Enabling Coexistence of Cognitive Vehicular Networks and IEEE 802.22 Networks via Optimal Resource Allocation

You Han*, Eylem Ekici*
*Department of Electrical and Computer Engineering
The Ohio State University, Columbus, Ohio 43210
Email: han.639@osu.edu, ekici@ece.osu.edu

Haris Kremo†, Onur Altintas †
†Department of Research and Development
Toyota InfoTechnology Center Co., Ltd, Tokyo, Japan
Email: {hkremo, onur}@jp.toyota-itc.com

***Abstract*—Many studies show that the Dedicated Short Range Communication band is insufficient to carry increasing wireless data traffic in vehicular networks. The release of large TV spectra by FCC for cognitive access provides additional spectrum resources to solve the spectrum scarcity problem. However, FCC allows fixed devices to use high transmitting powers, while requiring portable devices to use significantly lower powers. This power asymmetry policy leads to a challenging coexistence environment for portable (e.g., vehicular) and fixed (e.g., IEEE 802.22) networks. In this paper, we address the coexistence problem between vehicular and 802.22 networks via resource allocation. We show that the problem is an NP-hard mixed-integer nonlinear programming problem, to which we propose two algorithms. First, we convert it to a convex programming problem, and propose a near-optimal primal-dual algorithm. Next, we reformulate the problem as a packing problem, and present a constant-factor approximation algorithm. Finally, we evaluate the algorithms through numerical examples.**

I. Introduction

Recent studies show that the Dedicated Short Range Communication band allocated to vehicular networks is unlikely to meet the bandwidth demands of emerging wireless applications in vehicular networks [1]. Although the newly released TV White Space (or "TVWS") band is a promising spectrum expansion for vehicular networks, it also raised many novel challenges, one of which is the power asymmetry. Since portable TVWS devices (like vehicular devices) are only allowed to use at most 100 mW transmit power, while fixed devices (like IEEE 802.22 devices) can use up to $1W$ transmit power, the coexistence problem must be carefully resolved. Otherwise, 802.22 devices can easily starve vehicular devices in the TVWS spectrum.

To the best of our knowledge, this is the first work to deal with the coexistence issue between 802.22 networks (or Wireless Regional Area Networks) and cognitive vehicular networks (CVNs). So far, most existing works on TVWS access have been dedicated to the coexistence between Secondary User (SU) networks and Primary User (PU) networks, while the coexistence of heterogeneous SU networks has not received significant attention. A critical difference between SU-SU coexistence and PU-SU coexistence is that SUs have similar (if not identical) priorities for spectrum access in the former scenario while PUs have a dominant priority over the spectrum in the later scenario.

Although IEEE 802.22 includes an inter-network coexistence scheme, it is dedicated to separately managed 802.22 networks. IEEE 802.19.1 also proposed a framework for the coexistence of heterogeneous SU networks. However, it heavily relies on various coexistence entities, which makes it impractical to facilitate coexistence between mobile CVNs and fixed 802.22 networks. Even though some coexistence mechanisms have been proposed to enable the coexistence of heterogeneous networks in the ISM band [2], they neither deal with the power asymmetry problem, nor include cognitive functionality. Finally, novelty of our work also lies in that we consider features of the IEEE 802.22 standard as well as FCC's rules on TVWS access, which makes our work more relevant for implementation.

In this paper, we propose a 802.22-CVN coexistence framework via optimal resource allocation. Our work was motivated by the observation that, 802.22 customer premise equipment devices (CPEs) use relatively low transmit power in upstream frames, which leaves significant spectrum opportunities. Vehicles can spatially reuse the spectrum opportunities without causing unacceptable interference to 802.22 upstream transmissions. Therefore, in this paper, cooperative "coexistence" of CVN and 802.22 networks is defined as follows. The 802.22 network agrees to share its upstream scheduling information with the CVN such that vehicles would not cause unacceptable interference to its upstream transmissions. The CVN needs to know the 802.22 scheduling information, such that its own transmissions would not be starved by 802.22 transmissions. In return, CVN promises not to cause unacceptable interference to 802.22 upstream transmissions.

The theoretical contributions of our work are three-fold:

1) We formulate the coexistence problem between a

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CVN and 802.22 network as a resource allocation problem, which is proved to be an NP-hard Mixed-Integer Nonlinear Programming (MINLP) problem. Both features of the 802.22 standard and FCC's rules on TVWS access are considered in the problem formulation.

2) Since MINLP problems are usually intractable, our second contribution is to convert the MINLP problem to an equivalent convex programming problem. Then we propose a primal-dual method to acquire a near-optimal solution to the initial MINLP problem.
3) After reformulating the NINLP problem as a k-column-sparse packing problem, we present a constant-factor approximation algorithm with more favorable complexity.

The remainder of the paper is organized as follows. The System model and the problem formulation are described in Section II. In Section III, we propose a primal-dual method and analyze its performance. Then, in Section IV, the coexistence problem is reformulated as a k-column-sparse packing problem, to which a probabilistic constant-factor algorithm is developed. We present numerical results in Section V and conclude our work in Section VI.

II. System Model and Problem Formulation

The IEEE 802.22 standard was developed for wireless regional area networks, to provide broadband access in low population density areas using TVWS spectra. According to the standard, each 802.22 network consists of one Base Station (BS) and multiple fixed or portable CPEs. Each 802.22 network has an operating channel with 6, 7 or 8 MHz bandwidth. In time domain, each 802.22 frame has 10 milliseconds. A frame is further dynamically partitioned into a downstream sub-frame and an upstream sub-frame by the BS. In upstream frames, the BS usually allocates different power levels to the CPEs based on their locations to save energy and protect PUs.

Note that mobility of vehicles is not incorporated in our system model for the following reason. An 802.22 upstream frame is less than 10 milliseconds, and our scheduling is conducted during these frames. Due to shortness of the period, vehicles can be assumed to be static during this period. For example, in USA, the freeway speed limit is 121 km/h, and thus a car can only move at mot 0.336m during our scheduling period. Therefore, we can assume that system parameters do not change in each scheduling period. In this sense, our model also applies to the coexistence of 802.22 networks with other static or mobile low-power ad hoc networks in TVWS. Moreover, our scheduling model is practical since it is compatible with the IEEE standard for Wireless Access in Vehicular Environments, in which each scheduling period is 50 milliseconds [3].

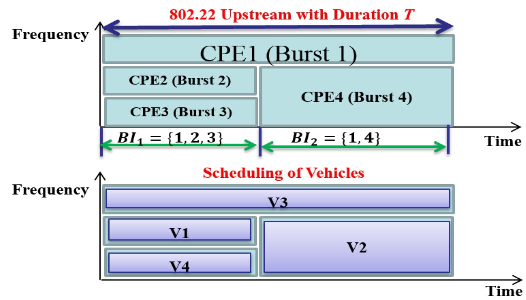

Figure 1. 802.22 upstream and scheduling of vehicles

Let N be the number vehicles and M be the number of CPEs in our system model. Our objective is to schedule a set of vehicles to every 802.22 upstream frame to maximize the total weighted throughput of the vehicular network. In the 802.22 standard, a "burst" is defined as a two-dimensional segment of OFDM sub-channels (frequency domain) and symbols (time domain). For example, in the upper figure of Fig. 1, there are four bursts allocated to four CPEs. According to the standard, there are two types of upstream bursts. Type 1 burst is mapped over the full upstream frame in time domain (e.g., burst 1 in Fig. 1), while normal type 2 burst is mapped over an interval with 7 upstream slots (e.g., burst 2 and 3 in Fig. 1). But if a type 2 burst is the last burst of an upstream, its duration can be 7 to 13 slots (e.g., burst 4 in Fig. 1) [4].

Based on the length of a normal type 2 burst, the upstream frame is split into "burst intervals" (BIs) in time domain, and let K be the number of BIs in the upstream frame. For example, there are two BIs in Fig. 1, where burst 1, 2 and 3 belong to BI_1 and burst 1, 4 belong to BI_2. Let $C_{jk} \in \{0, 1\}$ be a BI indicator, and $C_{jk} = 1$ if burst j belongs to BI k. FCC requires that total transmit power of portable SUs in a single TVWS channel must be less than a threshold (i.e., 100 mW/6MHz). Since in our system model, multiple vehicles would share a single 6 MHz channel, we require that total transmit power of vehicles must be less than that threshold in every BI. For example, in Fig. 1, let P_{max} be the power threshold, and thus both total transmit power of $V1, V3, V4$, and total transmit power of $V2, V3$ must be less than P_{max}. In our model, let T_j and B_j be the duration and bandwidth of burst j, respectively. In addition, let p^c_j be the transmit power of CPE j on burst j, and G_{ji} be the channel gain from CPE j to vehicle i. We assume that all channel gain information can be obtained by either vehicles' measurements or from the BS.

In IEEE 802.11p, vehicular packets are classified into four Access Categories with decreasing priority: $AC[0] \cdots AC[3]$. For example, safety related packets usually have higher priority than non-safety packets.

Therefore, in our model, we associate each priority class $AC[i]$ with a weight A_i subject to $A_i > A_j, \forall i < j$. In addition, we assume that Roadside Channel Monitors (RCMs) constantly sense and estimate usage patterns of PUs on all available TVWS channels. In our model, RCMs are also responsible for scheduling vehicles to 802.22 upstream frames. Although vehicular scheduling is not necessarily a 1-to-1 mapping to the 802.22 upstream scheduling, we assume that at most one vehicle can be allocated to each burst and the scheduled vehicle would transmit over the entire burst duration. This assumption is reasonable because: (1) vehicles need to transmit safety messages frequently; (2) the duration of each burst is rather short. In addition, the maximum transmit rate of vehicle i on burst j is determined by its transmitting power and the transmit power of CPE j, which can be calculated as $R_{ij} = B_j \log_2\left(1 + \frac{p_{ij}G_{ii}}{p_j^c G_{ji} + \sigma_0}\right)$, where p_{ij}, G_{ii} denote transmit power of vehicle i on burst j and channel gain from vehicle i to its receiver respectively, and σ_0 is the noise power spectrum density.

The TVWS occupancy of PUs is modeled through a random variable t_p, which is defined as residual time until the return of a PU to the 802.22 operating channel. We assume that probability density function (PDF) $f_j(t_p)$ of t_p is known to RCMs. Since exact behaviors of PUs are unknown to RCMs, vehicular transmissions scheduled on a burst can be interrupted by PU transmissions with non-zero probabilities. In our model, we assume that the transmissions of a vehicle before a PU returns are successful and all remaining packets are assumed to be lost (one possible reason is that high transmit power of PUs blocks vehicular transmissions). Therefore, for a vehicle scheduled to burst j, its expected transmission time is $\bar{T}_j = \int_0^\infty (T_j \cdot 1_{\{t_p \geq T_0 H_j\}} + (t_p - T_0(H_j - 1)) \cdot 1_{\{T_0(H_j-1) \leq t_p \leq T_0 H_j\}}) f(t_p) dt_p$, where T_0 is the length of a type 2 burst, and H_j denotes index of BI that burst j belongs to.

$$
\begin{aligned}
&\max_{\substack{x_{ij} \in \{0,1\} \\ p_{ij} \in [0, P_{max}]}} \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{1}{T} A_i x_{ij} R_{ij} \bar{T}_j \\
&\text{s.t.} \sum_{i=1}^{N} x_{ij} p_{ij} G_i^{BS} \leq \beta_j, \forall j \in \{1, 2 \cdots M\}, \\
&\sum_{i=1}^{N} \sum_{j=1}^{M} C_{jk} x_{ij} p_{ij} \leq P_{max}, \forall k \in \{1, 2 \cdots K\}, \\
&\sum_{j=1}^{M} x_{ij} \leq 1, \forall i; \sum_{i=1}^{N} x_{ij} \leq 1, \forall j,
\end{aligned} \tag{P.1}
$$

Given the above definitions, our objective is to maximize the weighted expected throughput (interchangeable with "utility") of vehicles via joint burst allocation and power control. Let $\{\mathbf{x} | x_{ij} \in \{0,1\}\}$ be assignment variables (i.e., $x_{ij} = 1$ if vehicle i is scheduled to burst j, and $x_{ij} = 0$ otherwise). The problem formulation is presented in P.1, where G_i^{BS} denotes the channel gain from vehicle i to the BS, and T represents the total length of the current upstream frame.

The first set of constraints mean that the interference caused by vehicular transmission on upstream transmission of CPE j is limited to β_j. The second set of constraint is to meet FCC's requirement that total transmit power of SUs on a TVWS channel must be less than a threshold in every BI. Recall that K denotes the number of BIs in the current upstream frame. The last two set of assignment constraints require that every vehicle be scheduled to at most one burst, and every burst be assigned to at most one vehicle. We can see that P.1 belongs to MINLP problems, which are generally NP-complete problems [5]. MINLP problems are usually solved by using numerical algorithms, such as branch and bound, Bender's decomposition, outer approximation and extended cutting plane [5]. Although most of these solutions are guaranteed to attain near-optimal solutions, their convergence rates are fairly slow.

III. Primal-Dual Method

The difficulty of MINLP problems usually results from integer constraints as well as non-convex nature of the problems. Given this observation, our proposed primal-dual method works as follows. First, all integer constraints of P.1 are relaxed to continuous constraints, i.e., $x_{ij} \in [0,1], \forall i, j$. Then the initial problem is converted to a convex programming problem (i.e., P.2) by replacing variables p_{ij} with new variables $y_{ij} = x_{ij} p_{ij}$. Then, P.2 is solved via a primal-dual method. More specifically, we first form the Lagrangian function of P.2 (i.e., Equation 3), and replace the last two sets of assignment constraints in P.1 with a domain constraint $\mathbf{X} := \left\{ \mathbf{x} | \sum_{j=1}^{M} x_{ij} \leq 1, \forall i; \sum_{i=1}^{N} x_{ij} \leq 1, \forall j; x_{ij} \in [0,1] \right\}$.

Then, we optimize over y_{ij} variables for given assignment variables as well as dual variables. By doing so, we are able to obtain the optimal power allocation policy, (i.e., all p_{ij}^* values). Then we plug these values back to the Lagrangian function, and proceed to optimize over all assignment variables x_{ij}. This problem turns out to be the linear programming version of the classic assignment problem, to which we are guaranteed to obtain an optimal "integer" solution in polynomial time. The last step is to optimize over all dual variables, i.e., solving dual problem of P.2. Since there is no closed-form solution to the assignment problem, the dual problem becomes a "non-smooth" convex optimization problem. By using an accelerated sub-gradient method [6], we are able to obtain a near-optimal solution to P.2 in polynomial time. Since x_{ij} are already integer solutions, the obtained solution is also a near-optimal solution to the P.1. Details of the primal-dual method are as follows.

A. Reformulate P.1 as a Convex Programming Problem

By relaxing the integer constraints and replacing all power variables with y_{ij}, we obtain the following new convex programming problem:

$$\max_{\substack{x_{ij}\in\mathbf{X}\\ y_{ij}\in[0,P_{max}]}} \sum_{i=1}^{N}\sum_{j=1}^{M}\frac{1}{T}A_i x_{ij} R_{ij}^{new}\bar{T}_j$$
$$\text{s.t.} \sum_{i=1}^{N} y_{ij}G_i^{BS} \le \beta_j, \forall j; \qquad \text{(P.2)}$$
$$\sum_{i=1}^{N}\sum_{j=1}^{M} C_{jk}y_{ij} \le P_{max}, \forall k,$$

where $y_{ij} = x_{ij}p_{ij}$, and R_{ij}^{new} are defined as

$$R_{ij}^{new} = \begin{cases} 0 & \text{if } x_{ij}=0\\ B_j\log_2\left(1+\frac{y_{ij}G_{ii}}{x_{ij}(p_j^cG_{ji}+\sigma_0)}\right) & \text{if } x_{ij}>0. \end{cases} \qquad (1)$$

Next we show some properties of P.2 by proving the following lemma.

Lemma 1: P.2 is a convex programming problem, and the Slater's condition holds.

Proof. First, we can see that domain of P.2 (i.e., $\mathbf{X}$ defined in the first paragraph of this section and $\mathbf{Y} := \{\mathbf{y}|y_{ij}\in[0,P_{max}]\}$) forms a convex set. Second, all functions in the constraints are also convex (linear). Finally, we need to prove that objective function of P.2 is concave. Since the objective function is sum of single utility functions $f_{ij}(x_{ij},y_{ij}) = \frac{1}{T}A_i x_{ij}R_{ij}^{new}\bar{T}_j$, the problem is reduced to show the concavity of $f_{ij}(x_{ij},y_{ij})$. Furthermore, since $f_{ij}(x_{ij},y_{ij}) = 0$ at $x_{ij} = 0$, we only need to consider the case where $x_{ij} > 0$. In this case, $f_{ij}(x_{ij},y_{ij})$ is the perspective function of a concave function

$$g_{ij}(y_{ij}) = \frac{1}{T}A_i\bar{T}_jB_j\log_2\left(1+\frac{y_{ij}G_{ii}}{p_j^cG_{ji}+\sigma_0}\right). \qquad (2)$$

Therefore, $f_{ij}(x_{ij},y_{ij})$ is also concave, which proves the first part of this lemma. The proof of satisfying Slater's Condition is trivial, which can be done by simply letting $x_{ij} = 0, y_{ij} = 0, \forall i,j$. In this case, we obtain strict inequalities in all constraints of P.2. ■

Now, we continue to solve P.2 by using a primal-dual method. The Lagrangian function of P.2 is as follows:

$$\begin{aligned} L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) = &\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{1}{T}A_i x_{ij}R_{ij}^{new}\bar{T}_j\\ &-\sum_{j=1}^{M}u_j\left(\sum_{i=1}^{N}y_{ij}G_i^{BS}-\beta_j\right)\\ &-\sum_{k=1}^{K}v_k\left(\sum_{i=1}^{N}\sum_{j=1}^{M}C_{jk}y_{ij}-P_{max}\right), \end{aligned} \qquad (3)$$

where $\mathbf{u}\ge\mathbf{0}, \mathbf{v}\ge\mathbf{0}$.

B. Optimize over $\mathbf{y}$ Given Assignment and Dual Variables

In this step, we intend to find $\mathbf{y}^*$ that maximizes the Lagrangian function, given the assignment variables $\mathbf{x}$ and dual variables $\mathbf{u},\mathbf{v}$. First, let us consider the case where $x_{ij} = 0$. In this case, vehicle i is not scheduled to burst j, and thus $y_{ij}^* = x_{ij}p_{ij} = 0$. For the case where $x_{ij} > 0$, the optimal y_{ij} can be obtained by letting the partial derivative of the Lagrangian function over y_{ij} be zero, i.e., $\frac{\partial L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v})}{\partial y_{ij}} = 0$. To make the expression of y_{ij}^* more concise and intuitive, we first define the following notations which are constants given assignment and dual variables:

$$a_{ij} = \frac{1}{T\ln 2}A_iB_j\bar{T}_j, b_{ij} = u_jG_i^{BS}+\sum_{k=1}^{K}v_kC_{jk}, \qquad (4)$$
$$c_{ij} = p_j^cG_{ji}+\sigma_0.$$

Then y_{ij}^* can be written as:

$$y_{ij}^* = x_{ij}p_{ij}^* = x_{ij}\left(\frac{a_{ij}}{b_{ij}}-\frac{c_{ij}}{G_{ii}}\right). \qquad (5)$$

Therefore, we can further conclude that the optimal power allocation policy given assignment and dual variables is

$$p_{ij}^*(\mathbf{x},\mathbf{u},\mathbf{v}) = \begin{cases} 0 & \text{if } x_{ij}=0\\ \frac{a_{ij}}{b_{ij}}-\frac{c_{ij}}{G_{ii}} & \text{if } x_{ij}>0. \end{cases} \qquad (6)$$

C. Optimize over Assignment Variables Given $\mathbf{y}$ and Dual Variables

In this step, we first plug the optimal power values into the Lagrangian function, and continue to find the optimal assignment policy $\{x_{ij}^*\}$. Before we solve this problem, we first introduce some new notations to make the problem formulation more concise. Define d_{ij} and E as

$$\begin{aligned} d_{ij} &= \frac{1}{T}A_i\bar{T}_jR_{ij}(p_{ij}^*) - p_{ij}^*b_{ij};\\ E &= \sum_{j=1}^{M}u_j\beta_j+\sum_{k=1}^{K}v_kP_{max}. \end{aligned} \qquad (7)$$

Then the problem of optimizing over assignment variables can be formulated as

$$\max_{\mathbf{x}\in\mathbf{X}}\sum_{i=1}^{N}\sum_{j=1}^{M}x_{ij}d_{ij} - E \qquad \text{(P.3)}$$

where both $\{d_{ij}\}$ and E can be viewed as constants given dual variables. This problem turns out to be linear programming version of the classic assignment problem, the objective of which is to assign a set of people to different jobs to maximize total benefit. It is proved in [7] that this linear programming problem has the following remarkable property: if it has a feasible solution at all, then it has an optimal integer solution, and the set of its optimal solutions includes all the optimal assignments. In other words, by solving this problem, we are guaranteed to obtain an optimal integer solution $\mathbf{x}^*$. There are many potential algorithms to solve this problem, such as the Hungarian algorithm whose time complexity is $O(n^3)$ [7] (in our case, $n = \max\{M, N\}$). Note that even though this problem can be solved efficiently, no closed form solutions exist.

D. Solve the Dual Problem

Since we have optimized over all primal variables of P.2 given dual variables, we are able to formulate and solve the following dual problem.

$$\begin{aligned} &\min_{\mathbf{u},\mathbf{v}} q(\mathbf{u}, \mathbf{v}) \\ &\text{s.t.} \mathbf{u} \geq \mathbf{0}; \mathbf{v} \geq \mathbf{0}, \end{aligned} \tag{P.4}$$

where $q(\mathbf{u}, \mathbf{v}) = L(\mathbf{x}^*(\mathbf{u}, \mathbf{v}), \mathbf{y}^*(\mathbf{u}, \mathbf{v}), \mathbf{u}, \mathbf{v})$.

Since we are not able to obtain closed form solutions to P.3 (i.e., $\mathbf{x}^*(\mathbf{u}, \mathbf{v})$), P.4 becomes a non-smooth convex programming problem. Sub-gradient based algorithms have been widely used to solve non-smooth convex problems. Even though those algorithms are usually guaranteed to obtain near-optimal solutions, their convergence rate can be rather slow [6].

In this section, we use an accelerated sub-gradient method to solve P.4, based on the works [6] [8]. So far, two accelerated sub-gradient methods have been proposed, namely Dual-Averaging Method and Mirror-Decent Method, which have been unified in [6]. Before we describe our proposed algorithm, let us first look at fundamental ideas of the accelerated sub-gradient methods to solve the following general optimization problem:

$$\min_{\mathbf{x} \in Q} f(\mathbf{x}), \tag{P.5}$$

where $f(\mathbf{x})$ is a non-smooth convex function and Q is a nonempty closed convex set.

P.5 can be solved via a primal-dual method. Different from normal primal-dual methods, we need two different sequences of parameters, each of which is responsible for some processes in primal and dual spaces. In primal space, it is necessary to have a vanishing sequence of steps to guarantee the convergence of primal variable sequence. Let $\{\lambda_t\}$ be the sequence satisfying: $\lambda_t > 0, \lambda_t \to 0, \sum_{t=0}^{\infty} \lambda_t = \infty$. However, in the dual space (the space of linear functions), we would like to apply non-decreasing weights to speed up the convergence rate. Let $\{\theta_t\}$ be the sequence satisfying: $\theta_t = \gamma\theta'_t, \theta'_0 = 1, \theta'_{t+1} = \theta'_t + \frac{1}{\theta'_t}$, where γ is a constant positive step size. In addition, define $h(x)$ as any strongly convex and continuously differentiable function on Q. Let $g(x)$ be a sub-gradient of $f(x)$, and normal sub-gradient methods update the solution sequence of P.5 $\{x_k\}$ as follows $x_{k+1} = \operatorname{argmin}_{x \in Q}\{f(x_k) + g(x_k)(x - x_k)\}$. In contrast, the accelerated sub-gradient method updates the solution sequence as follows

$$\begin{aligned} x_{k+1} = \operatorname*{argmin}_{x \in Q} \sum_{t=0}^{k} \lambda_t[f(x_t) + g(x_t)(x - x_t)] \\ + \theta_t h(x). \end{aligned} \tag{8}$$

It has been proved in [6] that, the accelerated sub-gradient method achieves an ϵ-solution (i.e., $|f(\mathbf{x}) - f^*| \leq \epsilon$) within $O(\frac{J^2R^2}{\epsilon^2})$ iterations, where f^* is the optimal solution of P.5, $J := \sup\{||g(\mathbf{x})||\}$, and $R := \sup\{||f(\mathbf{x})||\}$. This method has also been proved to achieve the least complexity among all sub-gradient algorithms attaining ϵ-solutions [6]. Since the complexity of the Hungarian algorithm is $O(n^3)$, total complexity of the primal-dual algorithm is $O(\frac{J^2R^2n^3}{\epsilon^2})$, where $n = \max\{M, N\}$.

IV. Integer Programming Based Method

Since complexity of the primal-dual algorithms can be very high for large scale coexistence problems, in this section, we propose another method based on integer programming, which achieves significantly lower time complexity. More specifically, similar to [9], we assume that only a fixed number of transmit power levels are supported, which are $\{p^l | \forall l \in 1, 2 \cdots L\}$. Let p^l_{ij} be the transmit power of vehicle i on burst j, and r^l_{ij} be its corresponding transmit rate. This assumption is motivated by the fact that most real wireless systems only support a fixed number of transmission rates. Given this observation, P.1 can be reformulated as an Integer Programming (IP) problem as follows:

$$\begin{aligned} &\max_{x^l_{ij} \in \{0,1\}} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{l=1}^{L} \frac{1}{T} A_i x^l_{ij} r^l_{ij} \bar{T}_j \\ &\text{s.t.} \sum_{i=1}^{N} \sum_{l=1}^{L} x^l_{ij} p^l_{ij} G^{BS}_i \leq \beta_j, \forall j \in \{1, 2 \cdots M\}, \\ &\sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{l=1}^{L} C_{jk} x^l_{ij} p^l_{ij} \leq P_{max}, \forall k \in \{1, 2 \cdots K\}, \\ &\sum_{j=1}^{M} \sum_{l=1}^{L} x^l_{ij} \leq 1, \forall i; \sum_{i=1}^{N} \sum_{l=1}^{L} x^l_{ij} \leq 1, \forall j. \end{aligned} \tag{P.5}$$

P.5 belongs to the category of 3-dimensional assignment problems, which are NP-hard problems. In this part, we propose a randomized rounding algorithm to efficiently solve P.5. More specifically, we first reformulate P.5 to a k-column-sparse packing problem (k-CSP). Then we relax the integer constraints to obtain a linear programming (LP) problem. Then a fractional solution can be obtained by solving the LP. Finally, we round the fractional solution to an integer solution, and prove that the rounded solution attains at least a constant fraction of the optimum. Details of this integer programming based method are as follows.

A. Reformulation of P.5 to a k-CSP

Packing problems are usually formulated as the form:

$$\max_{\mathbf{x}\in\{0,1\}^n} \mathbf{w}^T\mathbf{x}, \textbf{s.t.} : A\mathbf{x} \leq \mathbf{b}, A \in R^{m\times n} \quad \text{(P.6)}$$

where all vectors and matrices are non-negative. A k-CSP is defined as follows.

Definition 1: A packing problem is a k-column-sparse packing problem if every item j participates in at most k constraints, i.e., each column of matrix A has at most k non-zero entries.

In this section, our first step is to convert P.5 to a k-CSP with the form of P.6. Hence it is necessary to first transform all three-dimensional variables in P.5 to one-dimensional variables in P.6, which can be done as follows: $x_{i'} = x_{ij}^l; w_{i'} = \frac{1}{T}A_i r_{ij}^l \bar{T}_j$, where $i' = [(i-1)\cdot M+j]\cdot L+l$. For example, x_{11}^1 and x_{22}^1 in P.5 are represented by x_1 and $x_{(M+2)L+1}$ in P.6, respectively. The conversion of constraints in P.5 to the form in P.6 can also be done similarly. Now, we show P.5 is a k-CSP problem.

Lemma 2: P.5 is a k-column-sparse packing problem with $k = K + 3$, where K is the number of burst intervals in the upstream frame.

Proof. This lemma can be proved by analyzing the physical meanings of the constraints in P.5, and details of the proof is omitted here due to space limit. ∎

B. Solve the LP Problem and Round Fractional Solutions to Integer Solutions

In this part, we follow the idea of "randomized rounding with alteration" which was proposed in [10]. The main algorithm is shown as follows.

Algorithm 1: Integer Programming Based Method

Require: A, b and w for P.6, α (random variable to be optimized)

1: Solve P.6 by relaxing integer constraints, and obtain $\mathbf{x}^*$
2: Select each item j independently with probability $\frac{x_j^*}{\alpha k}$, and let C_{old} be the set of selected items
3: **for** $j = 1$ **to** $M \times N \times L$ **do**
4: Delete item j if any of the two conditions is satisfied:
  - there exists an item $l \in C_{old}\backslash\{j\}$, which is "big" for constraint i, or
  - sum of sizes of items in C_{old} that are "small" for i is larger than 1.

5: **end for**
6: **return** Left items in C_{old} (denoted by C_{new})

Although our rounding idea is similar to the one proposed in [10], our work is a generalization of [10]. More specifically, in [10], the authors set the boundary between "big" and "small" items to $1/2$, without studying its impact on performance of the algorithm. In our work, we consider a more general case, where we set the boundary to β that is a random variable to be optimized (see **Definition 2**). The trade-off of deciding β is that if we increase β, the probability that item j is deleted due to D_j is decreased while the probability that item j is deleted due to E_j is increased, and vice versa. We prove that $1/2$ is the optimal value of β for the proposed rounding algorithm, and thus $\frac{1}{8}$ is the best approximation factor we can obtain from Algorithm 1 (see Theorem 1). Without loss of generality, we assume that $b_i = 1, \forall i$, and $a_{ij} \leq 1, \forall a_{ij} \in A$.

Definition 2: An item j is called "big" for constraint i if $a_{ij} > \beta$, and "small" if $a_{ij} \leq \beta$.

Before we prove the performance of this algorithm, we must show feasibility of the finally returned items C_{new}. Since the proof is similar to the one in [10], we omit it here due to space limit. Next, we prove two important lemmas. Given the selected set of items C_{old} in Step 2 of Algorithm 1, let D_j be the event that any item $j \in C_{old}$ is deleted due to the first condition in Step 4 of Algorithm 1, and E_j be the event that item j is deleted due to the second condition in that step. Then the total probability that item j is deleted is $Pr[D_j|j \in C_{old}] + Pr[E_j|j \in C_{old}]$. Now we proceed to bound the two probabilities respectively, by proving **Lemma 3** and **Lemma 4**. First, Let B_i be the set of items that are big for constraint i, and $F_i = \sum_{l\in B_i} x_l^*$, and we have the following lemma.

Lemma 3: $Pr[D_j|j \in C_{old}] \leq \frac{F_i}{\alpha k}$, i.e., the probability that item j is deleted from C_{old} due to the first condition in Step 4 is less than $\frac{F_i}{\alpha k}$.

Proof. Recall that for any item that is big for i, we have $a_{ij} > \beta$. Therefore,

$$F_i = \sum_{l\in B_i} x_l^* < \sum_{l\in B_i} \frac{a_{il}}{\beta} x_l^* = \frac{1}{\beta}\sum_{l\in B_i} a_{il}x_l^* \leq \frac{1}{\beta}, \quad (9)$$

where the last inequality is due to constraint i of P.6. Hence we have $F_i \in [0, \frac{1}{\beta}]$. Moreover, we have the following inequality.

$$\begin{aligned} &Pr[D_j|j \in C_{old}] = Pr[\exists l \in B_i\backslash\{j\}|j \in C_{old}] \\ &\leq \sum_{l\in B_i\backslash\{j\}} Pr[l \in C_{old}|j \in C_{old}] \\ &= \sum_{l\in B_i\backslash\{j\}} Pr[l \in C_{old}] = \sum_{l\in B_i\backslash\{j\}} \frac{x_l^*}{\alpha k} \leq \frac{F_i}{\alpha k}, \end{aligned} \quad (10)$$

where the first inequality is due to union bound, the second equality is due to mutual independence of choosing items, and the last inequality is due to definition of F_i. ∎

Similarly, let S_i be the set of items that are small for constraint i, and we continue to find the probability that item j is deleted due to the second condition in Step 4.

Lemma 4: $Pr[E_j|j \in C_{old}] \leq (1-\beta F_i)/[\alpha k(1-\beta)]$, i.e., the probability that item j is deleted from C_{old} due to E_j is less than $\frac{1-\beta F_i}{\alpha k(1-\beta)}$.

Proof. In this case, if item j is big for constraint i, then we have $S_i = S_i \backslash \{j\}$, and total size of items in S_i must be larger than 1. Furthermore, we have the following inequality:

$$\begin{aligned} &Pr[E_j|j \in C_{old}] = Pr\left[\sum_{l\in S_i} a_{il} > 1|j \in C_{old}\right] \\ &= Pr\left[\sum_{l\in S_i\backslash\{j\}} a_{il} > 1|j \in C_{old}\right] \\ &\leq Pr\left[\sum_{l\in S_i\backslash\{j\}} a_{il} > 1-\beta|j \in C_{old}\right], \end{aligned} \tag{11}$$

where the second equality is due to $S_i = S_i\backslash\{j\}$, and the last inequality is because $\beta \in [0,1]$. In contrast, if j is small for constraint i, then E_j occurs only if the total size of items in S_i must be larger than 1. Since $a_{ij} \leq \beta$, the total size of items in $S_i\backslash\{j\}$ must be larger than $(1-\beta)$. More specifically, we obtain the same result as in Equation 11. In other words, we finally have:

$$Pr[E_j|j \in C_{old}] \leq Pr\left[\sum_{l\in S_i\backslash\{j\}} a_{il} > 1-\beta|j \in C_{old}\right] \tag{12}$$

Furthermore, the expected total size of items in $S_i\backslash\{j\}$ can be calculated as

$$\begin{aligned} &E\left[\sum_{l\in S_i\backslash\{j\}} a_{il}|j \in C_{old}\right] = \sum_{l\in S_i\backslash\{j\}} a_{il}\frac{x_j^*}{\alpha k} \\ &\leq \frac{1}{\alpha k}\left(1-\sum_{l\in B_i} a_{il}x_j^*\right) \leq \frac{1}{\alpha k}\left(1-\beta F_i\right), \end{aligned} \tag{13}$$

where the last inequality is due to Equation 9. Given this inequality, we can further extend Equation 12:

$$\begin{aligned} &Pr[E_j|j \in C_{old}] \leq Pr\left[\sum_{l\in S_i\backslash\{j\}} a_{il} > 1-\beta|j \in C_{old}\right] \\ &\leq \frac{E\left[\sum_{l\in S_i\backslash\{j\}} a_{il}|j \in C_{old}\right]}{1-\beta} \leq \frac{1-\beta F_i}{\alpha k(1-\beta)}, \end{aligned} \tag{14}$$

where the second inequality is due to Markov Inequality, and the third inequality is due to Equation 13. ■

Finally, we show the efficacy of the rounding algorithm by proving the following theorem.

Theorem 1: Algorithm 1 is an $\frac{1}{8}\left(1-\frac{1}{k}+\frac{1}{k^2}\right)$-approximation algorithm to P.6, and the approximation factor becomes $1/8$ for large k values.

Proof. Given **Lemma 3, 4**, the total probability that item j is deleted in Step 2 is

$$\begin{aligned} &Pr[j \notin C_{new}|j \in C_{old}] = Pr[D_j|j \in C_{old}] \\ &+ Pr[E_j|j \in C_{old}] \leq \frac{F_i}{\alpha k} + \frac{1-\beta F_i}{\alpha k(1-\beta)} \\ &= \frac{1}{\alpha k}\left(\frac{1-2\beta}{1-\beta}F_i + \frac{1}{1-\beta}\right). \end{aligned} \tag{15}$$

Intuitively, to improve the approximation factor, we need to delete as few items as possible on condition that all constraints are satisfied. In other words, we should minimize the probability $Pr[j \notin C_{new}|j \in C_{old}]$ by choosing optimal β^*. Since $\beta \in [0,1]$, we consider the choice of β for two cases: (1) $\beta \in [0,1/2]$ and (2) $\beta \in [1/2,1]$, and show that $\beta^* = 1/2$.

Case 1: $\beta \in [0,1/2]$. Recall that $F_i \in [0,\frac{1}{\beta}]$. Hence $(1-2\beta) \geq 0$, and Equation 15 implies that

$$\begin{aligned} &Pr[j \notin C_{new}|j \in C_{old}] \leq \frac{1}{\alpha k}\left(\frac{1-2\beta}{1-\beta}F_i + \frac{1}{1-\beta}\right) \\ &\leq \frac{1}{\alpha k}\left(\frac{1-2\beta}{1-\beta}\cdot\frac{1}{\beta} + \frac{1}{1-\beta}\right) = \frac{1}{\alpha k}\cdot\frac{1}{\beta}, \end{aligned} \tag{16}$$

where the right hand side is minimized for $\beta = 1/2$.

Case 2: $\beta \in [1/2,1]$. In this case, we have $(1-2\beta) \leq 0$, and Equation 15 implies that

$$\begin{aligned} &Pr[j \notin C_{new}|j \in C_{old}] \leq \frac{1}{\alpha k}\left(\frac{1-2\beta}{1-\beta}F_i + \frac{1}{1-\beta}\right) \\ &\leq \frac{1}{\alpha k}\left(\frac{1-2\beta}{1-\beta}\cdot 0 + \frac{1}{1-\beta}\right) = \frac{1}{\alpha k}\cdot\frac{1}{(1-\beta)}, \end{aligned} \tag{17}$$

where the right hand side is also minimized for $\beta = 1/2$. Hence $\beta^* = 1/2$, and the preceding two equations imply

$$Pr[j \notin C_{new}|j \in C_{old}] \leq \frac{2}{\alpha k}. \tag{18}$$

Finally, the probability of choosing set C_{new} is

$$\begin{aligned} &Pr[j \in C_{new}] = Pr[j \in C_{old}]\cdot Pr[j \in C_{new}|j \in C_{old}] \\ &= Pr[j \in C_{old}]\cdot(1-Pr[j \notin C_{new}|j \in C_{old}]) \\ &\geq \frac{x_j^*}{\alpha k}\left(1-\frac{2}{\alpha}\right). \end{aligned} \tag{19}$$

It is easy to see that the right hand side is maximized for $\alpha = 4$, and thus $Pr[j \in C_{new}] \geq \frac{x_j^*}{8k}$. Let $\mathbf{x}^{OPT}$ be the optimal solution to P.6, and Equation 19 implies

$$\begin{aligned} \sum_{j\in C_{new}} w_j x_j^* &\geq \frac{1}{8k}\sum_{j\in C_{old}} w_j x_j^* \\ &\geq \frac{1}{8k}\left(k-1+\frac{1}{k}\right)\sum_{j=1}^{n} w_j x_j^{OPT} \\ &= \frac{1}{8}\left(1-\frac{1}{k}+\frac{1}{k^2}\right)\sum_{j=1}^{n} w_j x_j^{OPT} \end{aligned} \tag{20}$$

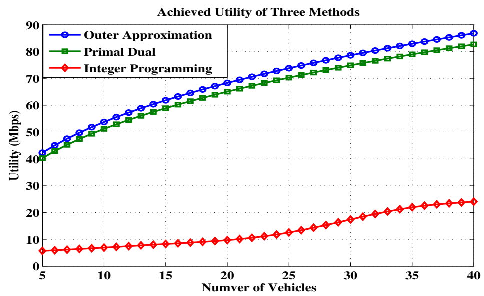

Figure 2. Total Utility of Vehicles of Three Methods

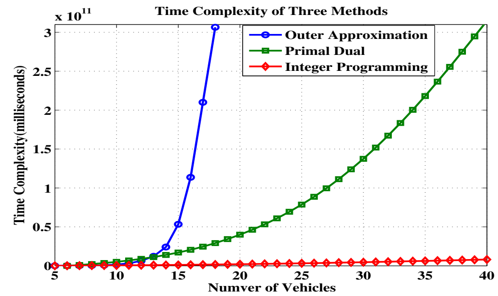

Figure 3. Time Complexity of Three Methods

where the second inequality is because LP relaxation for the k-set packing problem has an integrality gap of $k-1+\frac{1}{k}$ [10]. ■

Since there are totally n iterations, and each iteration contains $(m+n)$ operations. Hence complexity of the proposed algorithm is merely $O(n(m+n)) = O(M \cdot N \cdot L \cdot (2M+N+K+M \cdot N \cdot L))$.

V. Numerical Results

In this section, we consider the coexistence of a mobile CVN with an 802.22 network in a $20km \times 20km$ region, and the BS is placed at the center of the region. $M = 16$ CPEs are uniformly distributed in the whole region while vehicles move in the region by following a "Random Waypoint Mobility" model. In our simulations, mobility of vehicles would affect their channel gains to their receivers and the BS. A 6 MHz TVWS channel is used and residual idle time of the channel (i.e., t_p) is set to follow from a Gamma distribution with $k = 2$ and $\beta = 5$. CDF of the residual idle time can be calculated as $F(t_p) = 1 - e^{-5t_p} - 5t_pe^{-5t_p}$. Other simulation parameters are set as follows:$N \in \{5, 10 \cdots 40\}; A_i \in \{1, 2, 4, 8\}, \forall i \in \{1, 2 \cdots N\}; P_{max} = 100; \beta_j = -90dB; \sigma_0 = 100dBm/Hz; T = 9ms$.

For each N, we run the preceding two algorithms and the outer approximation algorithm [5] for 100 iterations, each iteration consisting of 100 upstream frames. Note that the outer-approximation algorithm is a near-optimal algorithm with time complexity of 2^n, where n is the number of variables of the MINLP [5]. We compare the performance of three algorithms in terms of total utility of vehicles and time complexity of the algorithms.

Fig. 2 shows achieved utility values of three algorithms with varying N. From this figure, we can see that both the outer-approximation and primal-dual algorithms are able to achieve near-optimal utility. Even though the outer-approximation algorithm is able to achieve slightly higher utility than the primal-dual algorithm, its time complexity is significantly higher than the latter one, which is shown in Fig. 3. Moreover, in Fig. 2, we can roughly see that the integer programming algorithm is able to achieve more than $\frac{1}{8}$ of the optimal utility, and the approximation factor increases with increasing N. Furthermore, complexity of the integer programming algorithm is significantly lower than the other two algorithms.

VI. Conclusion

In this paper, we studied the coexistence problem between a CVN and a 802.22 network. We show that the problem is NP-hard and propose both a primal-dual algorithm after converting the problem to a convex programming problem, and a constant-factor approximation algorithm by reformulating the problem to a k-column-sparse packing problem. The design of distributed algorithms with good approximation factors and low time complexity is an important topic for our future work.

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