To quit or to cruise? Modeling parking search decisions based on serious games

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Abstract. Knowing when a driver will quit cruising and either leave the area or park at an expensive off-street facility is critical for modeling parking search. We employ a serious game – PARKGAME for estimating the dynamics of drivers' decision making. 49 Participants of a game experiment were involved in three scenarios where they had to arrive on time to a fictional appointment or face monetary penalties, and to choose between uncertain but cheap on-street parking or a certain but costly parking lot. Scenarios diverged on the time to appointment and distance between the meeting place and parking lot locations. Players played a series of 8 or 16 computer games on a Manhattan grid road network with high on-street parking occupancy and nearby parking lot of unlimited capacity. Players' choices to quit or to continue search, as dependent on the search time, were analyzed with an accelerated-failure time (AFT) model. Results show that drivers are mostly risk-averse and quit on-street parking search very soon after potential loses begin to accumulate. The implications of game-based methods for simulation model development and sustainable parking policy are further discussed.

1 INTRODUCTION

Future automated vehicles will definitely simplify urban transportation and parking [1]. Until that happens, long search for parking is an inherent component of a car trip to the center of the city, with negative externalities including traffic congestion, and air and noise pollution [2]. Cruising typically involves time-to-money tradeoffs between certain but expensive parking at a paid and possibly distant off-street facility and uncertain yet usually cheaper on-street parking. Understanding driver behavior in response to on-street and off-street parking conditions and prices is a basic step on the way to sustainable parking policy.

Urban parking space is highly heterogeneous and adequate representation of drivers' parking search demands a high-resolution and spatially-explicit representation of cruising drivers and parking options. This can be achieved with Agent-Based models (ABM) [3]. Knowledge on individual driver parking behavior has the potential of turning ABM into a highly effective policy support tool. Significant efforts have been made to understand drivers' reaction to parking prices [6], yet we still lack a formal description of drivers' reaction to prominent factors such as the occupation rate, time stress and distance between parking place and destination.

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The goal of this paper is to experimentally establish models of individual parking search behavior in a highly occupied area, under the dilemma of the very uncertain and cheap on-street versus certain and expensive off-street parking. The models can be used to characterize agent-drivers in a spatially-explicit, empirically-based parking ABM, and thus improve our ability to study the collective consequences of parking policy. To this end, we study and analyze parking behavior based on gamified lab experiments with the PARKGAME Serious Game.

2 METHODOLOGY

2.1 PARKGAME serious game platform

Our experiments are performed with PARKGAME – a flexible serious game platform for studying parking search behavior and decision-making. The urban road and parking infrastructure in PARKGAME are represented by GIS layers of street links and parking lots in a standard *shapefile* format. On-street parking spots are constructed automatically by the game software at 4m distance from each other along the street links in line with the direction of traffic. Figure 1 presents the user interface of PARKGAME: On-street parking spots are presented to the player as green (vacant) or red (occupied) dots and a parking lot is represented by a larger circle, and the destination is marked by a red flag. A green arrow that appears above the car, represented by a blue rectangle, functions as a virtual compass and points the driver in the direction of the destination.

The player navigates – advances, accelerates, decelerates and takes turns using the keyboard arrow keys. The field of view is only 5 parking spaces ahead at any moment; spots further ahead remain colorless until the driver approaches them. Although other cars competing with the player for free spots are currently not included in the interface, the effect of other cruising drivers is indirectly represented in the game by a random turnover process whereby on-street parking spots are randomly occupied and vacated at a preset rate.

The player can only park at a vacant spot with a maximum speed of 12 km/h, similar to real-life conditions [5]. A slider on the top right corner of the screen changes from green to red when the speed is too high for safe parking. The player parks the car by pushing the SPACE button and this ends the game. The software then calculates the walking distance from the selected spot to the destination. Based on the preset walking speed, it then computes the walking time and adds it to the total time of the game.

In the game, players are expected to attend a fictional meeting in T_a minutes from the start of the game. They start the game with a fixed budget B, out of which the on-street C_{on} or lot C_{off} parking costs are deducted, based on the eventual parking choice. Lot parking is always available but, to roughly reflect local conditions, at double the price of parking on-street, $C_{off} \sim 2*C_{on}$. Players cruise for parking, park the car and walk to the destination at a constant speed of 3.6 km/h = 1m/sec. The total game time is calculated as the sum of the search time and walk to the destination. If players reach the destination later than T_a minutes from the start of the game, they are fined based on a per-minute lateness rate L_{minute}. Thus as in reality, the goal of the player is to find parking quickly and close to the destination. The maximum allowed cruising time is $T_m > T_a$ and the player that still cruises at T_m is considered to have decided to park at a lot at a moment T_m, in which case we overlook the time of driving to the lot. The on-street and lot parking costs C_{on} and C_{off}, as well as the remaining time until the meeting, the walking time from the current position of the car to the destination and the per-minute late fine L_{minute}, are presented to the player on the screen (figure 1). The game administrator's UI enables modifying key game parameters. The output of the game includes a detailed log of all the decisions taken by the player during the game.

2.2 Experiment design, participants and procedure

In pilot experiments, it became clear that in realistic irregular street layouts, cruising is strongly affected by the network topology and one-way traffic, resulting in confounding effects with topology-enforced wayfinding. For this reason the experiment was performed on a Manhattan-like city grid of 10X8 blocks. Each link is considered as two-way traffic, 90m long with 20 parking spots on each side. The on-street occupancy rate r was set very high, to r = 99.75%, enforcing long cruising for on-street parking. Every 15 seconds, several spots were assumed to be occupied by "other" drivers an identical number of randomly selected occupied spots were vacated. Lot parking was always available.

The starting point in all games is 315 meters from destination, equivalent to ca 1-minute drive at the maximum allowed speed of 30km/h (figure 2). This starting point is far enough from the destination to distinguish between the start of a game and start of the parking search, and close enough to avoid unnecessary navigation. The parking prices and lateness fee used in the experiments are presented in table 1.

The parking lot was always located down the road beyond the destination from the perspective of the player's starting position and direction. The maximum allowed search time T_m was 9 minutes in all scenarios.

Cruising behavior of drivers was tested in three scenarios (figure 2, table 2). In scenario A, the lot was located 45m from the destination, and the time until the expected meeting was 3:00 min. At a walking speed of 1m/sec, the walk between the parking lot and destination took 0:45 min. Scenarios B and C were devised for studying the influences of the parking lot's location on cruising behavior. The distance between the destination and the lot in these scenarios is 135m and thus W_{off} = 2:15 min. Players had less time to search for on-street parking in scenario B than in scenario A, while in scenario C, the additional walk is seemingly neutralized by increasing T_a from 3:00 to 4:30 minutes.

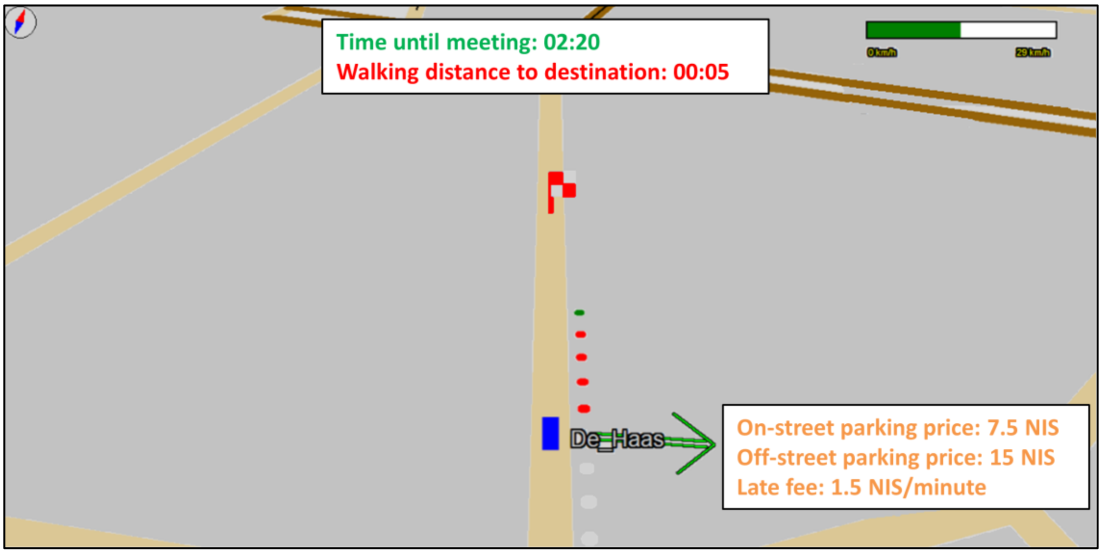

Figure 1. Game screenshot (translated from Hebrew): The player is about to reach the destination (red flag) in a Tel Aviv street, and observes a vacancy five parking places ahead (small green circle). Details important for decision-making are presented on the screen.

Table 1. Cost-related parameters used in the game experiments

	Parameters				Derived parameters	
	Initial budget B	Lot parking price C_{off}	On-street parking price C_{on}	Penalty for one minute delay L_{minute}	On-street parking maximum gain $G_{on,max} = B - C_{on}$	Off-street parking, Maximum gain $G_{off,max} = B - C_{off}$
Value (ILS)	20	15	7.5	1.5	20 - 7.5 = 12.5	20 - 12.5 = 7.5

Table 2. Description of PARGKAME scenarios and their parameter values

Scenario	Time of the meeting T_a	Distance from parking lot to destination D_{off} (m)	Safe (until fined) cruising duration T_s	Walk from the lot to destination w_{off} (min)	Number of participants
A	3:00	45	2:15	0:45	49
B	3:00	135	0:45	2:15	10
C	4:30	135	2:15	2:15	10

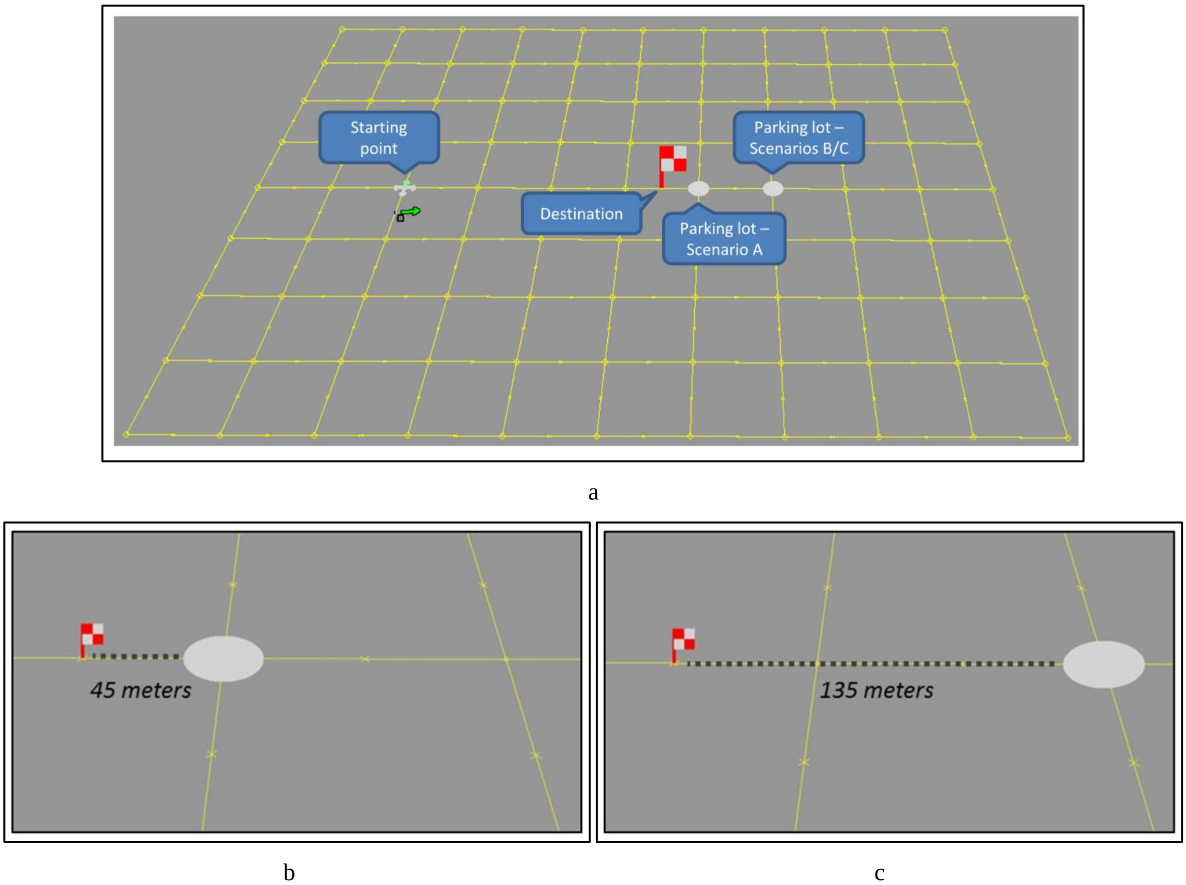

Figure 2. View of the game area (a) and zoom to the destination (red flag) and lot (grey circle) in Scenarios A (b) and B/C (c)

49 participants (30 men and 19 women) holding a valid driving license between the ages of 19 to 67 (Avg. = 32, STD = 11) were recruited through an online ad to participate in the experiment. The participants arrived at the lab after registering online and were randomly divided into game sessions of up to 4 players per session, based on their availability that day. The sessions took place in the computer lab and were run from August 2017 to May 2018.

After arriving to the computer lab, participants were provided a show up fee of ILS30 and signed a mandatory consent form. They sat individually, each in front of the computer with 21 inch screens. All 49 participants played scenario A. In addition, 10 randomly chosen players also played scenario B and another 10 participants played scenario C. Five players of each of the latter groups started out with scenario A and continued with B or C and five played in an opposite order - B or C and then A.

After a 10-minute oral briefing of the game mechanism and scenarios details, players filled in a pre-test questionnaire regarding their parking habits as well as basic socio-demographic data. Following this they participated in a training session of 4 consecutive games playing scenario A. The objectives of this session were to practice the use of the keyboard, and to get used to the on-street parking availability observed in the game. Players participating in pilot sessions were debriefed and shown videos of their movements in order to ensure that key decisions were correctly represented in the game.

Following the training session, the experimental session commenced. At the end of an experimental session, cumulative rewards were tallied up and granted to players.

3 RESULTS – TEMPORAL DECISION MAKING

Cruising drivers make two types of decisions at junctions. The first decision is whether to continue the search for the uncertain yet cheaper on-street parking or head to the certain but more expensive lot. The second decision is where to drive, that is whether to approach, remain at the same distance or recede from the destination and/or parking lot. In this paper we focus on the former choice, and leave the latter for later examination.

Overall, players parked at the lot in 59% (231 out of 392) of the games in scenario (A), 69% (55 of 80) in scenario (B) and 51% (41 of 80) in scenario (C). Every player parked at the lot at least once during the series of 8 games of a certain scenario. The minimal number of times a player parked at the lot in a scenario is 2 and the maximal is 8. On average, players park at the lot in 4.7 of 8 games in scenario A, 5.5 of 8 games in scenario B and 4.1 of 8 games in scenario C, with STDs of 1.2, 1.6 and 1.2 respectively. The differences between the distributions of the number of lot choices in the three scenarios are insignificant ($\chi^2 = 3.32$, $p > 0.1$) according to a Kruscal-Wallis test.

To study the dependence of the player's decision to quit cruising and park at the lot we applied actuarial survival analysis [6], where survival means continuing cruising for on-street parking. Formally, the survival is relevant for games where players parked at the lot before reaching the maximal game time T_m, whereas games concluding with on-street parking and the two games where players cruised for the entire game time T_m without succeeding to park are considered as "right-censored".

Figure 3a presents the Kaplan-Meier survival curves – i.e. the probability S(t) to continue cruising, in scenarios A, B and C. The horizontal axis represents the time t in seconds, and the vertical axis shows the fraction of drivers still cruising at time t. The log-rank test suggests the scenarios' survival curves differ substantially, ($\chi^2 = 39.98$, df = 2, $p < 0.001$). Figure 3b presents the corresponding kernel-smoothed instantaneous hazard function $h(t) = d/dt\ \log(S(t))$ for the three scenarios that reflects the instantaneous rate to park off-street at t if a player failed to park on street before.

As evident, the hazard rates h(t) are non-linear in all three scenarios: They grow from the start of the game and until shortly after the time of the meeting, and many games end with the player parking off-street and paying no fine or a minor fine. The hazard rates then decrease reflecting players who take the risk of being late and continuing cruising despite the fine time.

To assess the influence of scenario parameters on players' choice to quit on-street parking search we employ parametric hazard models. Namely, we fit an accelerated-failure time (AFT) model, the analytical form of which is $h(t|\boldsymbol{Z}) = h_0(t)e^{\boldsymbol{\beta Z}}$, where $\boldsymbol{Z}$ is a vector of covariates, $h_0(t)$ is the baseline hazard that is, the hazard function assuming all components of $\boldsymbol{Z}$ are zero, and β is a vector of coefficients to be estimated.

We compare four parameterizations of the basic hazard function $h_0(t)$: Lognormal, Log-logistic, Weibull and Exponential and consider the time until meeting and distance between parking lot and destination as covariates. Akaike's Information Criterion (AIC) is applied to compare goodness of fit for different parameterizations [7].

As can be seen in table 3, the log-logistic and the lognormal models provide the best and similar approximation of the experimental data and both generate hazard functions that fit very well to those presented in figure 3b. The Weibull model and the exponential model are the worst.

The analytical form of the hazard function that is based on the best approximating log-logistic hazard is as follows

$$h(t) = \frac{\lambda^{\frac{1}{\gamma}} t^{\left[\left(\frac{1}{\gamma}\right)-1\right]}}{\gamma\left[1+(\lambda t)^{\left(\frac{1}{\gamma}\right)}\right]} \qquad (1)$$

resulting in survival function of the form

$$S(t) = \left\{1 + (\lambda t)^{\frac{1}{\gamma}}\right\}^{-1} \qquad (2)$$

where $\lambda = e^{\boldsymbol{\beta Z}} = e^{\sum \beta_i Z_i}$, Z_i are covariates, and β_i and γ are estimated from the data. If $1/\gamma > 1$, the conditional hazard first rises and then falls, and if $1/\gamma < 1$, it declines monotonously.

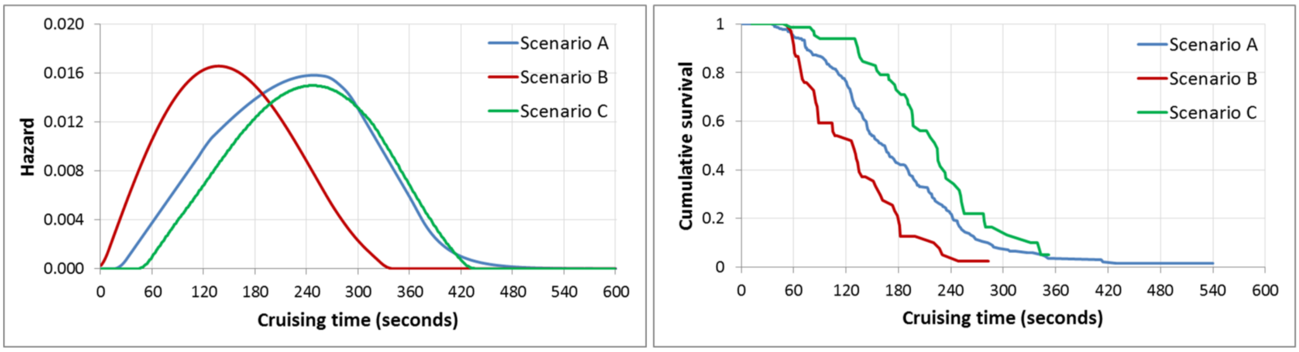

Figure 3. Kaplan-Meier curves (a) and kernel-smoothed hazard rates (b) scenarios A, B and C

Table 3. AIC value for four parametric survival models

Parameterization	Log likelihood	AIC
Lognormal	-308.8	625.5
Log-logistic	-307.9	623.8
Weibull	-321.9	651.7
Exponential	-472.7	951.3

Table 4. Log-logistic AFT regression model output

	Coef.	Std. Err.	Z	p
meet_time	0.0070	0.001	6.94	< 0.001
lot_dist	-0.0036	0.0007	-4.60	< 0.001
constant	3.988	0.1668	23.91	< 0.001
ln γ	-1.253	0.0443	-28.52	< 0.001

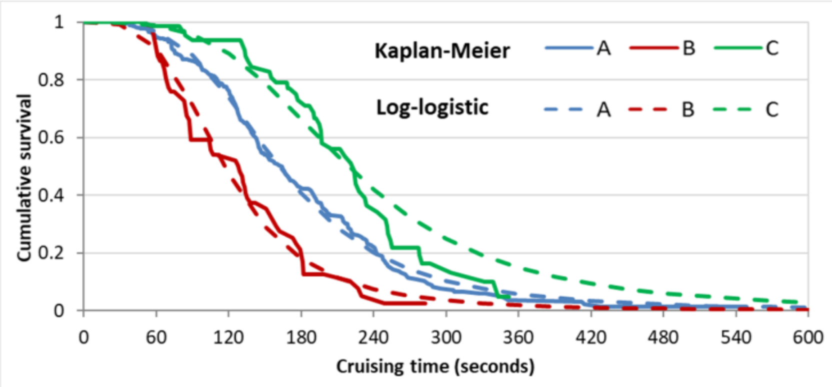

Figure 4. Kaplan-Meier curves (a) and kernel-smoothed hazard rates (b) scenarios A, B and C

The parameters' estimates for the log-logistic model are presented in table 4 and Wald statistic (z), indicates that the influence of both covariates is highly significant ($p < 0.001$).

As can be seen, $1/\gamma = e^{1.25282} \cong 3.5$, indicating non-monotonous unimodal hazard function, as in Figure 3b. The positive value of β for the *meet_time* indicates longer on-street search when the time between the start of the game and the meeting increases, while negative β for lot_dist covariate indicates shorter search in case the lot is farther away from the destination.

Given the Log-logistic hazard model, the empirical equation for λ is, thus

$$\lambda = e^{-(3.99823 - 0.0036 \times lot\_dist + 0.00701 \times meet\_time)} \quad (3)$$

and the overall survival function is

$$S(t) = \{1 + (\lambda t)^{3.5}\}^{-1} \quad (4)$$

while the hazard - conditional probability to decide to quit the on-street search and park at the lot, is given by

$$h(t) = \frac{\lambda^{3.5} t^{2.5}}{0.28571 \times [1 + (\lambda t)^{3.5}]} \quad (5)$$

Figure 4 shows the log-logistic fitted survival curves compared to the empirical Kaplan-Meier estimates. As evident the observed fit is very good.

According to (3), a marginal one-second increase in the time until meeting (*meet_time*) increases λ by $e^{0.0071}$ ~ 0.007 while a marginal one-meter increase in the distance between the lot and the destination (*lot_dist*) that is an additional one second walk decreases λ by only half i.e. $e^{-0.0036}$ ~ 0.0035. This makes sense as the distance between the lot and the destination is important in case of parking at the lot only, while the time until meeting always affects the game's outcome.

4 IS CRUISING FOR PARKING RISKY?

Experimental data make it possible to investigate an issue critically important for parking modeling: Do drivers decide to quit the search based on the instantaneous stress of being late or is there a general search strategy that they apply? To answer this question, we propose a theoretically optimal model of player behavior and compare it to the experimental results

Consider a series of games that start at a time moment 0, of duration T_m and assume that the player is cruising at the speed that is close to the maximal possible (30 km/h) and decreases the speed to the parking limit of 12 km/h immediately upon noticing a vacant on-street spot. In this case, the time necessary to traverse a 90m street link is close to 10 seconds and the appointment time T_a and maximum allowed game time T_m can be considered in 10-sec time steps. In the model below *on-street parking at t* is defined as "finding a vacant parking spot by the end of time step t" and *parking at a lot at t* is defined as "parking at the lot at the beginning of time step t".

Parameters of the model are as follows: initial game budget B, the cost of parking on street C_{on} and on the lot C_{off} and fine L_{10} per additional 10-second delay, $L_{10} = L_{minute}/6$. For the average occupation rate r, the probability to find on-street parking while traversing a random link with its 20 parking spots (that takes a time step of 10 seconds) can be estimated as

$$p = 1 - r^{20} \quad (6)$$

The accumulated late fine for arriving at the destination is denoted below as L(t), and counts down starting from the T_a. For the driver arriving at the destination at time-step t, it is

$$L(t) = \begin{cases} 0 & if\ t \le T_a \\ L_{10} \times (t - T_a) & if\ t > T_a \end{cases} \quad (7)$$

We assume that a player that cruises until the end of the game (T_m) and fails to park on-street, parks at the lot at the beginning of time step $T_m + 1$, pays the lot cost, walks to the destination from the lot and pays the maximal late fine calculated as $L(T_m + 1 + w_{off})$.

The probability of failing to find a vacant on-street parking spot during the time interval [0, t] is:

$$(1 - p)^t \quad (8)$$

The gain of a player who parked on-street, if cruised during the time interval [0, t] and parked by the end of a time step t is:

$$G_{on}(t) = B - C_{on} - L(t + w_{on}) \quad (9)$$

The gain of a player who parked at the lot, if cruising during a time interval [0, t] and then parked at the lot at the beginning of time step t + 1 is:

$$G_{off}(t) = B - C_{off} - L(t + w_{off}) \quad (10)$$

For our experiment design, the walk time after parking off-street is w_{off} = 45 sec = 4.5 time-steps in scenario A and w_{off} = 135 sec = 13.5 time steps in scenarios B and C. The value of w_{on} evidently varies between drivers and in what follows employ the rough value of w_{on} = 2 min = 12 time steps in all three scenarios.

4.1 Optimal strategy of a rational player

A perfectly rational player is assumed to choose a strategy, depending on the game parameters, on the cruising duration of for finding on-street parking that results in a maximal possible gain M. Based on (8) – (10), the gain M(t) from unsuccessful cruising during [0, t], and parking at the lot at t + 1 is:

$$M(t) = \left(\sum_{\tau=1}^{\tau=t} p \times (1 - p)^{\tau-1} \times G_{on}(\tau)\right) + (1 - p)^t \times G_{off}(t + 1) \quad (11)$$

In all three scenarios lot parking price exceeds on-street parking price. This is the major reason why $dM(t)/dt$ is always positive, and thus M(t) monotonously increases in all three scenarios. This holds true even if we assume the highest observed w_{on} of 200 sec. The optimal strategy of a rational player in all scenarios is therefore to cruise until the very end of the game. The optimal strategy is especially rewarding considering each player participated in a *series* of 8 or 16 games. For the exploited values of parameters, the average gain (11) of an optimally behaving player will be between 9 – 10 ILS over 8 games, depending on the scenario.

Players that did not follow the optimal strategy presented above may be considered as "myopic" that is, sensitive to the events during the game and deciding anew, depending on the course of a game, whether to continue cruising or quit and park at the lot. In the latter case, their decisions are based on the accumulated search time t and the experience gained during previous games.

To understand these players' choices, we consider a player who searched for on-street parking unsuccessfully during the time interval [0, t], $t < T_m$. The average gain K(t, Δt), Δt ≥ 1, $t + \Delta t \le T_m$, from the decision, at t, to cruise until t + Δt, and park at the lot at t + Δt + 1, is equal to:

$$K(t, \Delta t) = \left(\sum_{\tau=0}^{\tau=\Delta t-1} p \times (1 - p)^{\tau} \times G_{on}(\tau + 1)\right) + (1 - p)^{\Delta t} \times G_{off}(t + \Delta t + 1) \quad (12)$$

Dependence of $K(t, \Delta t)$ on t and Δt for the scenario A is presented in figure 5, and is similar for scenarios B and C.

In figure 5, each curve starts at a different t and represents $K(t, \Delta t)$ - the gain of a player, searching unsuccessfully until t, if they continue searching for additional time Δt. For each t, the entire curve $K(t + 1, \Delta t)$ is below the curve $K(t, \Delta t)$ and eventually $K(t, 0)$ becomes negative. In addition, for the values of t for which $K(t, 0)$ is negative, the time Δt_1 that is necessary to return to a positive-reward state of $K(t, \Delta t_1) > 0$, increases. That is, the gain from "cruising a bit longer" for on-street parking decreases throughout the course of the game. "Myopic" and, thus, bounded-rational players, unlike their rational and "strategic" counterparts, may interpret this as the potential reward from a long and unsuccessful search that gradually diminishes regardless of which course of action they choose. Eventually they become discouraged from very long cruising and head to the lot prematurely. According to the results presented in section 3, this is what indeed happens in our game experiments. Namely, the players' behavior is myopic and they cancel their search soon after the fine period starts (Figure 3). None of the players followed optimal strategy and only in 2 out of 552 games players played until the very end of the game.

5 DISCUSSION

As we have demonstrated, PARKGAME players' behavior can be considered risk-averse. They do not follow the optimal strategy that is to search until the end of the game. Instead, when the fine for being late starts to grow the probability to quit on-street search and park off-street grows as well. Shortly after that, the hazard function peaks and then starts to decline (figure 3). That is, despite general risk aversion tendency, some players in certain games may behave in a risk seeking (and optimal) manner and, despite accumulating losses, decide to search up to the very end of the game. No player behaved in this optimal risk seeking way over several games. Thus further experiments are needed to investigate the decline of the hazard function. It should be noted that this decline may well be considered a game artifact: players were aware that the total loss is limited and, thus, additional loss from searching to the last minute or two of a game was not substantially high. However, recognition of this effect demands a different organization of the experiment.

The choice of when to quit cruising on-street and head to a parking lot is well approximated by the accelerated-failure time model with the log-logistic hazard function. The parameter γ of the log-logistic function is essentially larger than 1 reflecting the hazard function with a maximum soon after the time at which a late fee for lot-parking starts to accumulate, while parameter λ of the accelerated-failure time model increases as the meeting time approaches and decreases if the distance between the destination

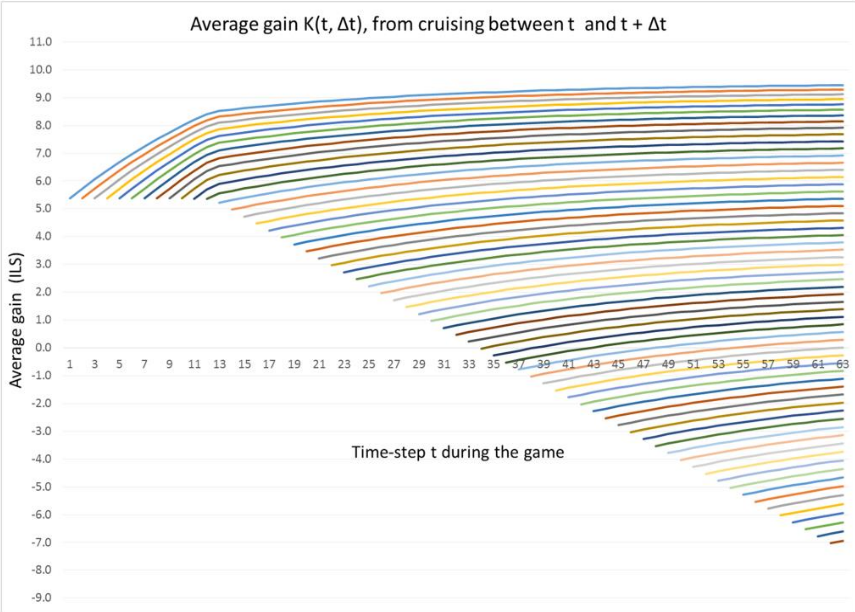

Figure 5. Average gain K(t, Δt), when cruising between t and t + Δt and then parking at the lot for scenarios A (a

and parking lot increases. That is, in a very intuitive manner, the shorter the time to the meeting and larger the distance between the destination and the off-street lot, the higher the probability becomes to quit cruising and park at the lot (figure 4).

The revealed rules of drivers' parking decisions can be incorporated into an agent-based parking simulation model. An advantage of the Log-logistic hazard (1) – (2) equations is in the estimated coefficients that can serve for the model's initial parameters. Then, the modeler can investigate the consequences of stronger or weaker reactions of drivers to the time- and distance related factors by varying the parameters of these analytical rules. This approach of game-based modeling can benefit the reliability of policies and services established using the model. It is especially relevant in the context of parking search, where empirical studies are scarce and little is known about the dynamics of the process.

The choice of whether and when to quit cruising and head to the expensive parking lot or continue searching for cheaper on-street parking is one of two major component of driver's parking behavior. The second major decision that of the search path. We leave it for an additional paper.

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REFERENCES

[1] A. Mllard-Ball, 'The autonomous vehicle parking problem', *Transport Policy*, **75**: 99-108, (2019).

[2] E. Inci, J.V. Ommeren, and M. Kobus, 'The external cruising costs of parking', *Journal of Economic Geography*, **17(6)**: 1301-1323, (2017).

[3] I. Benenson and P. Torrens, *Geosimulation: Automata-based modeling of urban phenomena*, John Wiley & Sons, Hoboken, 2004.

[4] S. Lehner and S. Peer, 'The price elasticity of parking: A meta-analysis', *Transportation Research Part A: Policy and Practice*, **121**: 177-191, (2019).

[5] S. Carrese, B. B. Bellés, and E. Negrenti, 'Simulation of the parking phase for urban traffic emission models', In *TRISTAN V – Triennial Symposium on Transportation Analysis*, Guadeloupe, (2004).

[6] D.G. Kleinbaum and M. Klein, *Survival analysis*, Springer, New York, 2010.

[7] H. Akaike, 'A new look at the statistical model identification', In *Selected Papers of Hirotugu Akaike*, 215-222, Springer, (1974).