

Supporting Material of the manuscript:

## **Towards optimal design of cancer nanomedicines: Multi-stage nanoparticles for the treatment of solid tumors**

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### **Description of the Mathematical Model**

We represent the tumor vasculature as a 2-dimensional percolation network at the percolation threshold, with one inlet and one outlet as shown in Supplementary Fig. 1<sup>1</sup>. These network structures have been found to represent tumor vasculature in animal models well<sup>10</sup>, though real angiography images could be used to represent additional heterogeneity present in unusual vascular networks or be applied to 3-dimensional structures. We repeated the simulations for five network realizations and the average values are presented.

### **Fluid Flow Equations**

The mathematical model requires coupling of fluid flow in the vascular and interstitial spaces. Blood flow rate in a vessel ( $Q_{vascular}$ ) is assumed to be axial and follows Poiseuille's equation<sup>18</sup>,

$$Q_{vascular} = -\frac{\pi d^4}{128\mu} \frac{\Delta p_v}{\Delta x}, \quad (1)$$

where  $d$  is the vessel diameter,  $\Delta p_v$  is the vascular pressure difference that corresponds to a vascular length  $\Delta x$  and  $\mu$  is the blood viscosity.

Fluid flow rate across the vessel wall ( $Q_{transvascular}$ ) follows Starling's approximation<sup>3</sup>,

$$Q_{transvascular} = L_p S (p_v - p_i), \quad (2)$$

where  $L_p$  is the hydraulic conductivity of the vessel wall,  $S$  is the surface area of the vessel and  $p_i$  is the interstitial fluid pressure. In the equation above, we neglect osmotic pressures.

Interstitial volumetric fluid flow rate ( $Q_{tissue}$ ) follows Darcy's law,

$$Q_{tissue} = -K_t A_C \frac{\Delta p_i}{\Delta x}, \quad (3)$$

where  $K_t$  is the hydraulic conductivity of the interstitial space and  $\Delta p_i$  is the interstitial pressure difference that corresponds to a tissue length  $\Delta x$ .  $\Delta x$  is the distance between two interstitial nodes (Supplementary Fig. 1 B), taken to be 50  $\mu\text{m}$ , the same as the vascular length in Eq. 1.  $A_c$  is the tissue cross-sectional area, which is related to the vascular density,  $S_v$ , and the diameter of the vessel,  $d$ , by  $A_C = \pi d / S_v$  (Ref. <sup>2</sup>).

## **Drug Delivery Equations**

Coupling of nanoparticle transport between the vascular and interstitial spaces is based on the following assumptions.

Inside the blood vessels diffusion is negligible and thus, the mass balance is convection dominated:

$$\frac{dc_v}{dt} = -u \frac{\Delta C_v}{\Delta x}, \quad (4)$$

where  $u$  is the axial blood velocity which is determined by dividing  $Q_{vascular}$  in Eq. 1 by the cross sectional area of the vessel,  $C_v$  is the intravascular concentration of the nanoparticle and  $\Delta C_v$  is the concentration difference that corresponds to a vascular length  $\Delta x$ .

The amount  $\Phi$  of nanoparticles (or drug for a single stage system) that is transferred across the vessel wall between the vascular and interstitial space is given by Starling's approximation as <sup>3</sup>

$$\Phi = L_p S (p_v - p_i) (1 - \sigma) \frac{C_v e^{Pe} - c_i}{e^{Pe} - 1} \quad \text{with } Pe = L_p (1 - \sigma) \frac{(p_v - p_i)}{P}, \quad (5, 6)$$

where  $Pe$  is the Péclet number across the vessel wall, and  $P$  is the vascular permeability of the nanoparticle through the pores of the wall. Therefore,  $\Phi$  is subtracted from Eq. 4 and added to the equation for the interstitial transport of the nanoparticles, as shown in Eqns. 12, 15 and 19 for the three delivery systems.

Using theory for transport of particles through cylindrical pores we calculate the hydraulic conductivity,  $L_p$ , vascular permeability,  $P$ , and reflection coefficient,  $\sigma$ , by the equations <sup>6</sup>:

$$L_p = \frac{\gamma r_o^2}{8\mu L}, \quad P = \frac{\gamma H D_o}{L}, \quad \sigma = 1 - W, \quad (7)$$

where  $\gamma$  is the fraction of vessel wall surface area occupied by pores,  $r_o$  is the pore radius,  $L$  is the thickness of the vessel wall, and  $D_o$  is the diffusion coefficient of the particle in free solution at 37 °C.

The parameters  $H$  and  $W$  account for hydrodynamic and electrostatic interactions and for dilute solutions are given by the equations <sup>6</sup>:

$$H = 2 \int_0^{1-\lambda} K^{-1} e^{-E/kT} \beta d\beta, \quad (8)$$

$$W = 4 \int_0^{1-\lambda} G(1 - \beta^2) e^{-E/kT} \beta d\beta, \quad (9)$$

where  $\lambda$  is the ratio of the particle size over the pore size,  $E$  is the electrostatic energy of interaction between the nanoparticle and the pore,  $k$  is Boltzmann's constant,  $T$  is the absolute temperature,  $K(\lambda, \beta)$  and  $G(\lambda, \beta)$  are hydrodynamic functions, and  $\beta$  is the

radial distance of the particle inside the pore divided by the pore radius to become dimensionless.

To calculate the hydrodynamic functions,  $K(\lambda, \beta)$  and  $G(\lambda, \beta)$ , the centerline approximation is employed, which suggests that use of the centerline values,  $K(\lambda, 0)$  and  $G(\lambda, 0)$  leads to reasonably accurate estimates of  $H$  and  $W$ . Therefore, Eqs 8-9 are written as:

$$H = 2K^{-1}(\lambda, 0) \int_0^{1-\lambda} e^{-E/kT} \beta d\beta, \quad (10)$$

$$W = 4G(\lambda, 0) \int_0^{1-\lambda} (1-\beta^2) e^{-E/kT} \beta d\beta. \quad (11)$$

Analytical expressions of the hydrodynamic coefficients  $K(\lambda, 0)$  and  $G(\lambda, 0)$  are given by Bungay and Brenner<sup>4,6</sup>. These expressions are composites of asymptotic centerline results for small and for closely fitting spheres, are valid for  $0 \leq \lambda < 1$  and are accurate to within 10% for all values of  $\lambda$  (6,7). Notice that in the absence of electrostatic interactions, which is the case accounted in this study,  $E=0$  and Eqs 10-11 are written as  $H = \phi K^{-1}(\lambda, 0)$  and  $W = \phi(2-\phi)G(\lambda, 0)$ , where  $\phi$  is the partition coefficient:  $\phi = (1-\lambda)^2$ .

*Interstitial transport:*

Single Stage (chemotherapy)

The equation system employed for the interstitial transport of the drug, its binding to cancer cells and its internalization by the cells is the following:

Free drug:

$$\frac{\partial C_F}{\partial t} = -v\nabla C_F + D_F \nabla^2 C_F - \frac{1}{\phi} K_{ON} C_{rec} C_F + K_{OFF} C_B + \Phi \quad (12)$$

Bound drug:

$$\frac{\partial C_B}{\partial t} = \frac{1}{\phi} K_{ON} C_{rec} C_F - K_{OFF} C_B - K_{INT} C_B \quad (13)$$

Internalized drug:

$$\frac{\partial C_I}{\partial t} = K_{INT} C_B \quad (14)$$

where,  $C_f$ ,  $C_b$  and  $C_i$  are the concentrations of the free, bound and internalized drug in the interstitial space,  $C_{rec}$  is the concentration of cell surface receptors,  $v$  is the interstitial fluid velocity given by Darcy's law,  $D$  is the diffusion coefficient,  $K_{on}$ ,  $K_{off}$  and  $K_{int}$  are the association (binding), dissociation and internalization rate constants, respectively and  $\phi$  is the volume fraction of tumor accessible to the drug and  $\Phi$  is the transvascular flux of the drug given by Starling's approximation.

### Two Stage System (nanocarrier + drug)

The primary nano-carrier is delivered to the tumor and releases the drug. In the equations below  $C_N$  is the concentration of the carrier,  $K_{el}$  the rate constant of release of the drug from the carrier and  $D_N$  the diffusion coefficient of the carrier. We assumed the nano-carrier to contain  $\alpha$  particles of chemotherapy. As we discuss in the main text, the choice of  $\alpha$  does not affect our calculations because we assume the same dose of chemotherapy to be delivered independently of the load capacity of the carrier.

$$\frac{\partial C_N}{\partial t} = -v\nabla C_N + D_N \nabla^2 C_N - K_{el} C_N + \Phi \quad (15)$$

$$\frac{\partial C_F}{\partial t} = \alpha K_{el} C_N - v\nabla C_F + D_F \nabla^2 C_F - \frac{1}{\varphi} K_{ON} C_{rec} C_F + K_{OFF} C_B \quad (16)$$

$$\frac{\partial C_B}{\partial t} = \frac{1}{\varphi} K_{ON} C_{rec} C_F - K_{OFF} C_B - K_{INT} C_B \quad (17)$$

$$\frac{\partial C_I}{\partial t} = K_{INT} C_B \quad (18)$$

### Multi-stage System:

$C_{N1}$  is the concentration of the primary nanoparticle and  $C_{N2}$  is the concentration of the secondary particle.  $K_{el1}$  is the rate constant for the release of the secondary particle from the primary and  $K_{el2}$  the rate constant for the release of the drug from the secondary particle.  $\alpha$  is the number of secondary particles released by the primary and  $\beta$  the number of the drug particles released by the secondary particle.

$$\frac{\partial C_{N1}}{\partial t} = -v\nabla C_{N1} + D_{N1} \nabla^2 C_{N1} - K_{el1} C_{N1} + \Phi \quad (19)$$

$$\frac{\partial C_{N2}}{\partial t} = \alpha K_{el1} C_{N1} - v\nabla C_{N2} + D_{N2} \nabla^2 C_{N2} - K_{el2} C_{N2} \quad (20)$$

$$\frac{\partial C_F}{\partial t} = \beta K_{el2} C_{N2} - v\nabla C_F + D_F \nabla^2 C_F - \frac{1}{\varphi} K_{ON} C_{rec} C_F + K_{OFF} C_B \quad (21)$$

$$\frac{\partial C_B}{\partial t} = \frac{1}{\varphi} K_{ON} C_{rec} C_F - K_{OFF} C_B - K_{INT} C_B \quad (22)$$

$$\frac{\partial C_I}{\partial t} = K_{INT} C_B \quad (23)$$

To reduce the model parameters,  $K_{el2}$  was taken to be the same as the  $K_{el1}$  and simulations were also run keeping the values of the one the same and varying the value of the other.

### **Fraction of killed cells**

The fraction of killed cells is calculated as  $1-S_F$ , where  $S_F$  is the fraction of surviving cells. The fraction of surviving cells is calculated from the equation:  $S_F = \exp(-\omega \cdot I_{peak})$  according to the work of Eikenberry <sup>7</sup>. Eikenberry fitted the

exponential decay equation to experimental data by Kerr et al. <sup>12</sup> for the fraction of non-small cell lung tumor cells surviving at different doses of chemotherapy (doxorubicin) in vitro. In this equation  $\omega=0.4938$  and  $I_{\text{peak}}$  is the peak intracellular concentration of chemotherapy used in <sup>12</sup> in  $\text{ng}/10^5$  cells. Assuming that a cell occupies  $4 \times 10^{-9} \text{ ml}^3$  and that 1 mole of doxorubicin is 579.98g, then 1  $\text{ng}/10^5$  cells is approximately  $10^{-6}$  moles/L. Therefore, the fraction of surviving cells was calculated as  $S_F = \exp(-\omega \cdot 10^6 \cdot C_I)$ , where  $C_I$  is concentration of the internalized doxorubicin.

## Boundary conditions

We modeled a bolus injection of the particles so that the concentration at the inlet of the vascular network decreases exponentially,  $C_v = C_o e^{-t/K_d}$ .  $C_o$  is the initial concentration at the inlet of the vascular network and  $K_d$  is a time constant that describes the exponential decay in the blood and corresponds to the blood half-life. In the case of the single stage system,  $C_v$  and  $K_d$  are the concentration and blood half-life of the chemotherapy; for the two-stage system,  $C_v$  and  $K_d$  are the corresponding values for the nano-carrier, and for the multi-stage system,  $C_v$  and  $K_d$  are the corresponding values for the primary particle. The concentration of the different delivery systems was matched so that the amount of chemotherapy to be the same in all cases. The initial amount of chemotherapy was determined as  $D \cdot A$  in <sup>9</sup>, where  $A=0.13 \text{ L}^{-1}$  and  $D=100\text{mg}$ . Converting  $D \cdot A$  in moles/L (using the fact that 1 mole doxorubicin is 579.98g) we get  $C_o=2.24 \times 10^{-5} \text{ M}$  chemotherapy.

In the inlet the vascular pressure is set to 30 mmHg and at the outlet is set to 5 mmHg <sup>3, 20</sup>. The concentrations and pressures at the boundaries of the interstitial space are set to zero as well (Supplementary Fig. S1).

## Numerical Solution Strategy

The equations of the fluid transport problem are summarized below:

Poiseuille's equation	$Q = \frac{\pi D^4}{128\mu} \cdot \frac{dP}{dz}$	$\Leftrightarrow$	$Q = G_v \cdot \Delta P_v$
Starling's approximation	$Q = L_p S (P_v - P_i)$	$\Leftrightarrow$	$Q = G_w \cdot \Delta P_w$
Darcy's equation	$Q = K_t A_c \frac{\Delta P_i}{\Delta x}$	$\Leftrightarrow$	$Q = G_t \cdot \Delta P_t$

where:

$$G_v = \frac{\pi D^4}{128\mu \cdot \Delta z} \quad \text{vascular conductivity}$$

$$G_w = L_p S \quad \text{transvascular conductivity}$$

$$G_t = \frac{K_t A_c}{\Delta x} \quad \text{interstitial conductivity}$$

We first solve the steady-state fluid problem to calculate the fluid pressures. The vascular and interstitial spaces are discretized by computational nodes (Supplementary Fig. 1B). Each node belonging to the vascular space is assigned a pore diameter taken randomly by a unimodal distribution with a given mean and standard deviation. Therefore, each of the "vascular" nodes has its own values of  $L_p$ ,  $P$ , and  $\sigma$  (Eq. 7). Conservation of fluid requires that at each node the volume of fluid entering the node is the same as the fluid exiting the node, i.e.,  $\sum_i Q_i = 0$  for each node  $i$ . This results in a linear system of equations of the form  $\mathbf{G} \cdot \mathbf{P} = \mathbf{b}$ , where  $\mathbf{G}$  is the conductivity matrix,  $\mathbf{P}$  is the vector of unknowns that contains the vascular and interstitial fluid pressures and the vector  $\mathbf{b}$  of the right hand side has zero elements unless a boundary condition is applied.

Subsequently, we solve the transient transport problem to calculate the concentration of the nanoparticles and the drug. The transient nanoparticle transport problem is solved with a finite difference scheme. Central differencing for diffusion, upwind differencing for convection and an explicit Euler method for time integration were used. The connectivity between vascular and interstitial nodes remains the same as in the fluid flow problem.

### One-dimensional Steady State Solution

Useful insights can be obtained from a simplified model in which the governing equations are considered under steady state conditions in a one-dimensional geometry. Here we seek the concentrations of the nanoparticles, free drug and bound drug as functions of the distance from the blood vessel wall located at  $x=0$ . A non-zero, but constant interstitial velocity is assumed that advects material away from ( $v > 0$ ) or toward the vessel wall ( $v < 0$ ). We further assume that the vascular concentration  $C_v$  varies slowly so that nearly steady state conditions develop near the blood vessel wall. The concentration of bound drug will serve as a proxy for the concentration of internalized drug which in turn determines the surviving fraction of cells. Careful organization of the resulting solutions yields analytical expressions for characteristic lengths scales that relate the penetration of each stage of the drug delivery system to the diffusion, advection, release rates and binding rates.

For the two-stage delivery system transport of the first stage is governed

$$D_{N1} \frac{d^2 C_{N1}}{dx^2} - v \frac{dC_{N1}}{dx} - K_{el1} C_{N1} = 0 \quad (24)$$

which we solve subject to the boundary conditions  $C_{N1}(0) = C_v$  and  $C_{N1}$  bounded for large  $x$  where we neglect the concentration difference across the vessel. This assumption affects the magnitude of the concentration in the tissue, but does not change the penetration distances. The solution is given by

$$C_{N1}(x) = C_v \exp\left(-\frac{x}{L_{N1}}\right) \quad (25)$$

where the characteristic length is given by  $L_{N1} = \frac{2D_{N1}}{(v^2 + 4K_{el1}D_{N1})^{1/2} - v}$  which reduces to  $L_{N1} = \left(\frac{D_{N1}}{K_{el1}}\right)^{1/2}$  in the absence of advection. We note that advection out of the vessel ( $v > 0$ ) increases the penetration of the first stage while advection toward the vessel ( $v < 0$ ) decreases penetration. After release from the primary nanoparticle transport of the free drug is governed by

$$D_F \frac{d^2 C_F}{dx^2} - v \frac{dC_F}{dx} - \frac{1}{\varphi} K_{on} C_{rec} C_F + \alpha K_{el1} C_{N1} = 0 \quad (26)$$

where we have assumed that return from the bound state is much slower than the binding rate. Further we define  $K_F \equiv \frac{1}{\varphi} K_{on} C_{rec}$ . We solve subject to  $C_{N2}(0) = 0$  and  $C_{N2}$  bounded for large  $x$ , that is, the concentration at the vessel wall is held low by rapid clearance by the blood. The concentration of the first stage is known from equation 25. We obtain

$$C_F(x) = A_F \left[ \exp\left(-\frac{x}{L_F}\right) - \exp\left(-\frac{x}{L_{N1}}\right) \right] \quad (27)$$

where the characteristic length scale is given by  $L_F = \frac{2D_F}{(v^2 + 4K_F D_F)^{1/2} - v}$  which reduces to  $L_F = \left(\frac{D_F}{K_F}\right)^{1/2}$  in the absence of advection and multiplicative coefficient is  $A_F = \frac{\alpha K_{el1} C_v}{D_F \left(\frac{1}{L_{N1}}\right)^2 + \frac{v}{L_{N1}} - K_F}$  which reduces to  $A_F = \frac{\alpha \frac{K_{el1} C_v}{K_F}}{\left(\frac{L_F}{L_{N1}}\right)^2 - 1}$  in the absence of advection.

In steady state the bound drug concentration may be estimated from

$$C_B = \frac{K_{on} C_{rec}}{\varphi(K_{off} + K_{int})} C_F \quad (28)$$

Analysis of multistage delivery follows a similar pattern. Equations 24 and 25 apply to the first stage as before. Transport of the second stage (secondary nanoparticle) is governed by

$$D_{N2} \frac{d^2 C_{N2}}{dx^2} - v \frac{dC_{N2}}{dx} - K_{el2} C_{N2} + \alpha K_{el1} C_{N1} = 0 \quad (29)$$

which we solve subject conditions similar to those for the free drug in the two-stage system  $C_{N2}(0) = 0$  and  $C_{N2}$  bounded for large  $x$ . The expression for the concentration for the second stage nanoparticle is formally similar to that for the free drug above, that is

$$C_{N2}(x) = A_2 \left[ \exp\left(-\frac{x}{L_{N2}}\right) - \exp\left(-\frac{x}{L_{N1}}\right) \right] \quad (30)$$

where  $L_{N2} = \frac{2D_{N2}}{(v^2 + 4K_{el2}D_{N2})^{1/2} - v}$  and  $A_2 = \frac{\alpha K_{el1} C_v}{D_{N2} \left(\frac{1}{L_{N1}}\right)^2 + \frac{v}{L_{N1}} - K_{el2}}$  which for no advection

reduce to  $L_{N2} = \left(\frac{D_{N2}}{K_{el2}}\right)^{1/2}$  and  $A_2 = \frac{\alpha \frac{K_{el1} C_v}{K_{el2}}}{\left(\frac{L_{N2}}{L_{N1}}\right)^2 - 1}$ , respectively. Now we add a third stage

which is the transport of the free drug after release from the secondary nanoparticle governed by

$$D_F \frac{d^2 C_F}{dx^2} - v \frac{dC_F}{dx} - \frac{1}{\phi} K_{on} C_{rec} C_F + \beta K_{el2} C_{N2} = 0 \quad (31)$$

For the distribution of the free drug we obtain

$$C_F(x) = -(A_{F1} + A_{F2}) \exp\left(-\frac{x}{L_F}\right) + A_{F1} \exp\left(-\frac{x}{L_{N1}}\right) + A_{F2} \exp\left(-\frac{x}{L_{N2}}\right) \quad (32)$$

where  $L_F = \frac{2D_F}{(v^2 + 4K_F D_F)^{1/2} - v}$ ,  $A_{F1} = \frac{\beta K_{el2} A_2}{D_F \left(\frac{1}{L_{N1}}\right)^2 + \frac{v}{L_{N1}} - K_F}$  and  $A_{F2} = \frac{-\beta K_{el2} A_2}{D_F \left(\frac{1}{L_{N2}}\right)^2 + \frac{v}{L_{N2}} - K_F}$

which reduces to  $A_{F1} = \frac{\beta \frac{K_{el2} A_2}{K_F}}{\left(\frac{L_F}{L_{N1}}\right)^2 - 1}$  and  $A_{F2} = \frac{\beta \frac{K_{el2} A_2}{K_F}}{1 - \left(\frac{L_F}{L_{N2}}\right)^2}$  without advection.



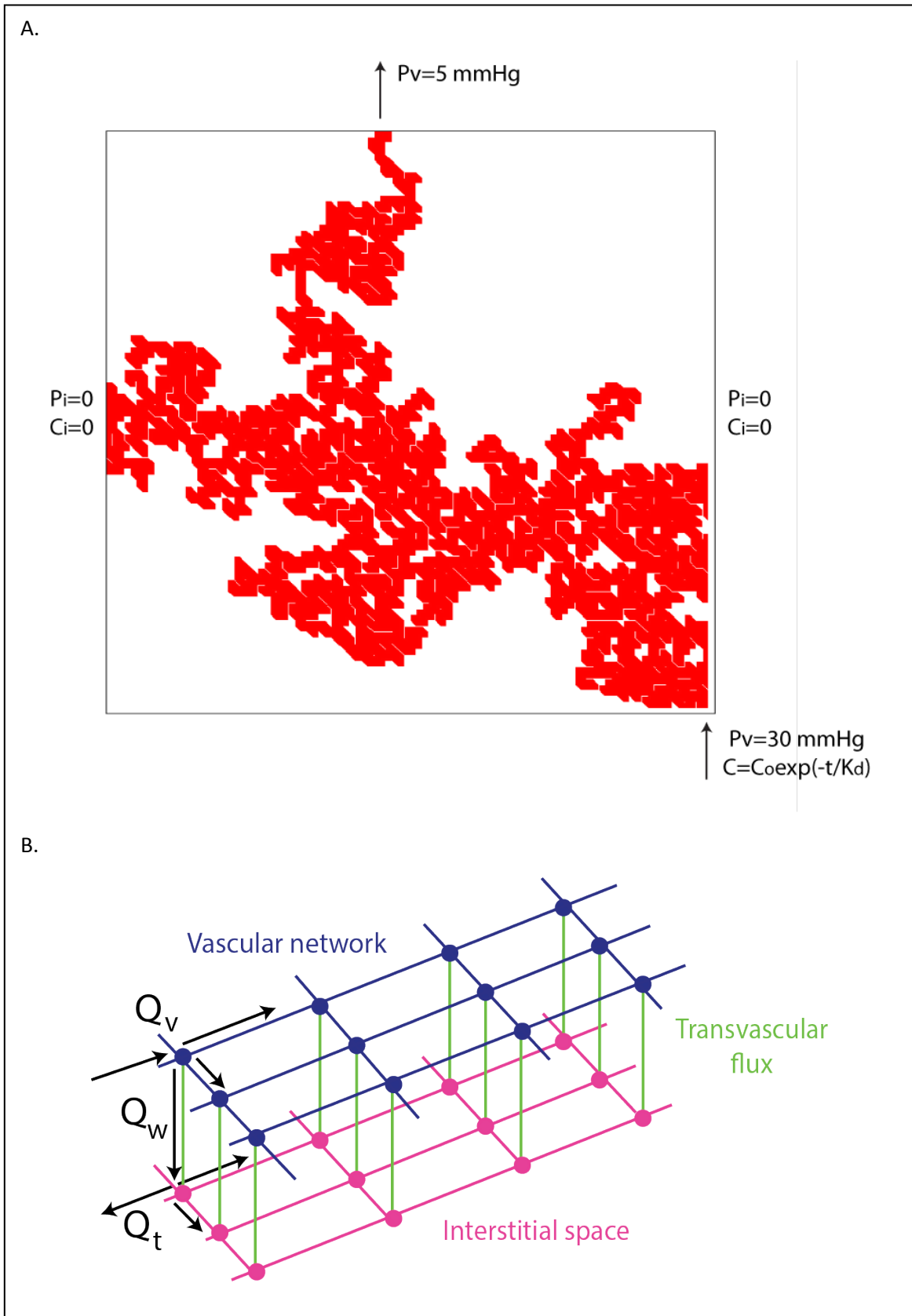
## Supplementary Tables and Figures

**Supplementary Table 1.** Values of model parameters employed in our analysis.

Parameter	Value	Reference
$K_d$ (blood circulation decay constant)	22 h for 20 nm particles 10 h for 100 nm particles	15, 17
$K_{el}$ (release rate constant)	$1.18 \times 10^{-4} \text{ s}^{-1}$ from data of Gelatin 100 nm multistage particles $1.5 \times 10^{-5} \text{ s}^{-1}$ fitting a first order kinetic to BIND-014 $2.1 \times 10^{-6} \text{ s}^{-1}$ fitting a first order kinetic to Doxil	11, 22
$K_{on}$ (binding rate constant)	$1 \times 10^1 - 1 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}$	19, 21
$K_{off}$ (unbinding rate constant)	$8 \times 10^{-3} \text{ s}^{-1}$	21
$K_{int}$ (cell uptake rate constant)	$5 \times 10^{-5} \text{ s}^{-1}$	21
Vessel wall pore diameter	$200 \pm 60 \text{ nm}$ (hyper-permeable tumor)	5
D (particle diffusion coefficient)	$5 \times 10^{-10} \text{ cm}^2/\text{s}$ for 100 nm particles $7 \times 10^{-8} \text{ cm}^2/\text{s}$ for 20 nm particles $3 \times 10^{-7} \text{ cm}^2/\text{s}$ for 10 nm particles $8 \times 10^{-7} \text{ cm}^2/\text{s}$ for 5 nm particles $3 \times 10^{-6} \text{ cm}^2/\text{s}$ for 1 nm particles	16
$\alpha, \beta$ (number of particles in the nanoparticle carrier)	$\alpha=800, \beta=10$ for a multistage 100 nm particle $\alpha=4, \beta=5$ for a multistage 20 nm particle $\alpha=10000$ for a two-stage 100 nm particle $\alpha=20$ for a two-stage 20 nm particle	14, 22
$\phi$ (tumor volume fraction accessible to drugs)	0.3 for 1 nm particles 0.1 for 5 nm particles 0.05 for 100 nm particles	19, 21
$C_{rec}$ (concentration of	$1 \times 10^{-5} \text{ M}$	13

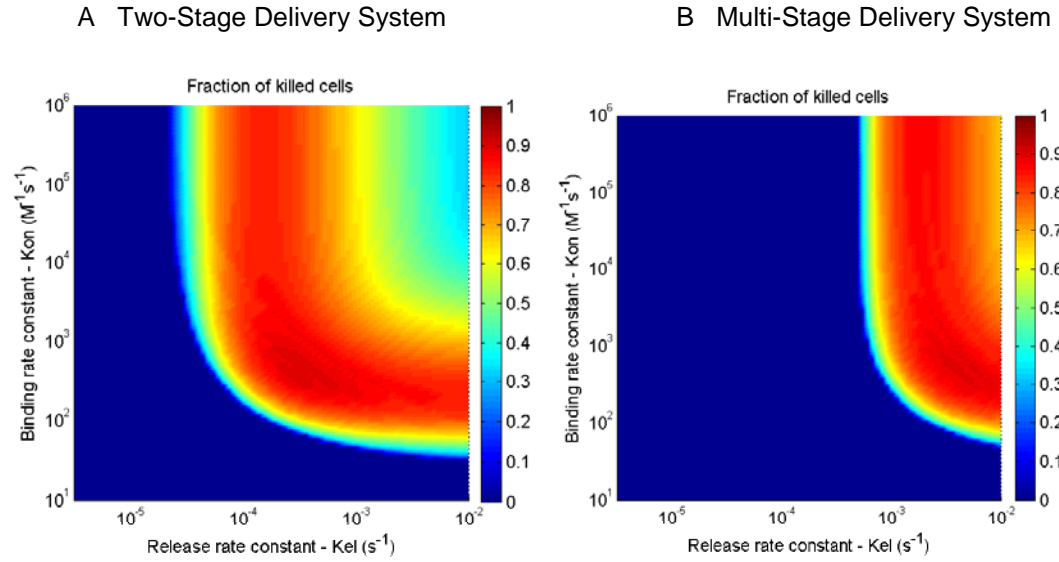
cell surface receptors)		
R (length of the domain)	0.5 cm	3
$K_t$ (interstitial space hydraulic conductivity)	$3 \times 10^{-8} \text{ cm}^2/\text{mmHg s}$	3
$P_v$ (pressure at the inlet)	30 mmHg	3
$S_v$ (vascular density)	$200 \text{ cm}^{-1}$	3
$\mu_{\text{blood}}$ (blood viscosity)	$3 \times 10^{-5} \text{ mmHg s}$	2
d (vessel diameter)	15 $\mu\text{m}$	2

**Supplementary Figure 1.** **A)** Computational domain showing the vascular percolation network (red color) and the boundary conditions. **B)** The domain is discretized in vascular and interstitial nodes, which are interconnected.  $Q_v$ ,  $Q_w$  and  $Q_t$  denote vascular, transvascular and interstitial fluid flow rate, respectively.

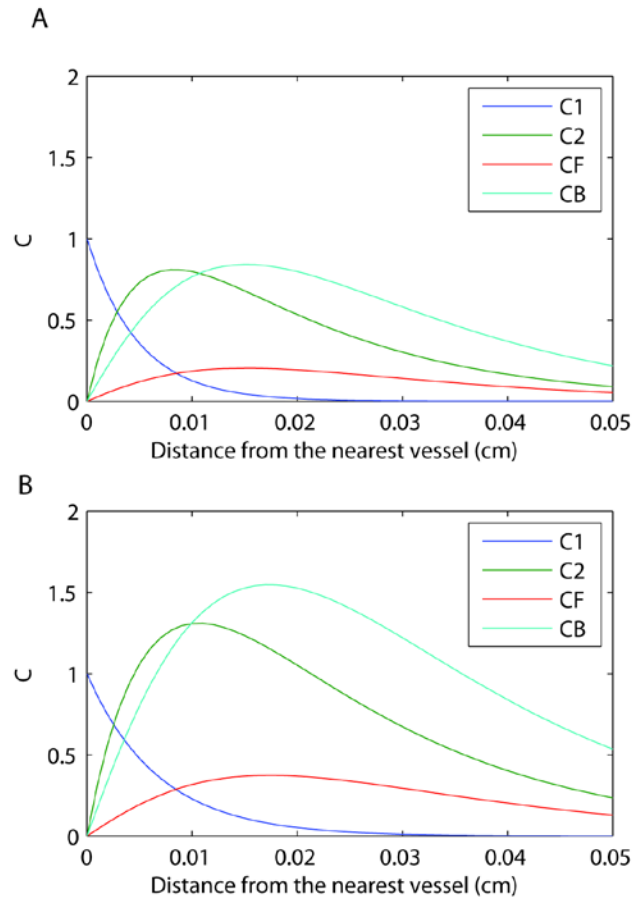


**Supplementary Figure 2.** Results from simplified 1-dimensional model.

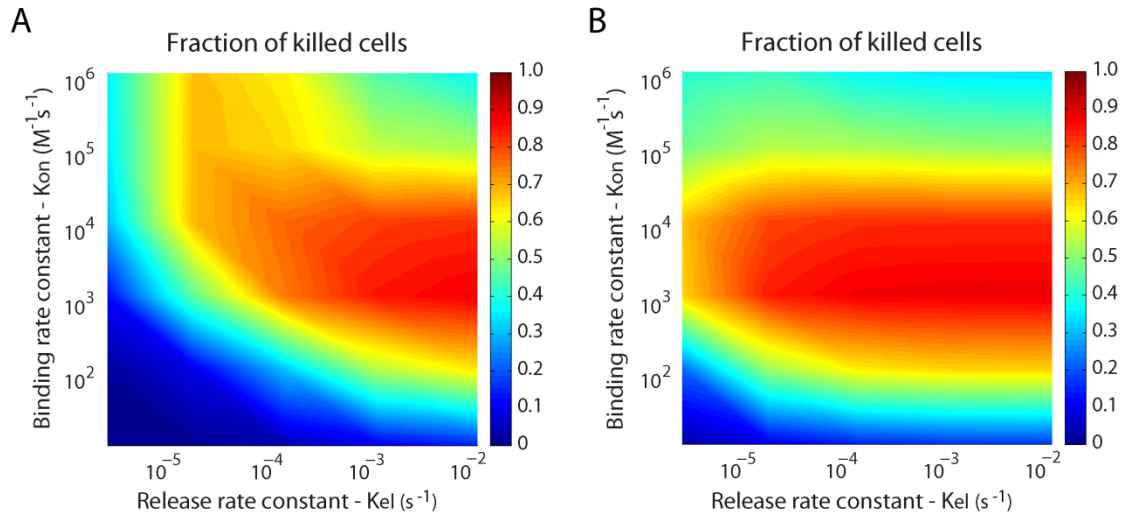
Optimization contour plots of the fraction of killed cells ( $C_B > 0.05C_v$ ) as a function of binding rate constant ( $K_{on}$ ) and rate constant of release ( $K_{el}$ ) for the 20 nm two-stage (A) and multi-stage (B) nanoparticles neglecting advection. The maximum distance from a blood vessel is 0.05 cm. Compare to Figure 5 in the main paper.



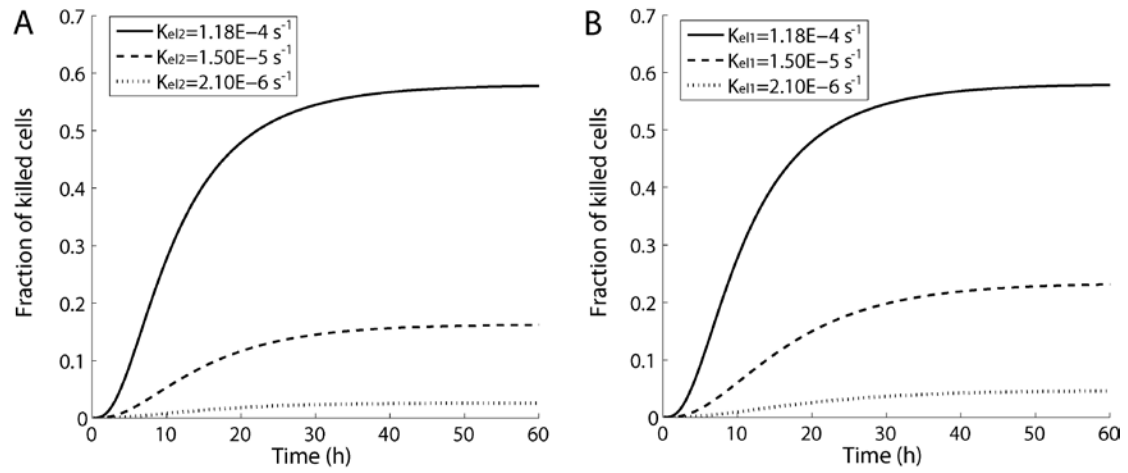
**Supplementary Figure 3.** Comparison of steady state multi-stage drug distribution without and with advection. Properties for a 20 nm first stage, a 5 nm second stage and a 1 nm free drug with  $K_{el1} = K_{el2} = 3 \times 10^{-3} \text{ s}^{-1}$  and  $K_{on} = 1 \times 10^3 \text{ M}^{-1} \text{ s}^{-1}$ . No advection (A),  $v = 0$  gives  $L_{N1} = 0.0048 \text{ cm}$ ,  $L_{N2} = 0.0163 \text{ cm}$  and  $L_F = 0.0095 \text{ cm}$ , while flow from the vessel (B),  $v = 1 \times 10^{-5} \text{ cm/s}$  gives  $L_{N1} = 0.0068 \text{ cm}$ ,  $L_{N2} = 0.0181 \text{ cm}$  and  $L_F = 0.0096 \text{ cm}$ . Advection by the interstitial fluid ( $v > 0$ ) can aid distribution of the drug.



**Supplementary Figure 4.** Optimization contour plots for a 20 nm, two-stage nanoparticle with the ability to bind to cancer cells. Two binding rate constants of the nanoparticle were used, (A) one for low binding affinity:  $1 \times 10^2 \text{ M}^{-1} \text{ s}^{-1}$  and (B) one for high binding affinity  $1 \times 10^4 \text{ M}^{-1} \text{ s}^{-1}$ . The contour plots are a function of the binding rate constant of the chemotherapy and the release rate constant from the nanoparticle. The blood half-life of the nanoparticle was taken to be  $K_d=10\text{h}$ .



**Supplementary Figure 5.** Fraction of killed cells for an 100nm, multistage nanoparticle. In panel (A) the release rate constant  $K_{el1}$  of the secondary particle was kept to  $1.18 \times 10^{-4} \text{ s}^{-1}$  and the release rate constant  $K_{el2}$  was varied. In panel (B) the release rate constant  $K_{el2}$  was kept to  $1.18 \times 10^{-4} \text{ s}^{-1}$  and  $K_{el1}$  was varied.  $K_{on}$  was set to  $1.5 \times 10^4 \text{ M}^{-1} \text{ s}^{-1}$  for both cases.



## References

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