Pnueli [33], several (propositional and first-order) LTL-based formalisms have been developed for applications such as automated planning [11, 12, 15–17], process modelling [1, 31] and (runtime) verification of programs [14, 24, 32–35]. Related research in knowledge representation has focussed on decidable fragments of first-order LTL [28], many of them obtained by combining LTL with description logics (DLs), and for this reason called *temporal description logics* (TDLs) [3, 4, 30, 36]. In this latter setting, the complexity of satisfiability problems can range from ExpSpace-complete, for the full-fledged combination of the description logic $\mathcal{ALC}$ with LTL [23, 27], down to ExpTime- or PSpace-complete, by restricting the application of temporal operators [10, 23], or even in NP or NLogSpace, by weakening both the temporal and the description logic components, for some TDLs used in temporal conceptual data modelling [5, 6].

Besides the usual LTL semantics defined over the natural numbers, rececently attention has been devoted to study LTL semantics based on *finite traces*, which are temporal structures having only a finite number of time points, with the resulting logic sometimes known as *linear temporal logic over finite traces* (LTL_f) [13, 18–22]. Indeed, for several of the already mentioned applications, the finiteness of the time dimension is a fairly natural restriction, since the processes involved are usually required to terminate within a finite amount of time. Motivated by

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similar applications, some recent work has considered TDLs in the context of runtime verification [8] and business process modelling [2]. Nonetheless, these attempts are still based on the infinite trace semantics for temporal operators (sometimes allowed only over axioms [8]).

On the way to overcome such limitations, in this work we combine DLs with LTL_f. In particular, we consider the logic $T^f_{\mathcal{U}}\mathcal{ALC}$ defined as the DL $\mathcal{ALC}$ extended with temporal operators on both concepts and formulas (but without temporalised or rigid roles), and interpreted over models with a finite time dimension. We show that the complexity of the satisfiability problem in $T^f_{\mathcal{U}}\mathcal{ALC}$ is ExpSpace-complete, as for the infinite case, and lowers down to NExpTime if we restrict to traces with a bound k on the number of instants, and even to ExpTime for a fragment with only globally interpreted TBoxes. We then consider two model-theoretical notions. Firstly, we show that $T^f_{\mathcal{U}}\mathcal{ALC}$ has the *bounded model property*, with the bound being double exponential in the size of the formula. We then study the notion of a formula being *insensitive to infiniteness*, considered in [19, 8, 13, 14] for dealing with the possibility to preserve the satisfiability of formulas from the finite to the infinite case and thus to reuse TDL algorithms developed for the infinite case.

This paper is organised as follows. In Section 2, we introduce $T_{\mathcal{U}}\mathcal{ALC}$ and we provide the semantics both over infinite and over finite traces. In Section 3 we study the complexity of the satisfiability problem for $T^f_{\mathcal{U}}\mathcal{ALC}$ formulas, while Section 4 refines these results for traces with fixed k instants. Section 5 investigates model-theoretical notions of $T^f_{\mathcal{U}}\mathcal{ALC}$ formulas. Finally, in Section 6, we conclude with some open problems and possible directions for future work.

2 Temporal Description Logics

In the following, we first define the temporal language $T_{\mathcal{U}}\mathcal{ALC}$ as a temporal extension of the description logic $\mathcal{ALC}$. The language $T_{\mathcal{U}}\mathcal{ALC}$ is obtained by extending $\mathcal{ALC}$ with the binary temporal operator *until* ($\mathcal{U}$). We then define the semantics based on *traces*, which are temporal structures having either a finite or an infinite number of time points. We denote by $T^f_{\mathcal{U}}\mathcal{ALC}$ the language $T_{\mathcal{U}}\mathcal{ALC}$ interpreted over finite traces. We further add the upperscript k and write $T^k_{\mathcal{U}}\mathcal{ALC}$ to indicate that the formulas are interpreted over finite traces with at most $k \in \mathbb{N}, k > 0$, time points.

Syntax. Let $\mathsf{N_C}$, $\mathsf{N_R}$ and $\mathsf{N_I}$ be countably infinite and pairwise disjoint sets of *concept names*, *role names*, and *individual names*, respectively. A $T_{\mathcal{U}}\mathcal{ALC}$ *concept* is an expression of the form:

$$C, D ::= A \mid \neg C \mid C \sqcap D \mid \exists R.C \mid C \,\mathcal{U}\, D,$$

where $A \in \mathsf{N_C}$ and $R \in \mathsf{N_R}$. For instance, the expression $A\,\mathcal{U}\,(B \sqcap \exists R.\neg A)$ is a $T_{\mathcal{U}}\mathcal{ALC}$ concept. A $T_{\mathcal{U}}\mathcal{ALC}$ *axiom* is either a *concept inclusion (CI)* of the form $C \sqsubseteq D$, or an *assertion* of the form $A(a)$ or $R(a, b)$, where C, D are $T_{\mathcal{U}}\mathcal{ALC}$ concepts, $A \in \mathsf{N_C}$, $R \in \mathsf{N_R}$, and $a, b \in \mathsf{N_I}$. $T_{\mathcal{U}}\mathcal{ALC}$ *formulas* are of the form:

$$\varphi, \psi ::= \alpha \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \,\mathcal{U}\, \psi,$$

where α is a $T_{\mathcal{U}}\mathcal{ALC}$ axiom.

Semantics. A *(TDL) interpretation* is a structure $\mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n \in I})$, where I is an interval of the form $[0, \infty)$ or $[0, l]$, with $l \in \mathbb{N}$, and each $\mathcal{I}_n$ is a classical DL interpretation with domain $\Delta^{\mathfrak{I}}$: we have $A^{\mathcal{I}_n} \subseteq \Delta^{\mathfrak{I}}$, $R^{\mathcal{I}_n} \subseteq \Delta^{\mathfrak{I}} \times \Delta^{\mathfrak{I}}$, and $a^{\mathcal{I}_i} = a^{\mathcal{I}_j} \in \Delta^{\mathfrak{I}}$ for all $a \in \mathsf{N_I}$ and $i, j \in \mathbb{N}$, i.e., the interpretation of individual names is fixed (in the following denoted as $a^{\mathcal{I}}$). We refer to a temporal interpretation $\mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n \in I})$ as a *finite trace* if $I = [0, l]$, and as an *infinite trace* if $I = [0, \infty)$. The stipulation that all time points share the same domain $\Delta^{\mathfrak{I}}$ is called the *constant domain assumption* (meaning that objects are not created or destroyed over time), and it is the most general choice in the sense that increasing, decreasing, and varying domains can all be reduced to it [23]. We now define the semantics of $T_{\mathcal{U}}\mathcal{ALC}$ considering $I = [0, \infty)$. The semantics of $T^f_{\mathcal{U}}\mathcal{ALC}$ is defined in the same way except that I is of the form $[0, l]$, for some $l \in \mathbb{N}$. For a fixed $k \in \mathbb{N}, k > 0$, we define the semantics of $T^k_{\mathcal{U}}\mathcal{ALC}$ considering $I = [0, l]$ with $l \leq k - 1$. The interpretation of concepts at instant n of interpretation $\mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n \in I})$ is defined as follows:

$$(\neg C)^{\mathcal{I}_n} = \Delta^{\mathfrak{I}} \setminus C^{\mathcal{I}_n}, \quad (C \sqcap D)^{\mathcal{I}_n} = C^{\mathcal{I}_n} \cap D^{\mathcal{I}_n},$$
$$(\exists R.C)^{\mathcal{I}_n} = \{d \in \Delta^{\mathfrak{I}} \mid \exists d' \in C^{\mathcal{I}_n} : (d, d') \in R^{\mathcal{I}_n}\},$$
$$(C \,\mathcal{U}\, D)^{\mathcal{I}_n} = \{d \in \Delta^{\mathfrak{I}} \mid \exists m \in I, m > n \colon d \in D^{\mathcal{I}_m} \wedge \forall i \in (n, m) \colon d \in C^{\mathcal{I}_i}\}.$$

We say that a concept C is *satisfied in* $\mathfrak{I}$ if $C^{\mathcal{I}_0} \neq \emptyset$. *Satisfaction* of a formula φ in $\mathfrak{I}$ at time point $n \in I$ (written $\mathfrak{I}, n \models \varphi$) is inductively defined as follows:

$$\begin{array}{lll lll}
\mathfrak{I}, n \models C \sqsubseteq D & \text{iff } C^{\mathcal{I}_n} \subseteq D^{\mathcal{I}_n}, & & \mathfrak{I}, n \models \neg\phi & \text{iff not } \mathfrak{I}, n \models \phi, \\
\mathfrak{I}, n \models A(a) & \text{iff } a^{\mathcal{I}} \in A^{\mathcal{I}_n}, & & \mathfrak{I}, n \models \phi \wedge \psi & \text{iff } \mathfrak{I}, n \models \phi \text{ and } \mathfrak{I}, n \models \psi, \\
\mathfrak{I}, n \models R(a, b) & \text{iff } (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}_n}, & & \mathfrak{I}, n \models \phi \,\mathcal{U}\, \psi & \text{iff } \exists m \in I, m > n \colon\ \mathfrak{I}, m \models \psi, \\
 & & & & \quad \text{and } \forall i \in (n, m) \colon \mathfrak{I}, i \models \phi
\end{array}$$

We say that φ is *satisfied in* $\mathfrak{I}$, writing $\mathfrak{I} \models \varphi$, if $\mathfrak{I}, 0 \models \varphi$. A formula φ *logically implies* a formula ψ if every $\mathfrak{I}$ that satisfies φ satisfies also ψ, and we write $\varphi \models \psi$. Moreover, φ is said to be $T^f_{\mathcal{U}}\mathcal{ALC}$ *satisfiable* (or *satisfiable over finite traces*) if it is satisfied in some finite trace. Given a concept C and a formula φ, we say that C *is satisfiable over finite traces with respect to* φ if there is a finite trace $\mathfrak{I}$ such that φ is satisfied in $\mathfrak{I}$ and C is satisfied in $\mathfrak{I}$, in symbols $\varphi \not\models C \sqsubseteq \bot$. Concept satisfiability w.r.t. φ can be reduced to formula satisfiability by checking whether the following formula is satisfiable: $\varphi \wedge C(a)$, for a fresh $a \in \mathsf{N_I}$.

We will use the following standard equivalences for concepts: $\bot \equiv A \sqcap \neg A$, $\top \equiv \neg\bot$; $\forall R.C \equiv \neg\exists R.\neg C$; $(C \sqcup D) \equiv \neg(\neg C \sqcap \neg D)$; $\bigcirc C \equiv \bigcirc^1 C \equiv \bot \,\mathcal{U}\, C$; $\bigcirc^n C \equiv \bigcirc \bigcirc^{n-1} C$, with $n > 1$; $\Diamond C \equiv \top \mathcal{U} C$; $\Box C \equiv \neg\Diamond\neg C$. Similar abbreviations also hold for formulas. We often write $\mathfrak{F} = (\Delta^{\mathfrak{F}}, (\mathcal{F}_n)_{n \in [0,l]})$ to indicate that a trace is finite and $\mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n \in I})$ when it is infinite.

3 Satisfiability over Finite Traces

In the following we show how to reduce the formula satisfiability problem over finite traces to the same problem over infinite traces. Similar to the encoding proposed in [20] for propositional LTL, we introduce a fresh concept name E to capture the finiteness of the temporal dimension. The fundamental properties regarding the *end of time* can be described as follows. *(i)* There is a least one instant before the end of time (E is initially false). *(ii)* The end of time comes for all objects (E is unavoidable). *(iii)* The end of time comes at the same time for every object (E is synchronous). *(iv)* The end of time never goes away (E is persistent). We axiomatise the above end of time properties:

$$\psi_{f_1} = \top \sqsubseteq \neg E, \quad \psi_{f_2} = (\top \sqsubseteq \neg E)\,\mathcal{U}\,(\top \sqsubseteq E), \quad \psi_{f_3} = \Box(E \sqsubseteq \bigcirc E).$$

Intuitively, ψ_{f_1} captures Point (i), ψ_{f_2} captures Points (ii), (iii), and ψ_{f_3} captures Point (iv). In the following, we use $\psi_f = \psi_{f_1} \wedge \psi_{f_2} \wedge \psi_{f_3}$.

We now show that satisfiability in $T^f_{\mathcal{U}}\mathcal{ALC}$ is ExpSpace-complete. We start by characterising, in Lemma 1, models satisfying the *end of time formula*, ψ_f. Our characterisation uses the following notion of an extension of a finite trace. Let $\mathfrak{F} = (\Delta^{\mathfrak{F}}, (\mathcal{F}_n)_{n\in[0,l]})$ and $\mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n\in[0,\infty)})$ be, respectively, a finite and an infinite trace s.t. $\Delta^{\mathfrak{F}} = \Delta^{\mathfrak{I}}$ (with a small abuse, we will write just Δ) and $a^{\mathcal{F}} = a^{\mathcal{I}}$, for all $a \in \mathsf{N_I}$. We denote by $\mathfrak{F}\cdot\mathfrak{I} = (\Delta^{\mathfrak{F}\cdot\mathfrak{I}}, (\mathcal{F}\cdot\mathcal{I}_n)_{n\in[0,\infty)})$ the *extension of* $\mathfrak{F}$ *with* $\mathfrak{I}$ defined as the infinite trace with $\Delta^{\mathfrak{F}\cdot\mathfrak{I}} = \Delta$, $a^{\mathcal{F}\cdot\mathcal{I}} = a^{\mathcal{F}}$, for all $a \in \mathsf{N_I}$, and for $n \in \mathbb{N}$:

$$X^{\mathcal{F}\cdot\mathcal{I}_n} = \begin{cases} X^{\mathcal{F}_n}, & \text{if } n \in [0,l] \\ X^{\mathcal{I}_{n-(l+1)}}, & \text{if } n \in [l+1,\infty) \end{cases}, \quad E^{\mathcal{F}\cdot\mathcal{I}_n} = \begin{cases} \emptyset, & \text{if } n \in [0,l] \\ \Delta, & \text{if } n \in [l+1,\infty) \end{cases},$$

where $X \in (\mathsf{N_C} \setminus \{E\}) \cup \mathsf{N_R}$ and E is the concept name encoding the end of time. Clearly, the interpretation of E characterises the satisfiability of ψ_f.

Lemma 1. *For every infinite trace $\mathfrak{I}$, $\mathfrak{I} \models \psi_f$ iff $\mathfrak{I} = \mathfrak{F}\cdot\mathfrak{I}'$, for some finite trace $\mathfrak{F}$ and some infinite trace $\mathfrak{I}'$.*

We now introduce a translation $\cdot^{\dagger}$ of $T^f_{\mathcal{U}}\mathcal{ALC}$ concepts and formulas into $T_{\mathcal{U}}\mathcal{ALC}$ concepts and formulas, respectively, such that, together with the end of time formula, ψ_f, captures those concepts and formulas satisfiable over finite traces. More formally, a concept C (formula φ) of $T^f_{\mathcal{U}}\mathcal{ALC}$ is satisfiable if and only if its translation $C^{\dagger}$ ($\varphi^{\dagger}$) is satisfiable in a $T_{\mathcal{U}}\mathcal{ALC}$ model that also satisfies the formula ψ_f. Our translation $\cdot^{\dagger}$ is defined as:

$$\begin{array}{rlrl} (\alpha)^{\dagger} & \mapsto \alpha & (\neg\beta)^{\dagger} & \mapsto \neg\beta^{\dagger} \\ (C \sqcap D)^{\dagger} & \mapsto C^{\dagger} \sqcap D^{\dagger} & (\phi \wedge \psi)^{\dagger} & \mapsto \phi^{\dagger} \wedge \psi^{\dagger} \\ (\exists R.C)^{\dagger} & \mapsto \exists R.C^{\dagger} & (C \sqsubseteq D)^{\dagger} & \mapsto C^{\dagger} \sqsubseteq D^{\dagger} \\ (C\,\mathcal{U}\,D)^{\dagger} & \mapsto C^{\dagger}\,\mathcal{U}\,(D^{\dagger} \sqcap \neg E) & (\phi\,\mathcal{U}\,\psi)^{\dagger} & \mapsto \phi^{\dagger}\,\mathcal{U}\,(\psi^{\dagger} \wedge \neg(\top \sqsubseteq E)) \end{array}$$

where α can be an assertion or a concept name $A \in \mathsf{N_C}$, β can be a concept C or a formula φ, and $R \in \mathsf{N_R}$. Lemma 2 states the correctness of $\cdot^{\dagger}$.

Lemma 2. *Let $\mathfrak{F}$ be a finite trace, and let $\mathfrak{F} \cdot \mathfrak{I}$ be an extension of $\mathfrak{F}$. Then, for every $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ, we have: $\mathfrak{F} \models \varphi$ iff $\mathfrak{F} \cdot \mathfrak{I} \models \varphi^\dagger$.*

From the previous lemmas, we obtain a reduction of the $T^f_{\mathcal{U}}\mathcal{ALC}$ satisfiability problem to the corresponding problem for $T_{\mathcal{U}}\mathcal{ALC}$.

Theorem 3. *Let φ be a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula. Then φ is satisfiable iff $\varphi^\dagger \wedge \psi_f$ is a satisfiable $T_{\mathcal{U}}\mathcal{ALC}$ formula.*

Since it is known that the $T_{\mathcal{U}}\mathcal{ALC}$ formula satisfiability problem is ExpSpace-complete [23, Theorem 14.15], the above theorem gives us an ExpSpace upper bound for checking satisfiability in $T^f_{\mathcal{U}}\mathcal{ALC}$. The lower bound can be proved using similar ideas as those used to prove hardness of $T_{\mathcal{U}}\mathcal{ALC}$. We are now ready to state the main theorem of this section.

Theorem 4. *Satisfiability in $T^f_{\mathcal{U}}\mathcal{ALC}$ is* ExpSpace*-complete.*

4 Traces with at Most *k* Time Points

In this section we study the satisfiability in $T^k_{\mathcal{U}}\mathcal{ALC}$, where we restrict the problem to traces with at most k time points, with k given in binary, as part of the input. We provide for this case a lower NExpTime complexity result which further reduces to ExpTime when just global CIs are allowed. To prove our upper bound for the satisfiability problem in this setting we use *quasimodels*, which have been used to prove the satisfiability of various TDLs [5, 9, 23, 25, 26, 36]. In particular, we show that there is a model with at most k time points iff there is a quasimodel with a sequence of quasistates of length at most k. Then, our upper bound is obtained by guessing an exponential size sequence of sets of types which serves as a certificate for the existence of a quasimodel (and therefore a model) for the input formula.

Let φ be a $T^k_{\mathcal{U}}\mathcal{ALC}$ formula. Assume w.l.o.g. that φ does not contain abbreviations (i.e., it only contains the logical connectives $\neg$, $\sqcap$, $\wedge$, the existential quantifier $\exists$, and the temporal operator $\mathcal{U}$). Denote by $\mathsf{cl}^\mathsf{c}(\varphi)$ and $\mathsf{cl}^\mathsf{f}(\varphi)$ the closures under single negation of the sets of all concepts and formulas occurring in φ, respectively. Also, let $\mathsf{ind}(\varphi)$ be the set of individuals occurring in φ. A *concept type for* φ is any subset t of $\mathsf{cl}^\mathsf{c}(\varphi) \cup \mathsf{ind}(\varphi)$ such that:

T1 $\neg C \in t$ iff $C \notin t$, for all $\neg C \in \mathsf{cl}^\mathsf{c}(\varphi)$;
T2 $C \sqcap D \in t$ iff $C, D \in t$, for all $C \sqcap D \in \mathsf{cl}^\mathsf{c}(\varphi)$; and
T3 t contains at most one individual name in $\mathsf{ind}(\varphi)$.

Similarly, we define *formula types* $t \subseteq \mathsf{cl}^\mathsf{f}(\varphi)$ for φ with the conditions:

T1′ $\neg\phi \in t$ iff $\phi \notin t$, for all $\neg\phi \in \mathsf{cl}^\mathsf{f}(\varphi)$; and
T2′ $\phi \wedge \psi \in t$ iff $\phi, \psi \in t$, for all $\phi \wedge \psi \in \mathsf{cl}^\mathsf{f}(\varphi)$.

We may omit 'for φ' if there is no risk of confusion. A concept type describes one domain element at a single time point, while a formula type expresses constraints on all domain elements. If $a \in t \cap \mathsf{ind}(\varphi)$, then t describes an named element. We may write t_a to indicate this and call it a *named type*.

The next notion captures how sets of types need to be constrained so that the DL dimension is respected. We say that a pair of concept types (t, t') is *R-compatible* if $\{\neg F \mid \neg\exists R.F \in t\} \subseteq t'$. A *quasistate* for φ is a set S of concept or formula types for φ such that:

Q1 S contains exactly one formula type t_S;
Q2 S contains exactly one named type t_a for each $a \in \mathsf{ind}(\varphi)$;
Q3 for all $C \sqsubseteq D \in \mathsf{cl}^{\mathsf{f}}(\varphi)$, we have $C \sqsubseteq D \in t_S$ iff $C \in t$ implies $D \in t$ for all concept types $t \in S$;
Q4 for all $A(a) \in \mathsf{cl}^{\mathsf{f}}(\varphi)$, we have $A(a) \in t_S$ iff $A \in t_a$
Q5 $t \in S$ and $\exists R.D \in t$ implies there is $t' \in S$ such that $D \in t'$ and (t, t') is R-compatible;
Q6 for all $R(a, b) \in \mathsf{cl}^{\mathsf{f}}(\varphi)$, we have $R(a, b) \in t_S$ iff (t_a, t_b) is R-compatible.

A *(concept/formula) run segment* for φ is a finite sequence $\sigma = \sigma(0) \ldots \sigma(n)$ composed exclusively of concept or formula types, respectively, such that:

R1 for all $a \in \mathsf{ind}(\varphi)$ and all $i \in (0, n]$, we have $a \in \sigma(0)$ iff $a \in \sigma(i)$;
R2 for all $\alpha\,\mathcal{U}\,\beta \in \mathsf{cl}^*(\varphi)$ and all $i \in [0, n]$, we have $\alpha\,\mathcal{U}\,\beta \in \sigma(i)$ iff there is $j \in (i, n]$ such that $\beta \in \sigma(j)$ and $\alpha \in \sigma(m)$ for all $m \in (i, j)$,

where cl^* is either cl^{c} or cl^{f} (as appropriate), and **R1** does not apply to formula run segments. Intuitively, a concept run segment describes the temporal dimension of a single domain element, whereas a formula run segment describes constraints on the whole DL interpretation.

Finally, a *quasimodel* for φ is a pair $(S, \mathfrak{R})$, with S a finite sequence of quasistates $S(0)S(1) \ldots S(n)$ and $\mathfrak{R}$ a non-empty set of run segments such that:

M1 $\varphi \in t_S$ where t_S is the formula type in $S(0)$;
M2 for every $\sigma \in \mathfrak{R}$ and every $i \in [0, n]$, $\sigma(i) \in S(i)$; and, conversely, for every $t \in S(i)$, there is $\sigma \in \mathfrak{R}$ with $\sigma(i) = t$.

By **M2** and the definition of a quasistate for φ, $\mathfrak{R}$ always contains exactly one formula run segment and one named run segment for each $a \in \mathsf{ind}(\varphi)$.

Every quasimodel for φ describes an interpretation satisfying φ and, conversely, every such interpretation can be abstracted into a quasimodel for φ. We formalise this notion for finite traces with the following lemma.

Lemma 5. *There is a finite trace satisfying φ with at most k time points iff there is a quasimodel for φ with a sequence of quasistates of length at most k.*

We devise a non-deterministic algorithm to check satisfiability of a $T^k_{\mathcal{U}}\mathcal{ALC}$ formula φ in NExpTime. First we compute in exponential time w.r.t. $|\varphi|$ the sets of all concept and formula types for φ, i.e., satisfying conditions **T1-T3** and conditions **T1′-T2′**, respectively. We then guess a sequence $S(0), \ldots, S(n)$ of sets of (concept/formula) types for φ of length $n \leq k$; and for each type at position i in this sequence we also guess a sequence of types of length n.

Denote by $\mathfrak{R}$ the set of such sequences of types. We now check (1) whether each $S(i)$ satisfies conditions **Q1-Q6**; (2) whether each sequence in $\mathfrak{R}$ satisfies conditions **R1-R2** (where **R1** only applies for concept run segments); and (3) whether φ is in the formula type in $S(0)$ (condition **M1**) and each sequence in $\mathfrak{R}$ satisfies condition **M2**. All these conditions can be checked in exponential time w.r.t. $|\varphi|$ and $|k|$. The algorithm returns 'satisfiable' iff all conditions are satisfied. Since the conditions exactly match the definition of a quasimodel for φ, their satisfaction implies that $(S, \mathfrak{R})$ is a quasimodel for φ. Hardness can be proved by adapting Lemma 6.2 by Baader et al. [10], for $\mathcal{ALC}$-LTL with rigid concepts. We are now ready to state our main result in this section.

Theorem 6. *Satisfiability in $T^k_{\mathcal{U}}\mathcal{ALC}$ is* NExpTime*-complete.*

We end this section considering the satisfiability problem in $T^k_{\mathcal{U}}\mathcal{ALC}$ *restricted to global CIs*, defined as the fragment of $T^k_{\mathcal{U}}\mathcal{ALC}$ in which formulas can only be of the form $\mathcal{T} \wedge \Box(\mathcal{T}) \wedge \phi$, where $\mathcal{T}$ is a conjunction of CIs and ϕ does not contain CIs. We write $T^k_{\mathcal{U}}(g)\mathcal{ALC}$ to denote this fragment interpreted over finite traces with at most k time points. We establish that satisfiability in $T^k_{\mathcal{U}}(g)\mathcal{ALC}$ is in ExpTime, which is tight since satisfiability in $\mathcal{ALC}$ is already ExpTime-hard [7]. Our upper bound is based on *type elimination* [23, 26, 30].

Theorem 7. *Satisfiability in $T^k_{\mathcal{U}}(g)\mathcal{ALC}$ is* ExpTime*-complete.*

We leave satisfiability in $T^f_{\mathcal{U}}\mathcal{ALC}$ *restricted to global CIs*, analogously defined as the fragment $T^f_{\mathcal{U}}(g)\mathcal{ALC}$ of $T^f_{\mathcal{U}}\mathcal{ALC}$ with only formulas of the form $\mathcal{T} \wedge \Box(\mathcal{T}) \wedge \phi$, as an open problem. It is known that the complexity of the satisfiability problem in this fragment over infinite traces is ExpTime-complete [9, 30][1]. Though, the end of time formula ψ_f is not in $T^f_{\mathcal{U}}(g)\mathcal{ALC}$. We cannot use the same strategy of defining a translation for the semantics based on infinite traces, as we did in Section 3. The upper bound in [30] is based on type elimination. The main difficulty of devising a type elimination procedure for $T^f_{\mathcal{U}}(g)\mathcal{ALC}$ is that the number of time points is not fixed and the argument in [30] showing that there is a quasimodel iff there is a quasimodel $(S, \mathfrak{R})$ such that $S(i+i) \subseteq S(i)$, for all $i \geq 0$, is not applicable to finite traces. A type with a concept equivalent to $\neg \bigcirc \top$ can only be in the last quasistate of the quasimodel. So it is not clear whether one can show that if there is a quasimodel then there is a quasimodel with an exponential sequence of quasistates, as in $T^k_{\mathcal{U}}(g)\mathcal{ALC}$ (Theorem 7).

5 Model-Theoretical Properties

In this section we discuss some model-theoretical properties of $T^f_{\mathcal{U}}\mathcal{ALC}$. In particular, we show that formulas of $T^f_{\mathcal{U}}\mathcal{ALC}$ enjoy the *bounded* model property: if there is a finite trace (which can be arbitrarily large) satisfying a formula φ

[1] In [30, Theorem 3] the authors show that satisfiability in $\mathrm{LTL}_{\mathcal{ALC}}$ w.r.t. TBoxes with constant domains is ExpTime-complete. An argument in [9, Theorem 1] extends this result to $T_{\mathcal{U}}\mathcal{ALC}$ formulas, which include assertions.

then there is a finite trace with at most *double exponentially* many time points w.r.t. the size of φ. Moreover, the bound on the number of time points implies a bound on the size of the domain, in the DL dimension of the model. Finally, we discuss *insensitivity to infiniteness*, a model-theoretical property introduced by De Giacomo et al. [19]. We lift to $T^f_{\mathcal{U}}\mathcal{ALC}$ a characterisation for propositional LTL [19]. We then discuss a limitation of this notion and propose a new related notion of insensitivity to infiniteness.

5.1 Bounded Model Property

We first show that if there is a finite trace which satisfies a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ then there is a finite trace with at most k time points, with k double exponentially large w.r.t. the size of φ. This bound is obtained by (1) showing that if there is a quasimodel for φ then there is a quasimodel for φ where there is no repetition of quasistates, except for the last quasistate; and (2) applying Lemma 5 from Section 4, which correlates the length of a sequence of quasistates with the number of time points in a finite trace. Since the number of formula types is bounded by $\mathsf{tp^f}(\varphi) = 2^{|\mathsf{cl^f}(\varphi)|}$ and the number of concept types is bounded by $\mathsf{tp^c}(\varphi) = |\mathsf{ind}(\varphi)| \cdot 2^{|\mathsf{cl^c}\varphi|}$, there are at most $\mathsf{tp^f}(\varphi) \cdot 2^{\mathsf{tp^c}(\varphi)}$ quasistates for φ: each quasistate contains one formula type and a subset of the set of concept types. The next lemma formalises Point (1) above and can be proved with an argument similar as that of Lemma 11.27 by Gabbay et al. [23].

Lemma 8. *There is a quasimodel for φ iff there is a quasimodel for φ of the form $(S, \mathfrak{R})$ where S is a sequence of length at most $\mathsf{tp^f}(\varphi) \cdot 2^{\mathsf{tp^c}(\varphi)} + 1$.*

Lemmas 5 and 8 directly imply that $T^f_{\mathcal{U}}\mathcal{ALC}$ enjoys the bounded model property. We formalise this result with the following theorem.

Theorem 9. *Satisfiability of a formula φ in $T^f_{\mathcal{U}}\mathcal{ALC}$ implies satisfiability of φ in $T^k_{\mathcal{U}}\mathcal{ALC}$ with k bounded by $\mathsf{tp^f}(\varphi) \cdot 2^{\mathsf{tp^c}(\varphi)}$.*

We now show that if a formula is satisfiable in $T^k_{\mathcal{U}}\mathcal{ALC}$ then there is a model where the size of the domain is exponential in k. Since satisfiability in $T^f_{\mathcal{U}}\mathcal{ALC}$ implies satisfiability in $T^k_{\mathcal{U}}\mathcal{ALC}$ for some $k > 0$, the formula:

$$\bigcirc A(a) \land \Box(A \sqsubseteq \Box(\neg A \sqcap \exists R.A))$$

which only admits models with an infinite domain [30] (recall that in this paper we use the irreflexive until) is unsatisfiable over finite traces. We formalise the bounded domain property with the following theorem.

Theorem 10. *Satisfiability of a formula φ in $T^k_{\mathcal{U}}\mathcal{ALC}$ implies the existence of a model with domain size bounded by $|\mathsf{tp^c}(\varphi)|^k$.*

5.2 Insensitivity to Infiniteness

In this subsection, we investigate the notion of insensitivity to infiniteness, introduced by De Giacomo et al. [19] for propositional LTL formulas. This property is meant to capture those LTL_f formulas that can be equivalently interpreted over infinite traces, provided that, from a certain instant, these traces satisfy an end event forever, falsifying all other atomic propositions. Firstly, we lift this notion of insensitivity to our TDL setting, providing a characterisation analogous to the propositional one [19]. Secondly, we propose a different definition for insensitivity, closer to other notions from the the literature on runtime verification [8, 13, 14] and motivated by the following idea: a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula should be considered as insensitive whenever being satisfiable on a finite trace implies that it is satisfiable on *every* infinite trace extending the finite one, provided that it satisfies the end of time formulas.

We introduce the following definitions. Let Σ be a finite subset of $\mathsf{N_C} \cup \mathsf{N_R}$. Assume w.l.o.g. that $T^f_{\mathcal{U}}\mathcal{ALC}$ concepts and formulas we mention in this subsection have concept and role names in Σ and that Σ contains the end concept E. Given an infinite trace $\mathfrak{I}$, the *Σ-reduct of* $\mathfrak{I}$ is the infinite trace $\mathfrak{I}_{|\Sigma}$ coinciding with $\mathfrak{I}$ on Σ and such that $X^{\mathcal{I}_{|\Sigma n}} = \emptyset$, for $X \notin \Sigma$ and $n \in [0, \infty)$ [29]. Let $\mathfrak{F} = (\Delta^{\mathfrak{F}}, (\mathcal{F}_n)_{n \in [0,l]})$ be a finite trace, and let $\mathfrak{E} = (\Delta^{\mathfrak{E}}, (\mathcal{E}_n)_{n \in [0,\infty)})$ be the infinite trace such that $\Delta^{\mathfrak{E}} = \Delta^{\mathfrak{F}}$ (for simplicity, we may write just Δ), $a^{\mathcal{E}} = a^{\mathcal{F}}$ for all $a \in \mathsf{N_I}$, and for all $A \in \mathsf{N_C} \setminus \{E\}$, $R \in \mathsf{N_R}$, $A^{\mathcal{E}_n} = \emptyset$ and $R^{\mathcal{E}_n} = \emptyset$ while $E^{\mathcal{E}_n} = \Delta$, where $n \in [0, \infty)$. The extension of $\mathfrak{F}$ with $\mathfrak{E}$ (see its definition in Section "Satisfiability over Finite Traces"), $\mathfrak{F} \cdot \mathfrak{E}$, will be denoted as the *end extension of* $\mathfrak{F}$. Recalling the definition of ψ_f, we define, $\chi_f = \psi_f \wedge \chi_{f_1} \wedge \chi_{f_2}$, where

$$\chi_{f_1} = \Box(E \sqsubseteq \bigsqcap_{A \in \Sigma \setminus \{E\}} \neg A), \quad \chi_{f_2} = \Box(E \sqsubseteq \bigsqcap_{R \in \Sigma} \neg \exists R.\top).$$

The following two lemmas correspond to Theorems 2 and 3 in [19], respectively. In particular, Lemma 11 characterises infinite traces of the form $\mathfrak{F} \cdot \mathfrak{E}$, for a finite trace $\mathfrak{F}$.

Lemma 11. *For every infinite trace $\mathfrak{I}$, $\mathfrak{I} \models \chi_f$ iff $\mathfrak{I}_{|\Sigma} = \mathfrak{F} \cdot \mathfrak{E}$, for some finite trace $\mathfrak{F}$.*

Lemma 12, which specialises Lemma 2, shows the equivalence between $T^f_{\mathcal{U}}\mathcal{ALC}$ satisfiability of a formula φ, and satisfiability of its $T_{\mathcal{U}}\mathcal{ALC}$ translation $\varphi^{\dagger}$ (see Section 'Satisfiability over Finite Traces') on end extensions of finite traces.

Lemma 12. *Let $\mathfrak{F}$ be a finite trace and let φ be a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula. We have that: $\mathfrak{F} \models \varphi$ iff $\mathfrak{F} \cdot \mathfrak{E} \models \varphi^{\dagger}$.*

We are now ready to state the first definition of insensitivity to infiniteness. Given a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ, we say that φ is *insensitive to infiniteness* if:

I1 for every finite trace $\mathfrak{F}$, $\mathfrak{F} \models \varphi$ iff $\mathfrak{F} \cdot \mathfrak{E} \models \varphi$.

The next characterisation result, obtained from Lemmas 11 and 12, corresponds to Theorem 4 in [19].

Theorem 13. *A $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ is insensitive to infiniteness **I1** iff the following $T_{\mathcal{U}}\mathcal{ALC}$ logical implication holds: $\chi_f \models \varphi \leftrightarrow \varphi^{\dagger}$.*

Thus, insensitive **I1** formulas allow us to blur the distinction between finite and infinite traces, provided that the infinite ones are as described in Lemma 11.

We now analyse some features of this property. Firstly, notice that $T^f_{\mathcal{U}}\mathcal{ALC}$ formulas without any occurrence of temporal operators, are insensitive **I1**. Moreover, given a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ, this property is preserved under boolean operators [19, Theorem 5]: if φ is insensitive **I1**, $\neg\varphi$ is insensitive as well, and similarly for conjunction. It is also possible to capture several standard temporal patterns derived from the declarative process modelling language DECLARE [1, 19]. For instance, given a $T^f_{\mathcal{U}}\mathcal{ALC}$ assertion α, we have that $\Diamond\alpha$ is insensitive **I1**. On the other hand, a formula of the form $\Box\alpha$ (which is not included among the standard DECLARE patterns) is not **I1**. However, we have also that $\Diamond\neg\alpha$ is not insensitive **I1**, while $\Box\neg\alpha$ is, due to the condition on the interpretation of concept and role names after the end of time. Therefore, in general, if a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ is **I1**, neither $\Diamond\varphi$ nor $\Box\varphi$ are. The same holds for formulas of the form $\varphi\,\mathcal{U}\,\psi$.

In order to overcome this temporal asymmetry, and so to obtain a notion of insensitivity to infiniteness that is preserved by the application of the until operator, we propose the following definition. Given a $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ, we say that φ is *insensitive to infiniteness* if:

I2 for every finite trace $\mathfrak{F}$ and every extension $\mathfrak{F}\cdot\mathfrak{I}$, if $\mathfrak{F}\models\varphi$, then $\mathfrak{F}\cdot\mathfrak{I}\models\varphi$.

We will use similar notational conventions as for **I1**. Notice that this definition matches the first condition of the *impartiality maxim* introduced by Bauer et al. [14]. It is possible to provide an analogous characterisation of insensitive **I2** formulas, as we did above for the **I1** case.

Theorem 14. *A $T^f_{\mathcal{U}}\mathcal{ALC}$ formula φ is insensitive to infiniteness **I2** iff the following $T_{\mathcal{U}}\mathcal{ALC}$ logical implication holds: $\psi_f\models\varphi^{\dagger}\to\varphi$.*

As a consequence of Theorems 13 and 14, checking **I1** or **I2** can be done in ExpSpace using the same algorithms developed for TDLs on infinite traces [30].

As before, $T^f_{\mathcal{U}}\mathcal{ALC}$ formulas without any occurrence of temporal operators are clearly insensitive **I2**. However, being insensitive **I2** is not preserved under negation. For instance, we have that $\Diamond\alpha$ (with α a $T^f_{\mathcal{U}}\mathcal{ALC}$ assertion) is insensitive **I2**, but its negation, $\neg\Diamond\alpha\equiv\Box\neg\alpha$, is not insensitive **I2**. The theorem below states that the property of being insensitive **I2** is preserved under conjunction and the until operator. Thus, differently from Theorem 5 in [19], we are now able to provide a recursive sufficient condition of insensitivity also in the case of temporal operators.

Theorem 15. *Let φ and ψ be insensitive **I2** $T^f_{\mathcal{U}}\mathcal{ALC}$ formulas. Then $\varphi\land\psi$ and $\varphi\,\mathcal{U}\,\psi$ are insensitive **I2**.*

In particular, as a consequence of the above statement, both $\Diamond\varphi$ and $\bigcirc\varphi$ are insensitive **I2**, if φ is, thus overcoming the temporal asymmetry discussed for insensitivity **I1**. On the other hand, formulas of the form $\Box\varphi$ should not be considered, in general, as insensitive **I2**, since the infinite interpretation of such formulas, when lifted from a finite trace, depends on how the finite trace is

Formula	**I1**	**I2**
φ (φ no temporal operators)	Y	Y
$\neg\varphi$ (φ insensitive)	Y	N
$\varphi \wedge \psi$ (φ, ψ insensitive)	Y	Y
$\varphi \,\mathcal{U}\, \psi$ (φ, ψ insensitive)	N	Y
$\Box\varphi$ (φ insensitive)	N	N

Table 1. Comparison between **I1** and **I2**

extended. Notice, however, that concepts and formulas equivalent to (respectively) $\Box\top$ and $\Box(\top \sqsubseteq \top)$, where $\top$ and $\top \sqsubseteq \top$ are clearly insensitive **I2**, are insensitive **I2** as well. Table 1 summarizes differences and similarities between **I1** and **I2**. Finally, notice that both **I1** and **I2** definitions can be easily adapted to finite traces with fixed length $k > 0$. This allows us to compare the cases of nested next operators. For example, we have that $\bigcirc^n \neg A(a)$ is **I1** iff $n < k$. On the other hand, $\bigcirc^n \neg A(a)$ turns out to be insensitive **I2** for both $n < k$ and $n \geq k$, since in the latter case the **I2** condition is vacuously satisfied.

6 Conclusion

We presented $T^f_{\mathcal{U}}\mathcal{ALC}$, a TDL obtained by combining $\mathcal{ALC}$ with LTL_f. Firstly, we have focussed on the complexity of the $T^f_{\mathcal{U}}\mathcal{ALC}$ formula satisfiability problem. Despite a result of ExpSpace-completeness for the general case, coinciding with the complexity of the same problem over infinite traces, we have shown how to improve it to NExpTime, when considering finite traces with a fixed number k of instants, and even to ExpTime, if additionally we consider the language restricted with only global CIs, namely $T^k_{\mathcal{U}}(g)\mathcal{ALC}$. We leave the complexity of satisfiability in $T^f_{\mathcal{U}}(g)\mathcal{ALC}$ as an open problem. In order to further refine these results, we plan to consider temporal *DL-Lite* logics interpreted over finite traces.

Concerning the section on model-theoretical properties, we have shown that the logic $T^f_{\mathcal{U}}\mathcal{ALC}$ has the bounded model property. Moreover, we have studied the notion of insensitivity to infiniteness, both as found in the literature (**I1**), and as re-defined here (**I2**). We have shown their characterisations, and sufficient conditions to preserve **I2**. As future work, we plan to refine our results, so to cover the case of CIs with temporal operators applied on concepts, that are nonetheless insensitive **I2** (such as $\top \sqsubseteq \Diamond A$). A characterisation of insensitive CIs could in turn be based on a suitably defined notion of insensitivity for concepts. We leave this task for future work, together with an in depth discussion of the interaction between temporal operators (including additional ones, as *weak next* $\bullet$, *release* $\mathcal{R}$, or *weak until* $\mathcal{W}$) and insensitivity.

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