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HOW LONG CAN ONE BLUFF IN THE DOMINATION GAME?

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Abstract

The domination game is played on an arbitrary graph G by two players, Dominator and Staller. The game is called Game 1 when Dominator starts it, and Game 2 otherwise. In this paper bluff graphs are introduced as the graphs in which every vertex is an optimal start vertex in Game 1 as well as in Game 2. It is proved that every minus graph (a graph in which Game 2 finishes faster than Game 1) is a bluff graph. A non-trivial infinite family of minus (and hence bluff) graphs is established. minus graphs with game domination number equal to 3 are characterized. Double bluff graphs are also introduced and it is proved that Kneser graphs $K(n, 2)$, $n \geq 6$, are double bluff. The domination game is also studied on generalized Petersen graphs and on Hamming graphs. Several generalized Petersen graphs that are bluff graphs but not vertex-transitive are found. It is proved that Hamming graphs are not double bluff.

Keywords: domination game, game domination number, bluff graphs, minus graphs, generalized Petersen graphs, Kneser graphs, Cartesian product of graphs, Hamming graphs.

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References

[1] B. Brešar, P. Dorbec, S. Klavžar and G. Košmrlj, *Domination game: Effect of edge- and vertex-removal*, Discrete Math. **330** (2014) 1–10. doi:10.1016/j.disc.2014.04.015

[2] B. Brešar, T. Gologranc, M. Milanič, D.F. Rall and R. Rizzi, *Dominating sequences in graphs*, Discrete Math. **336** (2014) 22–36. doi:10.1016/j.disc.2014.07.016

[3] B. Brešar, S. Klavžar, G. Košmrlj and D.F. Rall, *Domination game: extremal families of graphs for the* 3/5*-conjectures*, Discrete Appl. Math. **161** (2013) 1308–1316. doi:10.1016/j.dam.2013.01.025

[4] B. Brešar, S. Klavžar, G. Košmrlj and D.F. Rall, *Guarded subgraphs and the domination game*, Discrete Math. Theor. Comput. Sci. **17** (2015) 161–168.

[5] B. Brešar, S. Klavžar and D.F. Rall, *Domination game and an imagination strategy*, SIAM J. Discrete Math. **24** (2010) 979–991. doi:10.1137/100786800

[6] Cs. Bujtás, *Domination game on trees without leaves at distance four*, in: A. Frank, A. Recski and G. Wiener (Eds.), Proceedings of the 8th Japanese-Hungarian Symposium on Discrete Mathematics and Its Applications (Veszprém, Hungary, 2013), 73–78.

[7] Cs. Bujtás, *Domination game on forests*, Discrete Math. **338** (2015) 2220–2228. doi:10.1016/j.disc.2015.05.022

[8] Cs. Bujtás, *On the game domination number of graphs with given minimum degree*, Electron. J. Combin. **22** (2015) #P3.29.

[9] Cs. Bujtás and S. Klavžar, *Improved upper bounds on the domination number of graphs with minimum degree at least five*, Graphs Combin. **32** (2016) 511–519. doi:10.1007/s00373-015-1585-7

[10] Cs. Bujtás, S. Klavžar and G. Košmrlj, *Domination game critical graphs*, Discuss. Math. Graph Theory **35** (2015) 781–796. doi:10.7151/dmgt.1839

[11] Cs. Bujtás and Z. Tuza, *The disjoint domination game*, Discrete Math. **339** (2016) 1985–1992.
doi:10.1016/j.disc.2015.04.028

[12] P. Dorbec, G. Košmrlj and G. Renault, *The domination game played on unions of graphs*, Discrete Math. **338** (2015) 71–79.
doi:10.1016/j.disc.2014.08.024

[13] R. Frucht, J.E. Graver and M.E. Watkins, *The groups of the generalized Petersen graphs*, Proc. Cambridge Philos. Soc. **70** (1971) 211–218.
doi:10.1017/S0305004100049811

[14] I. Gorodezky, Dominating Sets in Kneser Graphs (Master Thesis, University of Waterloo, 2007).

[15] M.A. Henning and W.B. Kinnersley, *Domination game: A proof of the* 3/5*-conjecture for graphs with minimum degree at least two*, SIAM J. Discrete Math. **30** (2016) 20–35.
doi:10.1137/140976935

[16] M.A. Henning, S. Klavžar and D.F. Rall, *Total version of the domination game*, Graphs Combin. **31** (2015) 1453–1462.
doi:10.1007/s00373-014-1470-9

[17] M.A. Henning, S. Klavžar and D.F. Rall, *The* $\frac{4}{5}$ *upper bound on the game total domination number*, Combinatorica, to appear.
doi:10.1007/s00493-015-3316-3

[18] S.R. Jayaram, *Minimal dominating sets of cardinality two in a graph*, Indian J. Pure Appl. Math. **28** (1997) 43–46.

[19] W.B. Kinnersley, D.B. West and R. Zamani, *Game domination for grid-like graphs*, unpublished manuscript (2012).

[20] W.B. Kinnersley, D.B. West and R. Zamani, *Extremal problems for game domination number*, SIAM J. Discrete Math. **27** (2013) 2090–2107.
doi:10.1137/120884742

[21] G. Košmrlj, *Realizations of the game domination number*, J. Comb. Optim. **28** (2014) 447–461.
doi:10.1007/s10878-012-9572-x

[22] M.J. Nadjafi-Arani, M. Siggers and H. Soltani, *Characterisation of forests with trivial game domination numbers*, J. Comb. Optim. **32** (2016) 800–811.
doi:10.1007/s10878-015-9903-9

[23] S. Schmidt, *The* 3/5*-conjecture for weakly* $S(K_{1,3})$*-free forests*, arXiv:1507.02875v1 [math.CO] (2015).

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