Instantaneous Real-Time Kinematic Decimeter-Level Positioning with Galileo and BDS-3 Penta-Frequency Signals Over Long-Baseline

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Abstract

To take full advantages of Galileo and BDS-3 penta-frequency signals, a long-baseline RTK positioning method based on Galileo and BDS-3 penta-frequency ionosphere-reduced (IR) combinations is proposed. First, the high-quality signals with low-noise and weak-ionospheric delay characteristics of Galileo and BDS-3 are analyzed. Second, the multi-frequency extra-wide-lane (EWL)/ wide-lane (WL) combinations with long-wavelengths are constructed. Third, the IR-EWL combinations are calculated by geometry-free (GF) method, then the resolved IR-EWL combinations are used to constrain the IR-WL, of which the ambiguities can be obtained in a single epoch. There is no need to consider the influence of ionospheric parameters in the third step because the ionospheric delay factors of IR-EWL/WL combinations are close to 0. Compared with the estimated ionosphere model, the proposed method can improve the availability of positioning and reduce the number of parameters by half and the required operation time is greatly reduced. Therefore, it reduces the dimension of parameter estimation and is suitable for the use of multi-frequency and multi-system real-time RTK. The results using real data show that stepwise fixed model of the IR-EWL/WL combinations can realize long-baseline instantaneous decimeter-level positioning.

Keywords

Galileo, BDS-3, Penta-frequency, Ionosphere-reduced, RTK positioning, Long-baseline

1. Introduction

European Galileo Satellite Navigation System and China Beidou-3 Global Satellite Navigation System have been launched and broadcast penta-frequency signals. At present, Galileo broadcasts E1 (1575.42 MHz), E5a (1176.45 MHz), E5b (1207.14 MHz), E5 (1191.795 MHz) and E6 (1278.75 MHz) and BDS-3 broadcasts B1C (1575.42 MHz), B1I (1561.098 MHz), B3I (1268.52 MHz), B2a (1176.45 MHz), B2b (1207.14 MHz) [1]. Taking advantage of multi-system and multi-frequency signals, in addition to increasing observation redundancy, it can also construct some observation value combinations with excellent characteristics such as long wavelength, weak ionospheric delay and low noise factor. And the performance of the positioning solution can be greatly improved [2].

The application of multi-frequency signals combinations was proposed by Forssell and Jung. The basic principle is the classical GF method to fix the ambiguities of EWL, WL and narrow lane (NL) step by step according to the difficulty of ambiguity resolution [3]. Feng researched and gave the integer combination coefficients to reduce the influence of the ionosphere in the process of ambiguity resolution at all levels of GPS/Galileo/BDS-2 EWL/WL/NL combinations [5]. Gao studied the ionosphere-reduced NLs of BDS-3 and Galileo penta-frequency, the model strength and positioning

ICL-GNSS 2022 WiP, June 07–09, 2022, Tampere, Finland
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accuracy of ambiguity resolution are improved compared with the traditional dual-frequency ionosphere-free combination [6]. Based on the new frequency signals of Galileo and BDS-3, we studied the selection criteria for the combination of ionosphere-reduced observations suitable for long baselines. The optimal ionosphere-reduced combinations of Galileo and BDS-3 penta-frequency signals are analyzed. The estimated ionosphere model is used to compare the positioning performance with the IR model, and the decimeter-level positioning of Galileo and BDS-3 penta-frequency minimum number of parameters to be estimated for long-baseline RTK positioning is realized.

The purpose of this article is to study multi-frequency combination signals of Galileo and BDS-3 with ionospheric delay factor close to 0 and low combined observation noise. With the comparation of IR model and estimated ionosphere model, positioning performance and positioning efficiency of IR model is studied. In the following section, we define the conditions to be met by IR-EWL/WL combinations, and provide the geometric model required by the algorithm. Then experiments were conducted using a set of long baseline data. Finally, the experimental results are analyzed and summarized.

2. Penta-frequency Observation Combination Model of Galileo and BDS-3

2.1. Double Difference (DD) Mathematical Model

Ignore satellite systems, the observation equation of the basic pseudo-range and carrier-phase observations is:

$$\begin{cases} \Delta\nabla P_i = \Delta\nabla\rho + \eta_i \Delta\nabla I_1 + \Delta\nabla T + \Delta\nabla\varepsilon_P \\ \Delta\nabla\phi_i = \Delta\nabla\rho - \eta_i \Delta\nabla I_1 + \Delta\nabla T + \lambda_i \Delta\nabla N_i + \Delta\nabla\varepsilon_\phi \end{cases} \tag{1}$$

where, the symbol " $\Delta\nabla$ " represents DD operation; P_i and ϕ_i represent pseudo-range and carrier observations, respectively; ρ is the geometric distance between the satellite and the receiver; I_1 is the first-order ionospheric delay at the first frequency; η_i is the first order ionospheric scale factor; T indicates tropospheric delay; ε_P and ε_ϕ are observation noise of pseudo-range and carrier-phase respectively; N indicates integer ambiguity; λ is the carrier wavelength.

Correspondingly, the DD observation equation of the penta-frequency signals after basic observation equation linear combination is:

$$\Delta\nabla\phi_{(i,j,k,m,n)} = \Delta\nabla\rho - \eta_{(i,j,k,m,n)}\Delta\nabla I_1 + \Delta\nabla T + \lambda_{(i,j,k,m,n)}\Delta\nabla N_{(i,j,k,m,n)} + \Delta\nabla\varepsilon_{\phi(i,j,k,m,n)} \tag{2}$$

where, each parameter is expressed as:

$$\Delta\nabla\phi_{(i,j,k,m,n)} = \frac{i \cdot f_1 \cdot \Delta\nabla\phi_1 + j \cdot f_2 \cdot \Delta\nabla\phi_2 + k \cdot f_3 \cdot \Delta\nabla\phi_3 + m \cdot f_4 \cdot \Delta\nabla\phi_4 + n \cdot f_5}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4 + n \cdot f_5} \tag{3}$$

$$\eta_{(i,j,k,m,n)} = \frac{f_1^2 \left(i/f_1 + j/f_2 + k/f_3 + m/f_4 + n/f_5 \right)}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4 + n \cdot f_5} \tag{4}$$

$$\lambda_{(i,j,k,m,n)} = \frac{c}{i \cdot f_1 + j \cdot f_2 + k \cdot f_3 + m \cdot f_4 + n \cdot f_5} \tag{5}$$

$$\mu_{(i,j,k,m,n)} = \frac{\sqrt{(if_1)^2 + (jf_2)^2 + (kf_3)^2 + (mf_4)^2 + (nf_5)^2}}{f_{(i,j,k,m,n)}} \tag{6}$$

where, i,j,k,m,n is the combination coefficient; Correspondingly, the calculation of pseudo-range combination $\Delta\nabla P_{(i,j,k,m,n)}$ is similar to that of $\Delta\nabla\phi_{(i,j,k,m,n)}$; $\eta_{(i,j,k,m,n)}$ is the ionospheric scalar factor of the combined signal; $\lambda_{(i,j,k,m,n)}$ is the wavelength of combined observations; $\mu_{(i,j,k,m,n)}$ is the noise coefficient of combined observations; c is the speed of light.

2.2. Selection of optimal IR-EWL combination

The combination of penta-frequency EWL combination can construct infinite combined observations according to different coefficient values, but most signals do not have the characteristics of low-noise and weak-ionospheric scale factor. Referring to literature [6], this paper selects the IR-EWL/WL combinations for long-baseline positioning, and makes the following constraints on the characteristics of combined observations:

(1) The influence of ionospheric delay on ambiguity resolution is less than 0.02 cycles in unit, which can be expressed as:

$$\beta_{(i,j,k,m,n)} = \frac{f_1^2\left(i/f_1 + j/f_2 + k/f_3 + m/f_4 + n/f_5\right)}{c} \tag{7}$$

The ionospheric delay corresponding to 5 m has less than 0.1 cycles on ambiguity resolution. If the impact on ranging is less than 5 cm, it is required. In fact, the ionospheric residuals for the 100-500 km baseline double-differences are less than 1.5m [5].

(2) If the combined noise is required to be small, the combined coefficient of the combination value should not be too large. Taking the GPS EWL combination (1, 6, -5) as the reference (103.80), the noise amplification coefficient shall not be greater than 110;

(3) The wavelength of the combined observation value shall not be too small or too large. The combination wavelength shall be between 0.8m and 10m with reference of GPS triple-frequency WL combination the twice (1, -1, 0) and (0, 1, -1).

Based on the above three conditions, take [-10, 10] as the search interval of combination coefficient, the characteristics of combinations meeting the above conditions are shown in Table 1. The sequence of corresponding Galileo and BDS-3 signal types in the Table 1 is: E1/E5a/E5b/E5/E6 and B1C/B1I/B3I/B2a/B2b. It can be seen from the table that there are five EWL/WL combinations of BDS-3 and four EWL/WL combinations of Galileo that meet the conditions, of which BDS-3 (-1, 2, -4, 1, 2) and Galileo (1, 4, 1, -3, 3) are EWL combinations and the rest are WL combinations. Therefore (-1, 2, -4, 1, 2) of BDS-3 and (1, 4, 1, -3, 3) of Galileo are selected as the optimal IR-EWL combinations.

Table 1
Ionosphere-reduced WL combinations for BDS-3 and Galileo

Combination Coefficient	$\lambda_{(i,j,k,m,n)}$ /m	$\beta_{(i,j,k,m,n)}$ / $\mathrm{cycle \cdot m^{-1}}$	$\eta_{(i,j,k,m,n)}$	$\mu_{(i,j,k,m,n)}$
BDS-3				
(-1, 2, -4, 1, 2)	4.7266	-0.0023	-0.0005	105.9863
(-1, 3, -6, 6, -2)	2.1236	0.0058	0.0027	83.2113
(6, -4, -7, 6, -1)	1.6651	-0.0048	-0.0029	79.9460
(-5, 8, -9, 8, -2)	1.5588	-0.0094	-0.0060	109.4091
(-2, 5, -10, 7, 0)	1.4653	0.0033	0.0023	84.5970
Galileo				
(1, 4, 1, -3, -3)	3.9074	-0.0012	-0.0003	66.5653
(2, 7, 1, -4, -6)	1.9537	-0.0057	0.0029	89.6363
(3, 2, -2, 7, -10)	1.3630	0.0094	0.0069	74.2670
(3, 1, -3, 9, -10)	1.3630	0.0062	0.0046	80.7461

2.2.1. Selection of optimal IR-EWL combinations using GF model

Equation (8) calculates the IR-WL by using the combination of GIF:

$$\Delta\nabla N_{(i,j,k,m,n)} = \left[\frac{\Delta\nabla P_{[a,b,c,d,e]} - \Delta\nabla\phi_{(i,j,k,m,n)}}{\lambda_{(i,j,k,m,n)}}\right] \tag{8}$$

$$\Delta\nabla P_{[a,b,c,d,e]} = a\Delta\nabla P_1 + b\Delta\nabla P_2 + c\Delta\nabla P_3 + d\Delta\nabla P_4 + e\Delta\nabla P_5 \tag{9}$$

where, $[\bullet]$ represents the rounding operator, $\Delta\nabla P_{[a,b,c,d,e]}$ is the linear combination form of DD observations of pseudo-range combination, and the combination coefficient of pseudo-range a,b,c,d,e is any real number. As shown in equation (10-12), considering that the sum of pseudo-range coefficients is 1, the sum of ionospheric scale factor of combined pseudo-range observations and IR is 0, and the combined noise is the smallest, the optimal pseudo-range coefficient can be calculated by the minimum norm method, as shown in Table 2. It should be noted that only up to five significant figures are displayed in the table.

$$a+b+c+d+e=1 \tag{10}$$

$$\beta_{(i,j,k,m,n)}+a+b\cdot\frac{f_1^2}{f_2^2}+c\cdot\frac{f_1^2}{f_3^2}+d\cdot\frac{f_1^2}{f_4^2}+e\cdot\frac{f_1^2}{f_5^2}=0 \tag{11}$$

$$a^2+b^2+c^2+d^2+e^2=\min \tag{12}$$

$$\sigma_{\Delta\nabla N_{(i,j,k,m,n)}}=\frac{\sqrt{\frac{(if_1)^2+(jf_2)^2+(kf_3)^2+(mf_4)^2+(nf_5)^2}{f_{(i,j,k,m,n)}^2}\sigma_{\Delta\nabla\phi}^2+(a^2+b^2+c^2+d^2+e^2)\sigma_{\Delta\nabla P}^2}}{\lambda_{(i,j,k,m,n)}} \tag{13}$$

where, $\sigma_{\Delta\nabla\phi}$ and $\sigma_{\Delta\nabla P}$ represent the DD noise of non-combined carrier observation value and pseudo-range observation respectively, and the values in this paper are 0.5 m and 5 mm respectively.

The optimal pseudo-range coefficient combination of each IR combination is brought into respectively, and the rounding success rate of ambiguities of IR-WL P_s is calculated by equation (14) [8].

$$P_s\left(-0.5<x<0.5\right)=\int_{-0.5}^{0.5}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(x-\delta\right)^2}{2\sigma^2}\right)dx \tag{14}$$

$$\delta_{GF}=\frac{K_{ijkmn,abdce}\Delta\nabla I}{\lambda_{(i,j,k,m,n)}},\quad K_{ijkmn,abdce}=\beta_{(i,j,k,m,n)}+\beta_{(a,b,c,d,e)} \tag{15}$$

where, δ_{GF} is the systematic deviation caused by unmodeled errors. When calculating the ambiguities of GIF EWL combinations, the first-order ionospheric delay, tropospheric delay and satellite orbit error are eliminated. Therefore, the ambiguity accuracy is only affected by observation noise and second-order ionospheric delay. Here, the influence of second-order ionospheric delay can be ignored, so δ_{GF} can be regarded as 0, while $K_{ijkmn,abdce}=0$.

It can be seen from Table 2 that BDS-3 (-1, 2, -4, 1, 2) combination can obtain a rounding success rate of 100% and Galileo (1, 4, 1, -3, 3) combination can obtain a rounding success rate of 99.56%. Therefore, (-1, 2, -4, 1, 2) and (1, 4, 1, -3, 3) combination is only affected by pseudo-range noise, and under the condition of good observation accuracy, IR-EWL combinations can be fixed by rounding in a single epoch.

Table 2

Optimal combination of pseudo-range coefficients of ionosphere-reduced WL combinations and success rate by rounding

Combination Coefficient	Pseudo-range Coefficient Combination a,b,c,d,e					Ambiguity Accuracy /cycle	Success Rate by Rounding/%
BDS-3							
(-1, 2, -4, 1, 2)	1.2140	1.1685	-0.1227	-0.7409	-0.5190	0.103	100.0
(-1, 3, -6, 6, -2)	1.2198	1.1742	-0.1245	-0.7463	-0.5232	0.228	97.16
(6, -4, -7, 6, -1)	1.2122	1.1668	-0.1221	-0.7392	-0.5178	0.290	91.50
(-5, 8, -9, 8, -2)	1.2089	1.1637	-0.1210	-0.7361	-0.5154	0.310	89.35
(-2, 5, -10, 7, 0)	1.2180	1.1724	-0.1239	-0.7446	-0.5219	0.330	86.98
Galileo							

(1, 4, 1, -3, -3)	2.2153	-0.6790	-0.3506	-0.5116	0.3260	0.175	99.56
(2, 7, 1, -4, -6)	2.2094	-0.6765	-0.3490	-0.5096	0.3256	0.351	84.63
(3, 2, -2, 7, -10)	2.2153	-0.6790	-0.3506	-0.5116	0.3260	0.351	84.59
(3, 1, -3, 9, -10)	2.2250	-0.6833	-0.3532	-0.5150	0.3266	0.504	67.92

2.2.2. Selection of optimal IR-EWL combinations using GB model

Selecting IR-EWL combinations can make full use of pseudo-range observation data while using GB model. Referring to literature [7], using the estimated ionosphere model to calculate EWL combinations. The corresponding model is:

$$A=\begin{bmatrix} 1 & \eta_1 & 0 \\ 1 & \eta_2 & 0 \\ 1 & \eta_3 & 0 \\ 1 & \eta_4 & 0 \\ 1 & \eta_5 & 0 \\ 1 & -\eta_{EWL} & \lambda_{EWL} \end{bmatrix}, R=\begin{bmatrix} \sigma_P^2 & & & & & \\ & \sigma_P^2 & & & & \\ & & \sigma_P^2 & & & \\ & & & \sigma_P^2 & & \\ & & & & \sigma_P^2 & \\ & & & & & \sigma_{EWL}^2 \end{bmatrix} \tag{16}$$

$$P = R^{-1}, Q = \left(A^T PA\right)^{-1} \tag{17}$$

where, A is the design matrix, and the corresponding estimation parameters are station satellite distance accuracy, ionosphere accuracy and ambiguity accuracy; R is the corresponding observation noise variance covariance matrix; $\sigma_{EWL}^2 = \eta_{EWL}^2 \sigma_\phi^2$ is the observation noise of EWL; The matrix Q is the variance covariance matrix of parameter estimation, which reflects the accuracy of parameter estimation, and its diagonal element is the variance of estimated parameters. The IR-EWL combinations is affected by the unmodeled atmospheric residual and orbit error when GB model used. Specifically, δ in equation (14) can be calculated by equation (18):

$$\delta_{GB} = \frac{\Delta\nabla orb + \Delta\nabla T - \eta_{(i,j,k,m.n)} \Delta\nabla I}{\lambda_{(i,j,k,m.n)}} \tag{18}$$

Referring to literature [5], it is assumed that under the conditions of medium and long-baseline, the tropospheric residuals are 10 cm and 15 cm respectively, and the first-order ionosphere residuals are 80 cm and 100 cm respectively. The corresponding ambiguity accuracy and success rate used GB model are shown in Table 3.

Table 3

BDS-3 and Galileo optimal GB-IR ambiguity accuracy and success rate by rounding

Combination Coefficient	Ambiguity Accuracy /cycle	Success Rate by Rounding/%		cm
		$\Delta\nabla T=0, \Delta\nabla I=0$	$\Delta\nabla T=10, \Delta\nabla I=80$	$\Delta\nabla T=15, \Delta\nabla I=100$
BDS-3				
(-1, 2, -4, 1, 2)	0.232	96.91	96.81	96.71
(-1, 3, -6, 6, -2)	0.494	68.84	68.49	68.16
(6, -4, -7, 6, -1)	0.634	56.97	56.69	56.41
(-2, 5, -10, 7, 0)	0.705	52.16	51.93	51.70
(-5, 8, -9, 8, -2)	0.717	51.44	51.14	50.85
Galileo				
(1, 4, 1, -3, -3)	0.333	86.68	86.52	86.36
(2, 7, 1, -4, -6)	0.653	55.62	55.43	55.24
(3, 2, -2, 7, -10)	0.934	40.74	40.56	40.38
(3, 1, -3, 9, -10)	0.940	40.52	40.35	40.18

Regarding of estimated ionosphere method, the GB model reduces the model strength, which is equivalent to the ionosphere-fixed model [8]. Although the BDS-3 IR-EWL combination (-1, 2, -4, 1,

2) or Galileo IR-EWL combination (1, 4, 1, -3, -3) cannot obtain 100% success rate by using GB model, if four linearly independent EWL combinations are found, the ambiguities of IR-EWL combinations can be obtained by using linear combinations. Unfortunately, only three groups of linearly independent EWL combinations with high ambiguity accuracy can be obtained. Therefore, the linear combination method is not suitable to provide the success rate of IR-EWL combinations.

2.3. Selection of calculation model for EWL/WL combinations

Therefore, the IR-EWL combinations can be obtained directly by rounding using GF model.

In this paper, the IR-EWL combinations of BDS-3 or Galileo is calculated by GF method. As a comparison, with reference to equation (19), which estimates ionospheric parameters with low-noise EWL combinations:

$$\begin{bmatrix} \boldsymbol{v}_{P_1} \\ \boldsymbol{v}_{P_2} \\ \boldsymbol{v}_{P_3} \\ \boldsymbol{v}_{P_4} \\ \boldsymbol{v}_{P_5} \\ \boldsymbol{v}_{EWL} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & \eta_1 \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & \eta_2 \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & \eta_3 \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & \eta_4 \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & \eta_5 \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & -\eta_{EWL} \boldsymbol{I}_s & \lambda_{EWL} \boldsymbol{I}_s \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{ion} \\ \boldsymbol{N}_{EWL} \end{bmatrix} - \begin{bmatrix} \boldsymbol{l}_{P_1} \\ \boldsymbol{l}_{P_2} \\ \boldsymbol{l}_{P_3} \\ \boldsymbol{l}_{P_4} \\ \boldsymbol{l}_{P_5} \\ \boldsymbol{l}_{EWL} \end{bmatrix} \tag{19}$$

After fixing EWL, the calculation of WL ambiguity is selected according to its combination characteristics. Generally, the fixed EWL combinations are used to restrict the ambiguity of WL combinations, as shown in equation (20).

$$\begin{bmatrix} \boldsymbol{v}'_{EWL} \\ \boldsymbol{v}_{WL} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & -\eta_{EWL} \boldsymbol{I}_s & \boldsymbol{0} \\ \boldsymbol{B} & -\eta_{WL} \boldsymbol{I}_s & \lambda_{WL} \boldsymbol{I}_s \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{ion} \\ \boldsymbol{N}_{WL} \end{bmatrix} - \begin{bmatrix} \boldsymbol{l}'_{EWL} \\ \boldsymbol{l}_{WL} \end{bmatrix} \tag{20}$$

The WL is constrained with IR-EWL by GB model, and ionosphere can be ignored because it is very little. Its estimation equation is:

$$\begin{bmatrix} \boldsymbol{v}'_{EWL} \\ \boldsymbol{v}_{WL} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{0} \\ \boldsymbol{B} & \lambda_{WL} \boldsymbol{I}_s \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{N}_{WL} \end{bmatrix} - \begin{bmatrix} \boldsymbol{l}'_{EWL} \\ \boldsymbol{l}_{WL} \end{bmatrix} \tag{21}$$

It can be seen that using the IR-EWL combination to restrict the IR-WL combinations does not need to estimate the ionospheric parameters, and the dimension of parameter estimation can be reduced.

2.4. Selection of IR-WL combinations

Based on equation (21), calculate the DD float ambiguities of the remaining four IR-WL combinations in Table 1 after ambiguities of BDS-3 IR-EWL combination (-1, 2, -4, 1, 2) and Galileo IR-EWL combination (1, 4, 1, -3, -3) are fixed. The solution accuracy and the fixed DD ranging accuracy are shown in Table 4.

Table 4

Ambiguity accuracy of IR-WL combinations constrained by fixed EWL combination BDS-3 IR-EWL combination (-1, 2, -4, 1, 2) and Galileo IR-EWL combination (1, 4, 1, -3, -3)

Combination Coefficient	$\lambda_{(i,j,k,m,n)}$ /m	WL/cycle	DD Range/m
BDS-3			
(-1, 3, -6, 6, -2)	2.124	0.161	0.381
(6, -4, -7, 6, -1)	1.665	0.360	0.342
(-5, 8, -9, 8, -2)	1.559	0.212	0.451
(-2, 5, -10, 7, 0)	1.465	0.161	0.381
Galileo			
(2, 7, 1, -4, -6)	1.954	0.047	0.317

(3, 1, -3, 9, -10)	1.363	0.408	0.318
(3, 2, -2, 7, -10)	1.363	0.362	0.319

It can be seen that the accuracy of WL combination Galileo (2, 7, 1, -4, -6) is the best and the float accuracy of WL is also the best, so it is selected as the optimal IR-WL combination of Galileo. DD Range of BDS-3 (6, -4, -7, 6, -1) is best after fixed, but the float accuracy of which is the worst, while the float accuracy of WL combination BDS-3 (-1, 3,-6, 6, -2), BDS-3 (-2, 5, -10, 7, 0) are best. Although DD range of WL combination BDS-3 (-1, 3,-6, 6, -2), (-2, 5, -10, 7, 0) is not optimal, it is close to optimal. In addition, the ionospheric scale factor of WL combination BDS-3 (-1, 3, -6, 6, -2) is smaller, so it is selected as the optimal IR-WL combination of BDS-3.

3. Experiment and analysis

In this paper, a group of 189.4 km long-baseline of IGS station are used for the experiment. The data comes from TIT2 and FFMJ stations of BKG data center. The observation date is UTC time, October 1, 2021 (24 hours), day of year is 274, and the sampling interval is 30 s. During the calculation, the cut-off angle of the satellite in the calculation is set to 15°.

The number of BDS-3 and Galileo satellites with five frequencies and their RDOP in this period are shown in Figure 1. The number of common view satellites of the two stations fluctuates in the range of 8-14 mostly. In the 2279th epoch, rdop increased sharply due to the small number of visible satellites. Figure 2 shows the sky plots of BDS-3 and Galileo various satellites in the experiment.

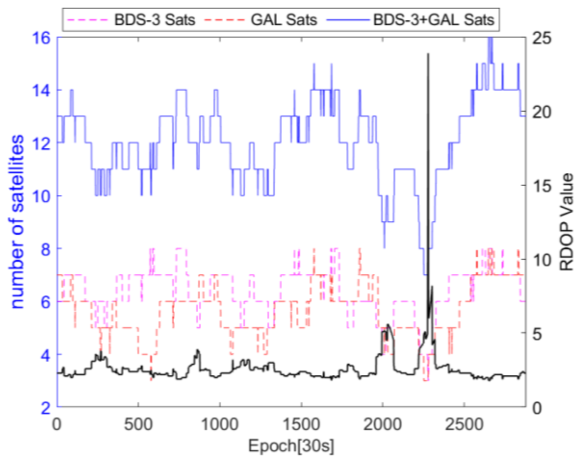
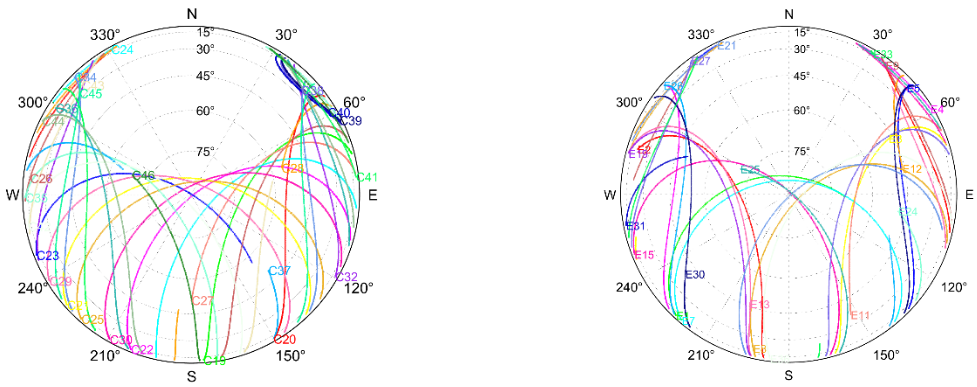


Figure 1: Number of satellites and RDOP value of BDS-3 and Galileo in full time



(a) (b)

Figure 2: Sky plots for the various satellites of BDS-3 (a) and Galileo (b)

Figure 3 and Figure 4 shows the fraction bias of EWL combinations ambiguities using estimated ionosphere model and IR-GF model, respectively. Different colors correspond to different satellite pairs.

It can be seen that the estimated ionosphere model can be all within 0.25 cycles and can be reliably rounded and fixed.

However, either BDS-3 or Galileo, the accuracy of IR-EWL ambiguities calculating by GIF method is poor because of greater pseudo-range noise, which affects the ambiguity accuracy. Therefore, the influence of the ionosphere can be properly ignored to reduce the noise of pseudo-range observations. This paper only analyzes the case without pseudorange noise reduction.

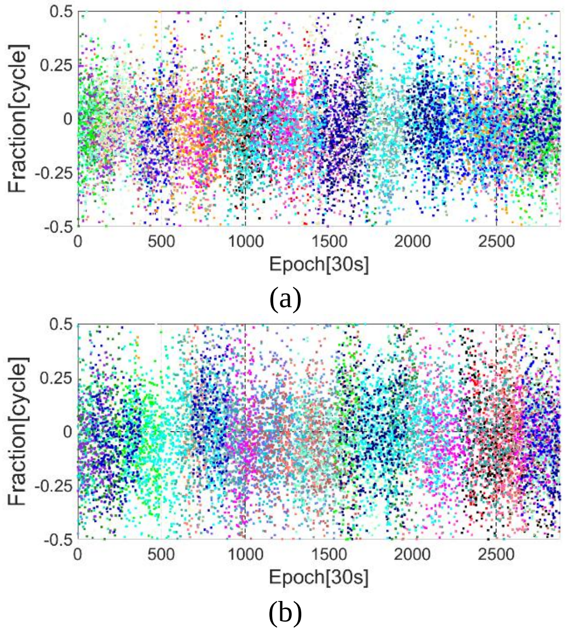


Figure 3: The fractions of EWL ambiguity of IR-GF model: BDS-3 (a) and Galileo(b)

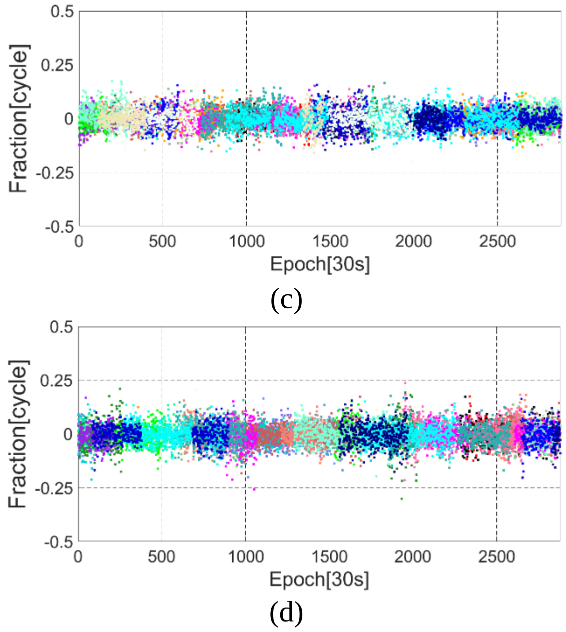


Figure 4: The fractions of EWL ambiguity of estimated ionosphere model: BDS-3 (c) and Galileo(d)

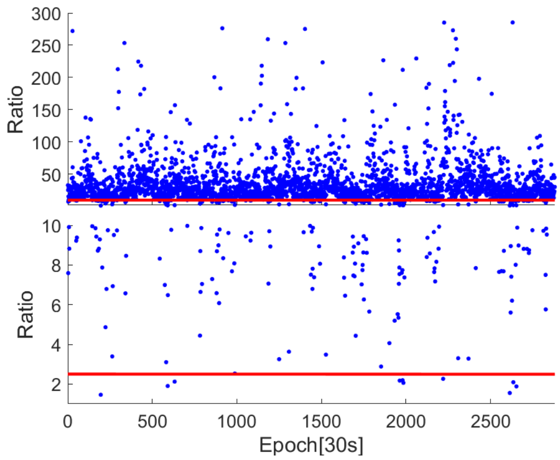
The true values of IR-EWL ambiguities are all obtained by multi-epoch filtering. It can be seen from the Table 5 that although the accuracy of IR-EWL ambiguity calculated by GIF model is not high, the rounding reaches 99.04% (BDS-3) and 97.89% (Galileo). In the experiment, the threshold of decimal deviation is set as 0.3 when rounding EWL.

Table 5

The ambiguity accuracy of BDS-3 and Galileo IR-EWL combinations calculated by pseudo-range with optimal coefficients.

Pseudo-range Coefficients					Noise Factor	EWL Fractions/cycle				%
a	b	c	d	e		>0.5	<0.5	<0.4	<0.3	<0.2
BDS-3										
1.214	1.169	-0.123	-0.741	-0.519	1.916	0.96	99.04	97.06	91.02	75.58
Galileo										
2.215	-0.679	-0.351	-0.512	0.326	2.420	2.11	97.89	94.64	86.77	70.07

The WL ambiguities are calculated by IR-GB model and estimated ionosphere model and the suboptimal/optimal ambiguity variance ratio (Ratio value) is shown in Figure 5 and Figure 6. In the figure, the values corresponding to the red lines in the upper and lower figures are 10 and 2.5 respectively. It can be seen that ratio value of IR-GB model is greater than that of estimated ionosphere model. The reason is that IR-GB model does not need to estimate the ionospheric delay, so the strength of the parameter estimation model is greater.



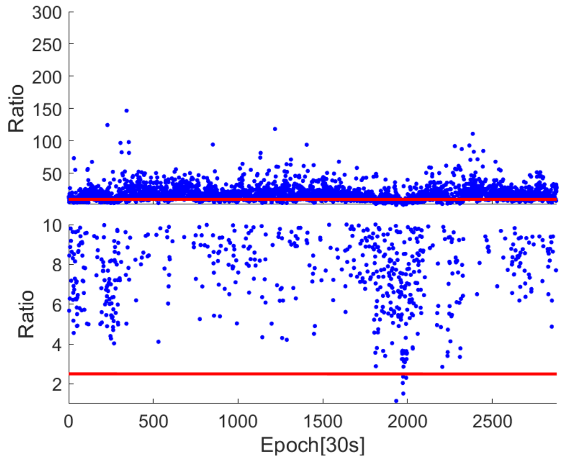
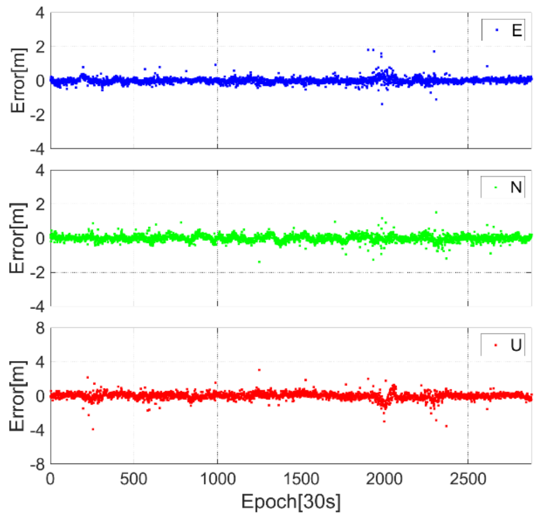


Figure 5: The Ratio of WL ambiguity in IR-GB model

Figure 6: The Ratio of WL ambiguity in estimated ionosphere model

The threshold for LAMBDA estimation of WL ambiguities is 0.2 cycles. If the number of WL ambiguity float solutions satisfying the condition is less than 4, the positioning result of this epoch is considered invalid. After the WL ambiguities are fixed, the observation equations are brought back to obtain the coordinate solution under the fixed solution. The positioning error of the corresponding solution coordinates in the East (E), North (N) and Up (U) directions is shown in Figure 7 and Figure 8.



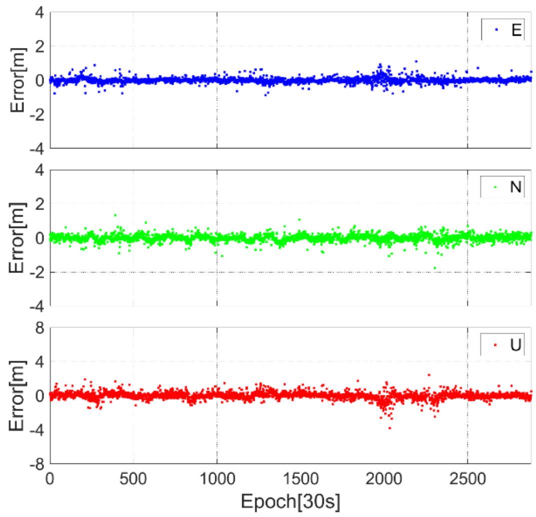


Figure 7: The positioning results in IR-GB model

Figure 8: The positioning results in estimated ionosphere model

Finally, the positioning accuracy statistics of the two methods are shown in Table 6. It can be seen that the accuracy of the IR-GB model and that of the estimated ionosphere model is basically the same. However, the IR does not need to estimate the ionospheric delay term, so it can achieve higher ambiguity calculation efficiency. Especially in the multi-level step-by-step resolution of multi-system ambiguity, it will be more obvious. Figure 9 and Table 7 shows the operation time of these two models. It can be seen that the operation time of the IR model is significantly lower than that of estimated ionosphere model, which is consistent with the analysis.

Table 6
Statistics of the positioning results with EWL/WL observations

Positioning Model	N/m	E/m	U/m	Positioning success rate %
Estimated ionosphere	0.159	0.206	0.408	98.4
Ionosphere-reduced	0.168	0.191	0.375	99.5

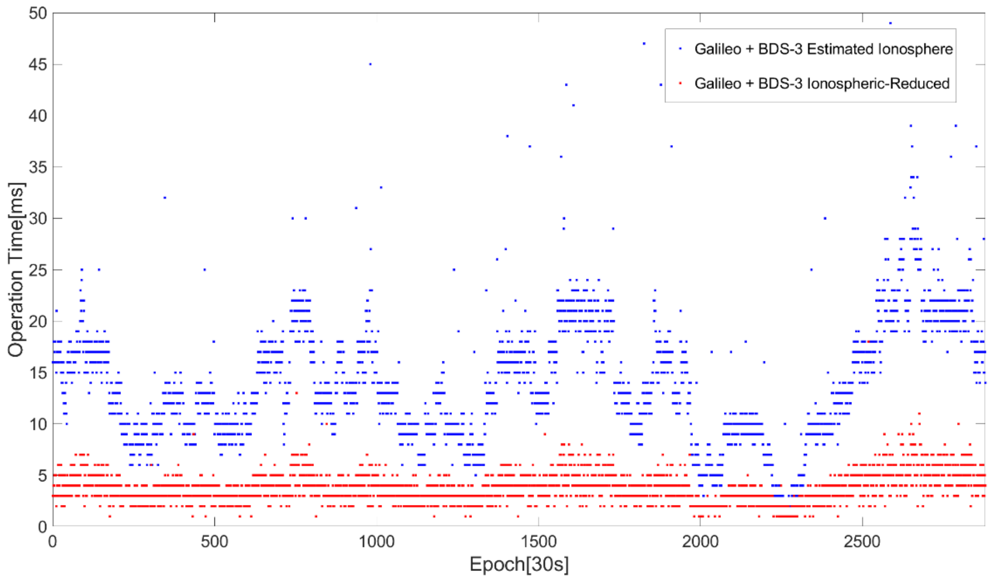


Figure 9: Operation time of the IR model (red) and estimated ionosphere model (blue)

Table 7
Statistics of operation time of the IR model and estimated ionosphere model

Positioning Model	Mean/ms	Max/ms	Min/ms
Estimated Ionosphere	13.8	67	1
Ionosphere-reduced	3.7	18	<1

4. Conclusions

In this paper, a step-by-step method for fixing the ambiguities of IR-EWL/WL combinations is proposed. First, the IR-EWL combinations are calculated by GF method, then the fixed IR-EWL combinations are used to constrain the IR-WL combinations, of which the ambiguities can be obtained in a single epoch. The proposed IR model does not need to estimate the ionospheric delay, so the strength of the parameter estimation model is greater.

The main difference between the ionosphere-reduced model and estimated ionospheric model is the accuracy of the WL float ambiguities. Once the EWL of the IR model is successfully fixed, a positioning performance comparable to that of the estimated ionosphere model can be obtained.

The experiment shows that the positioning performance of the IR model is comparable to that of estimated ionosphere model (0.168/0.191/0.375m vs. 0.159/0.206/0.408m), and positioning performence is more effective (99.5% vs. 98.4%) and the computation time is shorter (3.7ms vs. 13.7ms).

The performance of the ratio depends on the strength of the model. The ionosphere-reduced model is essentially an ionospheric-fixed model, which reduces the number of parameters to be estimated to improve the model and obtain a higher-precision float solutions.

5. Acknowledgements

IGS MGEX is gratefully acknowledged for providing Galileo and BDS-3 data. This research is supported by the National Key R&D Program of China (No. 2021YFC3000502); the National Natural Science Foundation of China (No. 41904022); and the Foundation of Laboratory of Science and Technology on Marine Navigation and Control, China State Shipbuilding Corporation (No. 2021010104).

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