A TECHNIQUE FOR ASSESSING VARIABILITY OF PERCEPTUAL SPAN*

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The processes by which human observers are able to detect signals in a background of noise have become a major topic of investigation in experimental psychology. To date the greater part of the work has been concerned with auditory signal detection, perhaps because of the many practical problems arising in the engineering of communication systems in which the receiver is a human operator faced with the task of discriminating signals from noise over an incoming channel, and with general theories of signal detectability.[1] Problems unique to visual signal detection have just begun to receive analysis.[2] The writer's interest in this line of investigation arose from the potential usefulness of a forced-choice detection technique for estimating perceptual span, that is, the number of elements perceivable from a visual display of very brief duration.

In preceding studies[2, 3] estimates of perceptual span were obtained from detection data in the following manner. A set of stimulus displays was prepared, each comprising a set of printed symbols in a random arrangement. Of the total set of symbols used in all of the displays, two, which we shall call A and B, were designated as critical elements and all the rest as noise elements. Each display included exactly one of the two critical elements plus a number of noise elements, and the position of the critical element was determined randomly. Over a series of trials, each of the two critical elements appeared equally often, and the task of the observer was to state following the 50-ms exposure of each display whether he believed the critical element present to have been A or B. In order to perform at better than the chance rate of 50 per cent correct, it would clearly be necessary for the observer to perceive at least a portion of the elements in each display. The term "perceive" is not entirely well defined, but here we take it to mean simply that the observer receives and processes information from an element of the display in such a way that he is able to classify it as signal or noise. Now if on a series of trials the observer is presented with displays of size D, then on all trials on which he perceives exactly k elements, his probability of correct detection of the critical element present will clearly be given by

$$P(C|k) = \frac{k}{D} + \frac{1}{2}\left(1 - \frac{k}{D}\right) = \frac{1}{2}\left(1 + \frac{k}{D}\right),$$

since he will be correct whenever the critical element falls in the set of k elements perceived (which has probability k/D) and will be correct with probability $^1/_2$ on trials on which the critical element is not among the k elements perceived (and the observer's response is simply a guess). Now, regardless of the way in which the number of elements perceived varies from trial to trial over a series, the over-all probability of a correct detection will be given by

$$P(C) = \sum_{k=0}^{D} P(C|k)\, f(k) = \frac{1}{2}\sum_{k=0}^{D}\left(1 + \frac{k}{D}\right) f(k),$$

where $f(k)$ denotes the probability of perceiving k elements from a display of size D on any trial. Thus we have the simple result

$$P(C) = \frac{1}{2}\left(1 + \frac{d}{D}\right),$$

where

$$d = E(k) = \sum_{k=0}^{D} kf(k).$$

Now a very simple estimate of d, the average number of elements perceived, is obtainable if we insert the observed proportion of correct responses over a series of trials in place of $P(C)$ in the preceding equation and solve for d, viz.,

$$\hat{d} = [2\hat{P}(C) - 1]D.$$

With a method in hand for estimating the mean value of perceptual span for any type of material and display size, we should like now to go further and find a way of evaluating the variability from trial to trial in number of elements perceived. From a practical viewpoint, such a result would enable us to determine how closely the human observer approximates optimal performance under a given set of conditions. From the theoretical standpoint, such estimates may be of critical value in narrowing down the class of theoretical models that might account for the perceptual process. For example, in a serial processing model proposed earlier for perceptual behavior in this situation,[2] the particular assumptions employed imply a geometric distribution of number of elements perceived per trial. If that model is correct, the standard deviation of the sample size should, then, be approximately equal to the mean. Thus, numerical estimates of the standard deviation of the span would enable us not only to provide a sharp test of that particular model, but also to set severe restrictions on the class of models which might prove adequate.

Within the original experimental situation, the problem is insoluble, for the data, being simply a sequence of binomial observations, suffice only to determine a single parameter, the mean of the distribution. Fortunately, however, a very simple extension of the original experiment provides a means for dealing with other aspects of the distribution of perceptual spans. This modification is to introduce redundant critical elements into the displays; that is, in the simplest extension of the experiment, the total set of elements used is exactly as before, but each display now includes two redundant critical elements (either two A's or two B's) plus noise elements, the positions of the critical elements again being randomly determined and the task of the observer again being simply to indicate following each 50-ms exposure which type of critical element was present.

It is easy to show, firstly, that optimal detection rate will result for a given mean span if the variability of span from trial to trial is zero. To show this, we note that for any given number k of elements perceived, the probability of a correct detection, given that there are m redundant critical elements in the display of size D, is given by

$$P(C|k) = 1 - \frac{1}{2}\frac{\binom{D-m}{k}}{\binom{D}{k}} = 1 - \frac{1}{2}\frac{(D-k)(D-k-1)\ldots(D-k-m+1)}{D(D-1)\ldots(D-m+1)}.$$

The ratio of combinatorial terms on the right side of the equation is, of course, simply the probability that the sample of k elements perceived does not include one of the redundant critical elements. Errors will occur on half of the trials on which the sample of k elements perceived does not include a critical element, and all other trials will be correct. Taking the expectation over all possible values of k, we have then for the over-all probability of a correct detection

$$P(C) = 1 - \frac{E[(D-k)(D-k-1)\ldots(D-k-m+1)]}{2D(D-1)\ldots(D-m+1)}.$$

Now it is readily seen that the following inequality obtains:

$$E[(D-k)(D-k-1)\ldots(D-k-m+1)] \geq (D-d)(D-d-1)\ldots(D-d-m+1),$$

since all of the factors in the product inside the expectation on the left are positively correlated, and as before d denotes the mean value of k. Thus, for any fixed value d of the mean perceptual span, $P(C)$ is maximal, and therefore performance is optimal, when the equality obtains, that is, when the span is constant from trial to trial.

To determine experimentally whether there is significant variability in perceptual span, we need only obtain data for the same observer for a series of trials run under the original experimental conditions (i.e., $m = 1$), from which we obtain an estimate of d, then a series of trials under any one other value of m. Using the estimate of d from the first part of the experiment, we can compute the predicted value of $P(C)$ for the other value of m and then determine whether the observed proportion of correct responses deviates significantly from the predicted value.

Further exploitation of the expression we have obtained for probability of correct detection with any number of redundant critical elements provides a simple and elegant means of estimating not only the mean but the variance, and in fact all higher moments, of the distribution of perceptual spans and thus of determining the form of the distribution to any desired degree of approximation. To illustrate, consider the case $m = 2$, for which

$$P(C|k) = 1 - \frac{1}{2}\frac{\binom{D-2}{k}}{\binom{D}{k}} = 1 - \frac{1}{2}\frac{(D-k)(D-k-1)}{D(D-1)},$$

and

$$P(C) = \sum_{k=0}^{D}\left[1 - \frac{1}{2}\frac{(D-k)(D-k-1)}{D(D-1)}\right]f(k)$$
$$= \frac{1}{2}\left[1 + \frac{(2D-1)d - E(k^2)}{D(D-1)}\right],$$

where

$$E(k^2) = \sum_{k=0}^{D} k^2 f(k).$$

Now the variance of k is by definition

$$\sigma_k^2 = E(k^2) - E^2(k) = E(k^2) - d^2.$$

Therefore, we can substitute into the preceding equation and write the proportion correct in terms of the mean and variance of the distribution of spans, viz.,

$$P(C) = \frac{1}{2}\left[1 + \frac{(2D - 1)d - \sigma_k^2 - d^2}{D(D - 1)}\right].$$

Finally, using this last result, we can insert an estimate of d obtained from an experiment on the same subject with $m = 1$, together with the observed value of $P(C)$ from a series with $m = 2$, and obtain an estimate of the variance of the k distribution:

$$\hat{\sigma}_k^2 = (2D - 1)\hat{d} - \hat{d}^2 - [2\hat{P}(C) - 1]D(D - 1).$$

Then, proceeding in the same manner, we could run a series with $m = 3$ on the same observer, from the data of which, together with the data for $m = 1$ and $m = 2$, we could now estimate the third moment of the k distribution, and so on for higher moments.

In this paper we have been concerned solely with the *number* of elements perceived during a brief stimulus exposure, without regard to the positions of the perceived elements in the display. For simplicity the derivations have proceeded on the assumptions that all subsets of the display which are of the same size are equally likely to be perceived. However, our results are not limited to this special case. So long as the positions of the critical elements in a display are assigned at random, the expressions derived above for probability of correct detections and for statistics of the distribution of perceptual spans are independent of the probability distributions of perceived subsets. To see this, consider an experiment in which m critical elements are assigned randomly to positions in a display of D elements. Given any fixed subset of k elements perceived, the probability that none of the critical elements are included is $\binom{D-k}{m}/\binom{D}{m}$, which is identically equal to the quantity $\binom{D-m}{k}/\binom{D}{k}$ utilized for probability of a "miss" in our derivation of $P(C)$. Therefore, our conclusions depend in no way upon knowledge as to whether or not k adjacent elements are more likely to be perceived simultaneously than k elements dispersed over the display field. By further exploitation of the basic method here introduced, it will be possible to test hypotheses concerning not only size, but also such properties as compactness and "shape" of the sample of elements perceived from a display.

Application of our technique for assessing variability of perceptual span can conveniently be illustrated in relation to a sample of data collected by Estes and Taylor[3] using the same apparatus and general procedure described in a previous report.[2] Eight subjects were tested on 16-element matrix displays containing 1, 2, or 4 redundant critical elements. Utilizing the observed proportions of correct detections on the 1 and 2 redundant-element displays, the values of d and σ_k^2 were estimated for each subject. Considering, as an example, Subject no. 1, his proportions correct were 0.854 and 0.875 for $m = 1$ and $m = 2$, respectively. From these values we obtain the estimates

$$\begin{aligned}\hat{d} &= (2 \times 0.854 - 1) \times 16 = 11.33,\\ \hat{\sigma}_k^2 &= (2 \times 16 - 1) \times 11 - 128.37 - (2 \times 0.875 - 1) \times 16 \times 15\\ &= 32.63,\end{aligned}$$

and

$$\hat{\sigma}_k = 5.7.$$

Proceeding similarly with the data of the remaining subjects, one obtains estimates of σ_k^2 equal to zero for four of the eight cases; in only one case does $\hat{\sigma}_k$ prove greater than half the value of $\hat{d}$. For the whole sample, the average of the σ_k estimates is less than 2 elements, whereas the average of the d estimates is approximately 7. No sweeping conclusions should be drawn from this result, since the assignments of pairs of redundant critical elements to positions in the display matrices were not entirely random. (The four corner positions were not used and one member of each pair of redundant elements was required to be on an edge of the matrix; these restrictions are eliminated in a study now in progress.) However, it seems likely, in the light of these preliminary data, that the assumption of a geometric distribution of perceptual spans embodied in a serial processing model for visual detection[2] will have to be modified.

Finally, it might be noted that the technique presented here offers possibilities of comparing perceptual spans for human and infrahuman subjects. It is well known that many animals, notably pigeons and monkeys, can be trained to attend to a viewing screen upon presentation of a signal and can learn discriminations involving symbols such as those used as critical elements in experiments on visual detection. By training animals to discriminate between displays including varying numbers of redundant critical elements per display, one can estimate statistics of the distribution of perceptual span, and thus in turn evaluate hypotheses as to how subjects of different species process information from visual displays.

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[1] Swets, J. A., ed., *Signal Detection and Recognition by Human Observers* (New York: John Wiley and Sons, 1964).

[2] Estes, W. K., and H. A. Taylor, "A detection method and probabilistic models for assessing information processing from brief visual displays," these PROCEEDINGS, **52**, 446–454 (1964).

[3] Estes, W. K., and H. A. Taylor, "Visual detection in relation to display size and redundancy of critical elements," Tech. Rept. No. 68, Stanford Univ., 1965.

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*CROSS-MODALITY MATCHING OF BRIGHTNESS AND LOUDNESS**

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In several experiments, observers have undertaken to match for apparent intensity the sensations aroused in two different sense modalities.[1] Despite the uncertainty sometimes expressed about what constitutes equal apparent magnitude in the face of a qualitative disparity (like the well-known difficulty of heterochromatic photometry), the method of cross-modality matching has demonstrated that subjective magnitude grows as a power function of stimulus intensity. The present study undertakes a cross-modality comparison of brightness and loudness—probably the two most important continua having to do with sensory intensity.