

DENSE ARBITRARILY PARTITIONABLE GRAPHS

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Abstract

A graph G of order n is called *arbitrarily partitionable* (AP for short) if, for every sequence (n_1, \dots, n_k) of positive integers with $n_1 + \dots + n_k = n$, there exists a partition (V_1, \dots, V_k) of the vertex set $V(G)$ such that V_i induces a connected subgraph of order n_i for $i = 1, \dots, k$. In this paper we show that every connected graph G of order $n \geq 22$ and with $\|G\| > \binom{n-4}{2} + 12$ edges is AP or belongs to few classes of exceptional graphs.

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