Seek and Decode: Random Access with Physical-Layer Network Coding and Multiuser Detection

Giuseppe Cocco[‡], Stephan Pfletschinger[‡*†] and Monica Navarro[†]

[‡] German Aerospace Center – DLR

[*] Offenburg University of Applied Sciences

[†] Centre Tecnològic de Telecomunicacions de Catalunya (CTTC)

mailto: giuseppe.cocco@dlr.de

Abstract

We present a novel scheme for Slotted ALOHA random access systems (RAS) that combines physical-layer network coding (PLNC) with multiuser detection (MUD). PLNC and MUD are applied jointly at the physical (PHY) layer in order to extract any linear combination of messages experiencing a collision within a slot. The set of combinations extracted from a whole frame is then processed by the receiver to recover the original packets. A simple pre-coding stage at the transmitting terminals allows the receiver to further decrease the packet loss rate. We present results for the decoding at the PHY layer as well as several performance measures at frame level, namely throughput, packet loss rate and energy efficiency. The results we present are promising and suggest that a cross layer approach leveraging on the joint use of PLNC and MUD can significantly improve the performance of RA systems in the presence of slow fading.

I. Introduction

Random access systems (RAS) can be regarded as an opportunity and a challenge at the same time. On the one side RAS require little coordination among the transmitters. This, among other

Part of the results presented in this article have been presented at ICC 2014 [1] and NetCod 2014 [2] conferences.

advantages, makes it possible to live together with large delays that are typical, for instance, of satellite communication networks. As a drawback, the lack of coordination brings about the issue of signals from different transmitters interfering at the receiver. So far different ways of tackling the problem of collisions in RAS have been proposed. These include exploiting the difference in the power of the received signals [3] or applying multiuser detection (MUD) methods as in code-division multiple-access (CDMA) systems [4]. Multi-packet reception, i.e., the capability for the receiver to decode more than one packet from a collision, has been and still is an active research field. In [5] an overview of the main MUD techniques is presented. The impact of multi-packets reception in Slotted ALOHA systems has been studied in [6]. Another approach proposed in the literature consists in having each transmitter sending multiple replicas of the same packet within a frame. The receiver tries to decode the packets that do not experience collision as proposed in [7] or, once such clean packets have been decoded, it subtracts the decoded packets from the slots where their replicas are [8] [9]. Recently, the possibility of decoding functions of colliding signals has been studied in [10] and [11]. In these works the linearity of error correction codes has been applied in the two-way relay channel (TWRC) to decode the XOR of messages experiencing a collision. Starting from the sum of the physical signals and assuming the same channel code at both end nodes is used, the corresponding XOR is calculated and exploited, through an adequate MAC protocol, as side information to recover the single messages. This approach is one of the possible implementations of the wider concept of physical-layer network coding (PLNC). The performance limits for the decoding of the sum of colliding signals have been studied from an information theoretical perspective and assuming lattice codes in [12] [13]. Most of the literature about PLNC focuses on the TWRC. In [14] [15] a quaternary decoding approach for the MAC phase of the two-way relay channel has been proposed, showing that there is an advantage in computing the XOR by combining the previously estimated individual messages, rather than directly decoding the sum from the analog signal. In [16] PLNC has been applied to random access systems by decoding the XOR of all colliding signals within a slot and then trying to recover all transmitted packets within a frame using matrix manipulations in $\mathbb{F}_2$. In [17] and [18], an enhanced scheme based on PLNC over extension fields has been proposed. An information theoretical analysis of the performance of physical-layer network coding in random access systems has been presented in [19]. Recent variants of coded random access schemes are presented in [20], focusing on MAC aspects and their asymptotic performance. Details on the theoretical analysis of the different proposals based on the coded Slotted ALOHA paradigm (with the respective frame and frameless approaches) can be found in the references therein, together with discussions on practical implementation aspects. In [21] a theoretical analysis of coded Slotted ALOHA systems is presented. In [22] and [23] a practical implementation of a system that makes use of both PLNC and MUD in the multiple-access channel of a wireless local area network is presented. Specifically, in [23] the case of two colliding signals is considered, a relaying setup is assumed and a joint detection (but not joint decoding) is performed. Practical solutions for the detection of active users and the estimation of channel state information parameters are also being actively investigated [24]. Some of these techniques are based on compressed-sensing or sparse multi-user detection. Since this is a common problem for coded random access solutions in general, we are not providing a complete overview of recent work. Some examples and related bibliography can be found in [25] [26].

In the present paper we propose two novel schemes to enhance throughput and packet loss rate (PLR) of Slotted ALOHA networks that leverage on a combination of PLNC and MUD. In the proposed schemes each information message undergoes a pre-coding stage at the transmitter before the channel encoding. The pre-coding consists in a simple multiplication by a coefficient drawn at random from an extension field. The receiver tries to decode at the physical (PHY) layer any linear combination in $\mathbb{F}_2$ from the set of colliding bursts within each slot. Once the whole frame has been processed at the PHY layer, the receiver uses the set of linear combinations available to retrieve all messages transmitted within the frame by using matrix manipulation techniques over the same extension field of the pre-coding stage. The use of an extension field in the pre-coding stage decreases the PLR of the system. At the PHY layer the receiver employs a hybrid between a PLNC decoder and a MUD. Two different MUD schemes are considered in combination with PLNC. One is a joint decoder (JD), in which all signals are decoded jointly [1]. The other MUD technique we combine with PLNC is successive interference cancellation (SIC). We present numerical results for the number of innovative (i.e., linearly independent)

[1]This differs significantly from a parallel interference cancellation scheme (PIC), since in this last one several decoders are employed in parallel estimating a different message each, while in a JD just one decoder is used, which decodes jointly all of the messages.

messages decoded within a slot as well as for throughput, packet loss rate and energy efficiency in a framed slotted ALOHA-like scenario. Our results show that, unlike the scheme presented in [18], the joint use of PLNC and joint decoding is robust against slow fading, which characterizes many scenarios of practical relevance.

The rest of the paper is organized as follows. In Section II we introduce the system model. In Section III the proposed approach is described while in Section IV we focus on the different decoding alternatives at the PHY layer. Section V contains the numerical results, while the conclusions are presented in Section VI.

II. System Model

Let us consider a random multiple-access network with an infinite population of terminals and one receiver Rx. Time is divided into slots. Transmissions are organized in frames of S slots each. We define a packet $\mathbf{u}$ as a block of RN information bits. The user population generates an aggregated offered traffic which is modelled as a Poisson process of intensity G packets per slot. Each time a packet $\mathbf{u}_i = [u_{i,1}, \ldots, u_{i,RN}]$ is generated at terminal T_i, it is channel encoded using an encoder of rate R, thus creating a codeword $\mathbf{c}_i = [c_{i,1}, \ldots, c_{i,N}]$ of N bits. The same channel code is used by all transmitting terminals. The codeword $\mathbf{c}_i$ is then mapped to a binary phase-shift keying (BPSK)-modulated burst $\mathbf{x}_i$ and transmitted over the channel. We consider BPSK modulation for simplicity, but other kinds of modulations can also be used. It is worth noting that the specific modulation considered can have a significant impact on the packet loss rate performance of PLNC. In [27] and [28] it was shown that finding the modulation which minimizes the message error rate is not trivial even for an uncoded system and collisions of size 2. How to optimally design the modulation constellation in a coded system and for a generic collision size in case a joint PLNC and MUD receiver is used is a challenging open problem which is out of the scope of the present paper. We also point out that the schemes proposed in the following rely on channel codes and modulations already in use in commercial standards and have the advantage of requiring little modification at the transmitter side. Most of the additional complexity if moved at the receiver which usually has less constraints in terms of computational capabilities with respect to the user terminals.

We assume that the burst duration is approximately equal to that of a slot. Let us now consider one of the S slots of the frame. In case of a collision of K packets (namely, collision of *size*

K) the n-th sample of the received signal can then be written as

$$y_n = \sum_{k=1}^{K} h_k x_{k,n} + w_n, \quad w_n \sim \mathcal{N}(0,1), \tag{1}$$

where the fading coefficients are real-valued and follow a certain probability distribution with $E\{|h_k|^2\} = \mathsf{SNR}$, $E\{x\}$ being the mean value of x. The fading coefficients are estimated at the receiver but are not known at the transmitters and are assumed to change in an independently and identically distributed (i.i.d.) fashion across terminals and time slots. We further assume that the transmitters are synchronized such that all signals transmitted within a slot add up with symbol synchronism at the receiver. At the receiver side, Rx first processes the frame at the physical level one slot at a time. The PHY processing consists into applying a combination of MUD and PLNC in order to decode as many linearly independent messages as possible. Once the processing at the PHY level is completed, Rx applies a second step of decoding, which takes place at the frame level. The linear combinations recovered from the PHY layer processing are then treated as a system of equations in $\mathbb{F}_q$, q being an extension field of the kind $q = 2^{n_{bc}}$, $n_{bc} \in \mathbb{N}$. The decoding process at frame level is detailed in Section III while the details of how different combinations are extracted from the same collision are given in Section IV.

III. Random Access with PLNC and MUD

In the present section we describe the proposed random access scheme named Seek and Decode (S&D). The transmitter side is the same as in [18]. The main innovation is in the decoding process at both slot level and frame level. We briefly recall the operations at the transmitter side presented in [18] and then move on to the description of the receiver side.

A. Transmitter Side

Each burst is transmitted more than once within a frame, i.e., several replicas of the same burst are transmitted. Assume that terminal i has a message $\mathbf{u}_i$ to deliver to Rx during a given frame, i.e., terminal T_i is an *active terminal* in that frame. Before each transmission, terminal i pre-encodes $\mathbf{u}_i$ as depicted in Fig. 1. The message to be transmitted is divided into groups of n_{bc} bits each. Each group of bits is mapped to a symbol in $\mathbb{F}_q$, $q = 2^{n_{bc}}$, and then multiplied by a coefficient $\alpha_{i,j} \in \mathbb{F}_q$. The coefficient $\alpha_{i,j}$, $j \in \{1, \dots, S\}$, is chosen at random in each time slot j while it is fixed for all symbols within a message. Note that the pre-coding does not have

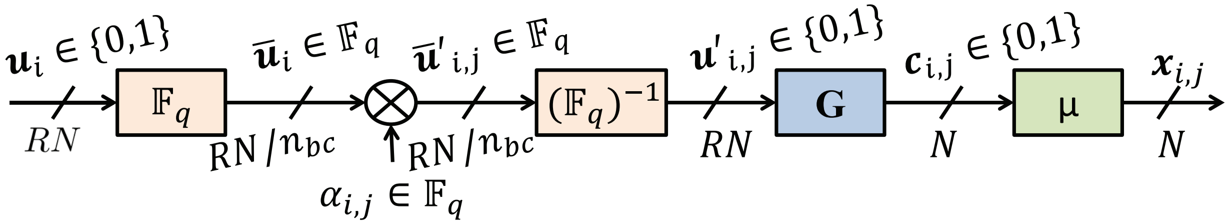

Fig. 1. Pre-coding, channel coding and modulation at the transmitter side. Pre-coding consists in mapping the message to a vector in $\mathbb{F}_q$, $q = 2^{n_{bc}}$, multiply each element of the vector by the same coefficient $\alpha_{i,j}$ randomly chosen in $\mathbb{F}_q$ and apply an inverse mapping $(\mathbb{F}_q)^{-1}$ from $\mathbb{F}_q$ to $\{0, 1\}$. The sub index j indicates the slot within a frame in which the replica of message $\mathbf{u}_i$ is transmitted. A different coefficient $\alpha_{i,j}$ is used for each replica.

any impact on the decoding process at the PHY layer and requires little increase in complexity with respect to a traditional scheme. The multiplication of $\mathbf{u}_i$ by $\alpha_{i,j}$ helps the decoding at frame level, which will be described later in this section. After the multiplication, the message is channel-encoded (block $\mathbf{G}$ in Fig. 1), a header is attached and the modulation takes place (block μ in Fig. 1). Pseudo-noise sequences with good cross-correlation properties can be used in the header in order to identify the user within the frame. Such identification allows the receiver to deduce the pre-coding coefficients used by each transmitter as described in the following. The coefficients $\alpha_{i,j}$ can be generated using a pseudo-random number generator. In a given frame the active terminal chooses a seed for the generator and takes from it as many outputs as the number of replicas to be transmitted. Each seed is associated to a certain header, which is detected by the receiver using the cross-correlation properties of the header[2]. The same header is used within a given frame by an active terminal. In this way the receiver can detect which slots a certain terminal is transmitting in and derive the coefficients used in the different replicas from the header. The header is also used to perform the channel estimation of each of the transmitters. A more detailed analysis of the issues related to header detection and channel estimation can be found in [18], [29], [30].

[2]Other PHY layer signatures can also be used by the terminals to allow Rx to identify the transmitters. This is a subject which has been extensively studied in literature and further discussion on this is out of the scope of the present work.

B. Receiver Side: Decoding at Frame Level

According to the literature related to random access systems, when two or more signals interfere at the receiver, this can either use some kind of interference cancelation or, as in physical-layer network coding, try to decode a function of the colliding signals. Most of the MUD techniques found in literature can be categorized as PIC or SIC. Often such methods are iterative and alternate a detection phase to an estimation phase. In the proposed scheme the receiver applies a *joint* decoder which tries to recover simultaneously all messages involved in the collision. An FFT-based belief propagation decoder over the vectorial combination of all message bits, which is described in detail in [31], is adopted. The decoder jointly estimates all the single messages and then calculates the XOR of any subset of the estimated messages. It is important to notice that, as shown in [15], the sum in $\mathbb{F}_2$ of a set of estimated messages can be correct even if the estimated messages taken individually contain errors. A cyclic redundancy check (CRC) can be used for error detection. Thanks to the linearity of the channel code, the XOR of the CRCs relative to a set of messages is a valid CRC for the XOR of the messages in the set. Here we assume ideal error detection at the receiver for ease of exposition. Given a slot with a collision of size K, the receiver tries to decode K independent linear combinations in $\mathbb{F}_2$ of the colliding signals. The total number of linear combinations that the decoder can try to recover is $\sum_{i=1}^{K} \binom{K}{i} = 2^K - 1$. Assuming the receiver is able to reliably estimate the random coefficients and the identity of the transmitters in each slot through the packet headers [18] [29], each decoded linear combination in $\mathbb{F}_2$ can be interpreted at the receiver, according to arithmetics of extension fields, as an equation in $\mathbb{F}_q$, $q = 2^{n_{bc}}$. Stacking together all equations the receiver ends up with a linear system having the form

$$\mathbf{A}^T \mathbf{U} = \mathbf{b}, \tag{2}$$

where $\mathbf{A}$ is the coefficient matrix having N^{tx} rows and a number of columns that depends on the number of combinations decoded at PHY layer, $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_{N^{tx}}]^T$ is a vector containing the information messages transmitted by the N^{tx} active terminals in the frame, $\mathbf{b}$ is a vector containing the output of the decoding at PHY layer and ^{T} is the transpose operator.

IV. Decoding at Slot Level

In Section III we described the proposed scheme assuming a joint decoder is applied at the PHY layer. As described in the previous section the joint decoder first estimates all the single messages involved in a certain collision. Afterwards, the S&D variant of the joint decoding is applied, in which the sum in $\mathbb{F}_2$ of any subset of the estimated messages is calculated. Since in some cases the sum of a set of estimated messages can be correct even if the estimated messages taken individually contain errors, the S&D approach increases the average number of useful packets decoded from a collision with respect to a normal joint decoder.

This approach is only one of the many in which MUD can be combined with PLNC. In fact other kinds of MUD can be adapted to the S&D scheme rather than joint decoding. Although some of them may lose in terms of performance with respect to the joint decoding approach, they can be attractive from a practical perspective for their lower complexity.

For completeness we recall that PLNC can be applied to recover the sum of all the collided packets for a generic collision size directly to the received signal, i.e., without first trying to estimate the individual messages. We do not report here the details for a matter of space and remaind the interested reader to [32] for further details.

In the present section we describe several alternative schemes while in Section V we numerically compare their performance in terms of the number of innovative packets decoded from a collision. Here we focus only on the decoding within a slot, while the performance at frame level is assessed in Section V.

A. Separate Decoding

The simplest approach is to decode each packet separately, considering all other packets as interference. As for all other schemes to follow, we assume channel state information (CSI) at the receiver for all transmitting terminals as well as knowledge at the receiver of the transmit alphabet, i.e. BPSK constellation [3]. With this, and assuming K bursts collide in a slot, we can write the log-likelihood value (L-value) of user $i, i \in \{1, 2, \ldots, K\}$ and symbol position $n, n \in \{1, 2, \ldots, N\}$, as:

[3] A further simplification would be to consider the interference as Gaussian noise, which would result in reduced performance and is therefore not considered here.

$$L_{i,n} \triangleq \ln \frac{P\left[c_{ki,n}=1 \mid y_n\right]}{P\left[c_{i,n}=0 \mid y_n\right]} = \ln \frac{P\left[x_{i,n}=1 \mid y_n\right]}{P\left[x_{i,n}=-1 \mid y_n\right]}. \tag{3}$$

According to Eqn. (1), y_n is a weighted sum of the n-th symbols of all K colliding signals. Since the received symbol y_n depends on all symbols, we need to marginalize over all other users' symbols. For this, we define the sets $\mathcal{X}_i^{(b)} \triangleq \left\{\mathbf{x}=\mu\left(\mathbf{d}\right): \mathbf{d} \in \mathbb{F}_2^K, d_i = b\right\}$ for $b \in \mathbb{F}_2$, with cardinality $\left|\mathcal{X}_i^{(b)}\right| = 2^{K-1}$. We can think of the variable $\mathbf{d}$ as the vector of the coded bits of all users at the same position, i.e. $\mathbf{d}_n = \left[c_{1,n}, c_{2,n}, \ldots, c_{K,n}\right]^{\mathrm{T}}$. We obtain for the L-values

$$\begin{aligned} L_{i,n} &= \ln \frac{\sum_{\mathbf{x}\in\mathcal{X}_i^{(1)}} P\left[\mathbf{x} \mid y_n\right]}{\sum_{\mathbf{x}\in\mathcal{X}_i^{(0)}} P\left[\mathbf{x} \mid y_n\right]} = \ln \frac{\sum_{\mathbf{x}\in\mathcal{X}_i^{(1)}} p\left(y_n \mid \mathbf{x}\right)}{\sum_{\mathbf{x}\in\mathcal{X}_i^{(0)}} p\left(y_n \mid \mathbf{x}\right)} \\ &= \ln \frac{\sum_{\mathbf{x}\in\mathcal{X}_i^{(1)}} \exp\left(-\left(y_n - \mathbf{h}^{\mathrm{T}}\mathbf{x}\right)^2\right)}{\sum_{\mathbf{x}\in\mathcal{X}_i^{(0)}} \exp\left(-\left(y_n - \mathbf{h}^{\mathrm{T}}\mathbf{x}\right)^2\right)} \\ &= \operatorname*{jacln}_{\mathbf{x}\in\mathcal{X}_i^{(1)}} \left\{-\left(y_n - \mathbf{h}^{\mathrm{T}}\mathbf{x}\right)^2\right\} \\ &\quad - \operatorname*{jacln}_{\mathbf{x}\in\mathcal{X}_i^{(0)}} \left\{-\left(y_n - \mathbf{h}^{\mathrm{T}}\mathbf{x}\right)^2\right\} \end{aligned} \tag{4}$$

where $\operatorname{jacln}\left\{x_1, \ldots, x_n\right\} \triangleq \ln \sum_{j=1}^{n} \exp\left(x_j\right)$ denotes the Jacobian logarithm, which can be computed recursively and for which computationally efficient approximations exist [33]. These L-values are input to a soft-input decoder, which typically is a Viterbi, a turbo or an LDPC decoder.

B. Successive Interference Cancellation (SIC)

A straightforward and well-known extension of basic single-user decoding is SIC: if a packet $\mathbf{u}_{k^*}$ is successfully decoded, its corresponding codeword $\mathbf{c}_{k^*}$ and symbol sequence $\mathbf{x}_{k^*}$ are known and can be subtracted from the received signal $\mathbf{y}_n$, creating a multiple-access channel (as defined in [34]) within a slot with $K-1$ terminals. This process can be repeated until decoding of all remaining packets fails. To avoid unneccessary computations, we can exploit the knowledge of the instantaneous SNRs and order the users accordingly: let π be a permutation of $\{1, 2, \ldots, K\}$ such that

$$h_{\pi(1)} \geq h_{\pi(2)} \geq \cdots \geq h_{\pi(K)}. \tag{5}$$

Then decoding starts with user $\pi(1)$. Apart from reducing computational complexity, this ordering is also useful to reduce the probability of undetected errors. To check the correct decoding of a packet, usually an additional error detection code, e.g. a CRC, has to be introduced into each message $\mathbf{u}_k$. Since there is a non-zero probability that an erroneous decoding is not detected, the number of decoding attempts with low probability of success should be kept to a minimum.

C. Seek & Decode with Successive Interference Cancellation (S&D+SIC)

For a coded Slotted ALOHA system, a further decoding step after SIC is possible. Assume that after the SIC procedure described above, $K-K_1$ packets have been correctly decoded, hence leaving $K_1 \in \{2, \ldots, K\}$ packets for which decoding failed. In this situation, the receiver can try to decode a combined packet, which is given by the sum of two or more of the packets that have not yet been decoded. In a typical SIC the decoding process would stop here. In the proposed S&D approach, instead, the receiver can try to decode the sum of a subset of $\{1, 2, \ldots, K_1\}$, e.g. given by $\mathcal{K} = \{k_1, k_2, \ldots, k_\ell\} \subset \{1, 2, \ldots, K_1\}$. If the decoding is successful, such decoded packet can be exploited as a side information in order to help the decoding of other packets within the same collision. The way in which such side information is exploited resembles the SIC process, even though the cancellation is not applied directly on the sampled signal. In the following we detail such mechanism more in depth. Let us assume that no user in the set $\{1, 2, \ldots, K_1\}$ could be decoded with the normal SIC. Then the receiver can try to decode the sum of a subset of $\{1, 2, \ldots, K_1\}$, e.g. given by $\mathcal{K} = \{k_1, k_2, \ldots, k_\ell\} \subset \{1, 2, \ldots, K_1\}$. For this subset we define the sets of constellation symbols for $\ell \geq 2$ as

$$\mathbb{X}_\ell^{(b)} \triangleq \left\{ \mathbf{x} = \mu\left(\mathbf{d}\right) : \ \mathbf{d} \in \mathbb{F}_2^\ell \text{ with } \sum_{i=1}^{\ell} d_i = b \right\} \ , b \in \mathbb{F}_2, \tag{6}$$

$\mu()$ being the mapping function from bits to constellation symbols, and obtain the corresponding L-values as

$$L_n^{\mathcal{K}} = \ln \frac{\sum\limits_{\mathbf{x} \in \mathbb{X}_\ell^{(1)}} \exp\left(-\left(y_n - \left[h_{k_1} h_{k_2} \cdots h_{k_\ell}\right] \mathbf{x}\right)^2\right)}{\sum\limits_{\mathbf{x} \in \mathbb{X}_\ell^{(0)}} \exp\left(-\left(y_n - \left[h_{k_1} h_{k_2} \cdots h_{k_\ell}\right] \mathbf{x}\right)^2\right)}. \tag{7}$$

These L-values $L_1^{\mathcal{K}}, L_2^{\mathcal{K}}, \ldots, L_N^{\mathcal{K}}$ are fed to the soft-input decoder, which, if successful, finds the corresponding codeword $\sum_{k \in \mathcal{K}} \mathbf{c}_k$ or message $\sum_{k \in \mathcal{K}} \mathbf{u}_k$. Note that the sum of messages or codewords is defined in the finite field $\mathbb{F}_2$, which is the same as the bit-wise XOR. This concept of packet combining is closely related to inter-flow network coding and it exploits the linearity of the code, which can be seen by the relation

$$\sum_{k\in\mathcal{K}} \mathbf{c}_k = \sum_{k\in\mathcal{K}} \mathbf{u}_k \mathbf{G}. \tag{8}$$

For error detection, since CRC codes are also binary linear codes, the same CRC can be used. For K_1 undecoded packets, there exist

$$\sum_{\ell=2}^{K_1} \binom{K_1}{\ell} = 2^{K_1} - K_1 - 1$$

combinations of two or more packets, for which a decoding attempt is possible from the L-values defined by (7). With this definition, note that the subsets $\mathbb{X}_\ell^{(b)}$ only depend on b and on the number of packets ℓ but not on their indices $k_1, \ldots, k_\ell$. After successful decoding of a packet sum, a subsequent idea is to re-apply interference cancellation with the packet combination. This, however, is not directly possible since the combined codeword $\mathbf{c}_\mathcal{K} = \sum_{k\in\mathcal{K}} \mathbf{c}_k$ does not correspond to any received symbol sequence $\mathbf{x}_k$ in (1) and the sum of codewords and symbol sequences are taken over different fields, namely $\mathbb{F}_2$ and $\mathbb{R}$. However, knowledge of a combined packet $\mathbf{c}_\mathcal{K}$ might still be useful for another decoding attempt: the cardinality of the sets $\mathbb{X}_\ell^{(b)}$ can be reduced by a factor of two by introducing the additional constraint of the known combined packet. Then, the L-values can be recomputed and new decoding attempts (including $\ell = 1$ for individual packets) can be undertaken. This approach brings about a slight additional complexity due to the constraint on the decoded combination. In this case, the sets $\mathbb{X}_\ell^{(b)}$ will additionally depend on n and hence have to be computed for each coded bit.

It is interesting how such approach has strong similarities with [35]. In [35] the decoder first tries to decode linear combinations of a subset of the colliding messages, and then uses the knowledge of such combination to help recovering others. In [35] as well as in the approach just presented, the knowledge of the first combination decoded can not be exploited by just subtracting it from the received signal, since it does not contain a waveform corresponding to the decoded combination. However, in both cases such side information can be exploited by the decoder. Although there are significant differences between the channel models in the two cases, a joint study of the two models may lead to interesting results from a practical perspective. The need for an in-depth analysis of the subject does not allow for an adequate assessment in the present paper and is left as a promising matter of study for future works.

D. Seek & Decode with Joint Decoding (S&D+JD)

From (1) we can observe that, for what concerns the detection, the received samples y_n depend on all coded bits $c_{k,n}$ at the same bit position but are independent of bits at other positions. The optimum decoding approach is therefore to consider the vectorial symbols $\mathbf{d}_n \triangleq [c_{1,n}, c_{2,n}, \ldots, c_{K,n}]^{\mathrm{T}}$ jointly. This can be done with a joint decoder which operates on the vectors $\mathbf{d}_n$ or on an equivalent integer representation $\bar{d}_n$ such that $\mathbf{d}_n = \mathrm{bin}(\bar{d}_n)$. The notation $\mathrm{bin}(b)$ denotes the binary representation of the non-negative integer b. For LDPC and for convolutional codes, such joint decoders are described in [31], [36]. The decoder input is given by the probability vector

$$\mathbf{p}_n \triangleq \begin{bmatrix} p_n(0) \\ p_n(1) \\ \vdots \\ p_n(2^K - 1) \end{bmatrix} \in \mathbb{R}^{2^K}, \tag{9}$$

where

$$p_n(b) \triangleq P\left[\mathbf{d} = \mathrm{bin}\left(b\right) \mid y_n\right] \propto p\left(y_n \mid \mathbf{x} = \mu\left(\mathrm{bin}(b)\right)\right), \tag{10}$$

for $b = 0, 1, \ldots, 2^K - 1$. Let $\bar{\mathbf{x}}_b = \mu\left(\mathrm{bin}(b)\right)$, then

$$\mathbf{p}_n = \alpha \begin{bmatrix} \exp\left(-\left(y_n - \mathbf{h}^{\mathrm{T}}\bar{\mathbf{x}}_0\right)^2\right) \\ \exp\left(-\left(y_n - \mathbf{h}^{\mathrm{T}}\bar{\mathbf{x}}_1\right)^2\right) \\ \vdots \\ \exp\left(-\left(y_n - \mathbf{h}^{\mathrm{T}}\bar{\mathbf{x}}_{2^K-1}\right)^2\right) \end{bmatrix}, \tag{11}$$

where α is a scaling factor which is irrelevant for the decoding algorithm. The decoder output is an estimate of all messages (or equivalenty of all codewords),

$$\hat{\mathbf{U}}_{\mathrm{slot}} = \begin{bmatrix} \hat{\mathbf{u}}_1 \\ \hat{\mathbf{u}}_2 \\ \vdots \\ \hat{\mathbf{u}}_K \end{bmatrix}. \tag{12}$$

Note that $\hat{\mathbf{U}}_{\mathrm{slot}}$ in Eqn. (12) refers to the packets transmitted within a slot, while $\hat{\mathbf{U}}$ in (2) refers to the packets transmitted in a whole frame. Making use of an error detecting code, the receiver checks all possible packet combinations, i.e. all $2^K - 1$ non-empty subsets of

$\{\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \ldots, \hat{\mathbf{u}}_K\}$ and builds the binary matrix $\mathbf{A}_{slot} \in \mathbb{F}_2^{\left(2^K-1\right)\times K}$. Matrix $\mathbf{A}_{slot}$ is such that its rows $\mathbf{a} = [a_1, a_2, \ldots, a_K]$ indicate the user indices which are contained in successfully decoded combinations. For instance, if the combined packet $\mathbf{c}_1 + \mathbf{c}_3 + \mathbf{c}_4$ is correctly decoded, the corresponding row is $\mathbf{a} = [1, 0, 1, 1, 0, 0]$ for $K = 6$. From this matrix, the number of innovative packets decoded from the collision of size K is calculated as its rank. This joint decoding approach reverses the order of the S&D+SIC method: while in S&D+SIC the packet combination is determined first and then a decoding attempt is carried out, joint decoding first tries to decode all packets jointly and then the receiver checks which combinations are correct.

In order to assess the performance of the different schemes considered so far, we count the number of *innovative* packets per slot. Innovative packets are either individually decoded packets or combinations of packets which cannot be obtained by combining other decoded packets. The number of innovative packets is the same as the number of linearly independent packet combinations, i.e., the rank of $\mathbf{A}_{slot}$ in $\mathbb{F}_2$ arithmetic.

Another benchmark we consider is joint decoding (JD), which consists in applying the joint decoder without PLNC. We adopt JD and SIC as benchmarks since they allow to measure the gains of the joint use of PLNC and MUD with respect to MUD only. The main features of the schemes presented in this section are summarized in Table I.

It is worth noting that many other MUD methods have been proposed and proved to achieve good performance with respect to simple SIC, such as Turbo MUD [37]. For a matter of space all such methods can not be compared in the present paper, and a full comparison is out of the scope of this work. Besides, our choice of the MUD schemes is motivated by the following. The JD is the optimal decoder, in that it jointly decodes the received messages, thus achieving better performance (average rank of $\mathbf{A}_{slot}$) than any other decoder. The SIC has been selected due to its low implementation complexity and practical importance. As a matter of facts, SIC is nowadays included in commercial communications standards such as the Digital Video Broadcasting - Return Channel to Satellite (DVB-RCS) [38].

E. Example

In the following we illustrate the S&D scheme with a toy example. Let us consider a frame with $S = 2$ slots and $N^{tx} = 4$ active terminals. Let us assume that terminals 1 and 2 transmit in both slots, each time choosing at random their pre-coding coefficients. Terminal 3 only transmits in the first slot while terminal 4 transmits only in the second, as illustrated in Fig. 2. The S$\&$D decoder is applied at the physical layer in one of the variants presented previously in the present section. As explained, the S$\&$D decoder consists of a combination of PLNC and MUD and, depending on the channel state of each of the transmitters, it may be able to decode from a single collision a number of linearly independent combinations up to the collision size. In the following example we assume that the decoder is able to output only two linear combinations from each of the two slots as shown in the picture. Starting from these combinations, the receiver tries then to recover all information messages $\mathbf{u}_1, \ldots, \mathbf{u}_4$ by applying another decoding stage, this time at packet level rather than at the PHY level. The decoding is possible if the coefficient matrix $\mathbf{A}$ in $\mathbb{F}_q$ (shown below) has rank equal to the number of active terminals

$$\mathbf{A}^T = \begin{pmatrix} \alpha_{1,1} & \alpha_{2,1} & 0 & 0 \\ \alpha_{1,1} & 0 & \alpha_{3,1} & 0 \\ \alpha_{1,2} & \alpha_{2,2} & 0 & 0 \\ 0 & \alpha_{2,2} & 0 & \alpha_{4,2} \end{pmatrix}.$$

In order to further clarify how the PHY decoder is able to obtain the system in Eqn. (2) starting from the analog superposition of the interfering signals, let us consider the decoding of slot 1 in the example of Fig. 2. The physical signal seen by the receiver is $\mathbf{y}_1$, which is the superposition of signals $\mathbf{x}_{1,1}$, $\mathbf{x}_{2,1}$ and $\mathbf{x}_{3,1}$ transmitted by terminal 1, 2 and 3, respectively, each weighted by the corresponding fading coefficient. By applying the S$\&$D decoder described previously in this section the decoder outputs the bit-wise XOR of two different pairs of messages, namely $\mathbf{u}'_{1,1} \oplus \mathbf{u}'_{2,1}$ and $\mathbf{u}'_{1,1} \oplus \mathbf{u}'_{3,1}$. We recall that $\mathbf{u}'_{i,1}$ is the packet that is transmitted by terminal T_i in slot 1 after channel encoding and modulation. Due to the pre-coding (multiplication times a random coefficient $\alpha_{i,1}$) we have $\mathbf{u}'_{i,1} = \alpha_{i,1}\mathbf{u}_i$, where the multiplication is done in $\mathbb{F}_q$, $q = 2^{n_{bc}}$. According to the arithmetics of extension fields, the bit-wise XOR (sum in $\mathbb{F}_2$) of $\mathbf{u}'_{1,1}$ and $\mathbf{u}'_{2,1}$ is equivalent to the equation $\alpha_{1,1}\mathbf{u}_1 + \alpha_{2,1}\mathbf{u}_2$ in $\mathbb{F}_q$. The pre-coding process adds little complexity to the transmitters and allows to achieve better results in terms of packet loss rate (PLR) as shown in the numerical results presented in Section V.

We recall from Section III that the coefficients in the matrix $\mathbf{A}^T$ above are chosen at random by the four transmitters (see Fig. 1). Specifically, transmitter $i, i = 1, \ldots, 4$, chooses coefficients $\alpha_{i,j}$, $j = 1, 2$. We see in the present example that coefficient $\alpha_{1,1}$ is present twice in the first

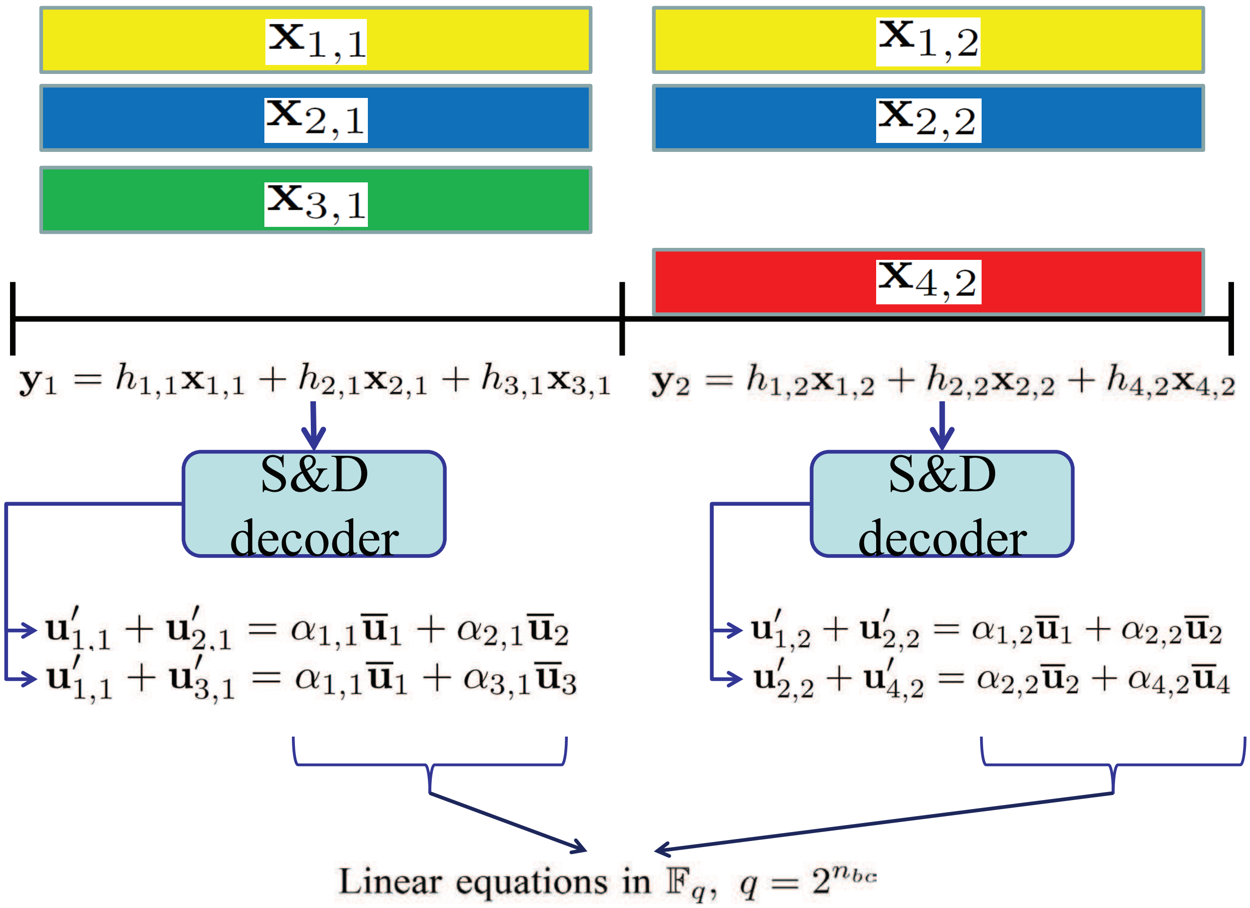

Fig. 2. Example of decoding at the PHY layer in S&D with a two-slots frame and four active terminals. Terminals 1 and 2 transmit in both slots, each time choosing at random their pre-coding coefficients. Terminal 3 only transmits in the first slot while terminal 4 transmits only in the second. We recall that, as shown in 1, $\bar{\mathbf{u}}_i$ represents the mapping of the information message $\mathbf{u}_i$ form an RN-dimensional vector in $\mathbb{F}_2$ to an RN/n_{bc}-dimensional vector in $\mathbb{F}_q$.

column of matrix $\mathbf{A}^T$. This is because the first two rows of the matrix correspond to equations obtained from the same slot. Note also that matrix $\mathbf{A}$ is rank deficient if coefficients are chosen in $\mathbb{F}_2$ (i.e., all coefficients shown in the matrix above are equal to 1), while it can be full rank if coefficients are chosen in some larger extension field, since the probability of obtaining a full rank matrix increases with the field size [39]. This motivates the inclusion of the pre-coding stage. We also note that in the example the average number of packets decoded per slot, if $\mathbf{A}$ is full rank, is 2.

Note that $\mathbf{A}$ contains information about the packet combinations within a given frame, but it is not the same as the matrix $\mathbf{A}_{slot}$ defined in Section IV. Matrix $\mathbf{A}$ is obtained by the receiver combining the $\mathbf{A}_{slot}$ matrices from all the slots in the frame and using the information relative to the pre-coding coefficients.

We stress the fact that the proposed scheme does much more than simply applying a MUD, since any linear combination of the colliding signals decoded at the PHY layer can be exploited in the second decoding phase at the frame level. We also note that, in principle, it would be possible to use the soft information extracted from each slot and combine it at the frame level. Although such approach would perform better than S&D, its complexity and memory requirements would be much larger with respect to the S&D scheme, which has the advantage of processing each slot only once and allows a lower complexity decoding at the frame level, since all operations are performed over a GF of size $2^{n_{bc}}$, which is suited to a digital implementation.

To conclude this section we recall that up to now we made the assumption that the receiver is able to estimate the fading coefficients starting from the preambles of the colliding signals. The practical feasibility of the channel estimation, as well as other practical issues, have been discussed in [29] and so are not dealt with explicitly here. The channel estimation based on the estimate-maximize algorithm presented in [29] has been enhanced in [30] exploiting the cross-correlation properties of the preamble as well as considering the presence of pilot symbols, that are foreseen by many standards, showing that the average channel estimation error in a MUD context can be kept reasonably low in a practical setup.

F. Complexity Considerations and Possible Combined Approaches

An important aspect in the different decoding approaches at the PHY layer is their performance-complexity tradeoff. For the basic separate decoding scheme, complexity can be reduced by ordering users according to their instantaneous SNR and stop decoding after the decoding of one user has failed. This will obviously cause a slight performance loss which depends mainly on the SNR differences and on the applied coding scheme, i.e. basically on the packet length. The same idea can be applied to both SIC techniques, while for S&D+SIC, a packet combination can be checked for linear independency *before* the decoding attempt. The complexity of S&D+SIC in the worst case is proportional to $2^K - 1$ decoding attempts. The complexity of joint decoding using LDPC codes is proportional to $K \cdot 2^K$ for belief propagation with transform-based check-node processing [40], [41]. This complexity can be reduced on the one hand by applying joint decoding after SIC and on the other hand by applying reduced-complexity decoding algorithms

[42].

V. Numerical Results

In this section we evaluate numerically the performance of the proposed schemes. First we compare the different PHY layer decoding approaches presented in Section IV in terms of number of innovative packets decoded from a single slot, then we move to the comparison of throughput, packet loss rate and energy efficiency at frame level for the S&D scheme and several benchmark systems.

A. Performance at Slot Level

We recall that innovative packets are either individually decoded packets or combinations of packets which cannot be obtained by combining other decoded packets. Figures 3 and 4 show the achieved number of innovative packets per slot with the described decoding techniques with 4 and 8 users, that correspond to the average rank of the matrix $\mathbf{A}_{slot}$ defined in Section IV. We can see that for both cases, S&D+JD performs best and its gain with respect to the others increases with the number of users. For a high number of users, the advantage of S&D+JD to all other techniques is dramatic. On the other hand, we point out that, unlike S&D+JD, the S&D+SIC scheme has the advantage that is does not require any modification at the decoder, since only the LLR calculation is modified with respect to a standard receiver. We further note that the advantage of S&D+SIC over pure SIC decreases with the number of users. For sufficiently high SNR, all methods benefit from collided packets, which can be most clearly seen in Fig. 3 for four users. At low SNR the average number of recovered packets per slot is close to the single-user case, while for medium to high SNR, on average more than one packet is recovered from a single slot. For all considered cases, the number of innovative packets tends to K as the SNR grows, i.e. for high SNR nearly all collided packets can be decoded.

B. Performance at Frame Level

We define the *normalized throughput* $\mathcal{T}$ as the average number of packets decoded within a slot averaged across the realizations. We further define the PLR as the ratio of the number of lost packets to the total number of packets transmitted (not counting repetitions). The following

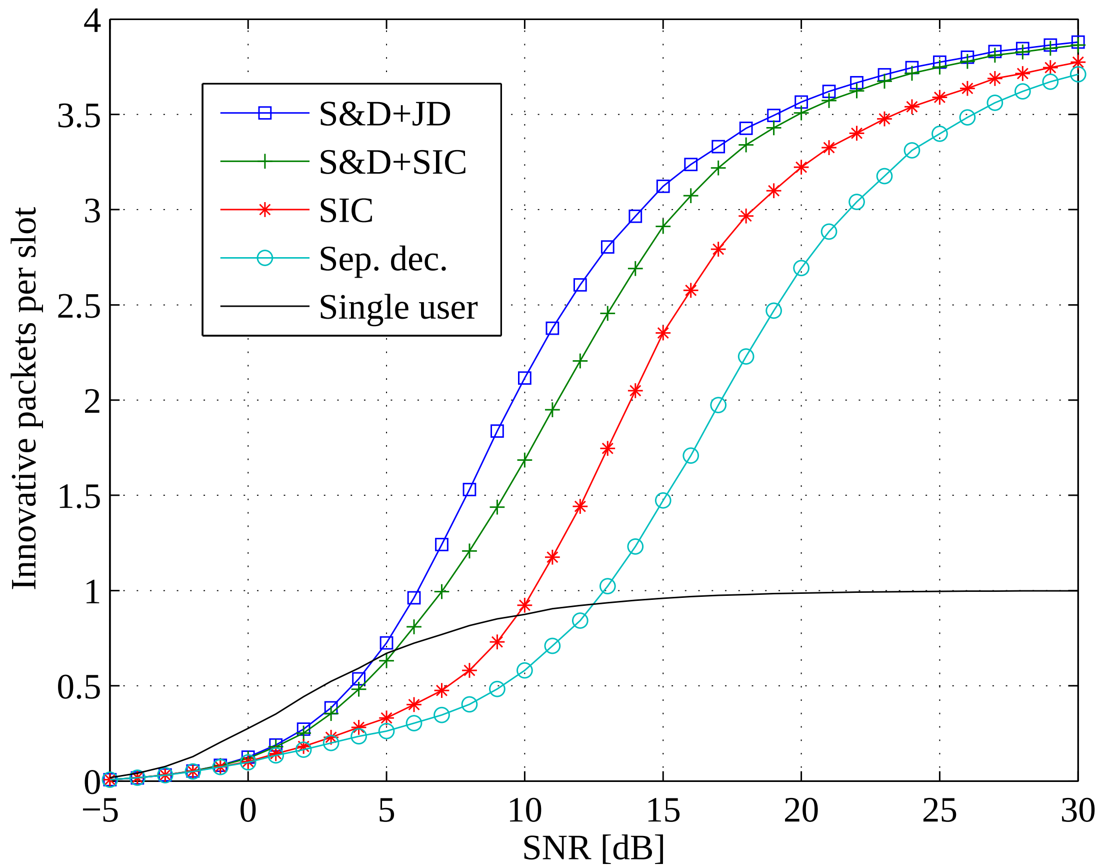

Fig. 3. Innovative packets decoded per slot versus average SNR in Rayleigh fading channel for a collision of size $K = 4$.

holds:

$$\mathcal{T} = G(1 - PLR). \tag{13}$$

Note that G, which represents the logical load of the network [8] [9], is independent of the number of times a message is repeated within a frame. The physical load on the network is larger than or equal to G. In particular, if 2 copies of the same packet are sent by each active terminal, then the physical load is twice as large as the logical load. Since the interaction between the frame and the PHY layers are of fundamental importance in the schemes considered here, in the simulations the whole decoding process has been implemented. The actual decoded combinations at the physical layer have been used as input to the decoder at the frame level. As suggested in Section III, if $rank(\mathbf{A}) < N^{tx}$, i.e. not all messages can be decoded in a frame, the receiver

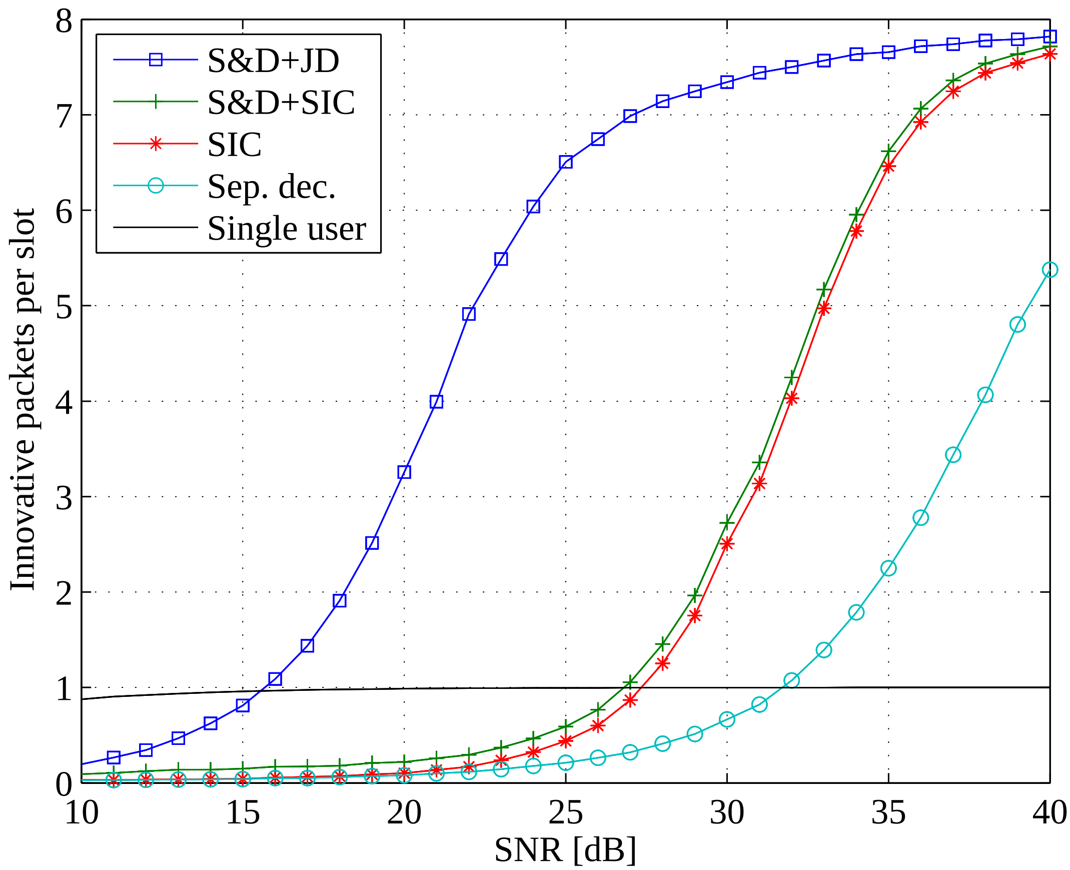

Fig. 4. Innovative packets decoded per slot versus average SNR in Rayleigh fading channel for a collision of size $K = 8$.

applies Gaussian elimination on $\mathbf{A}$ in order to extract as many packets as possible. In Fig. 5, 6 and 7 $\mathcal{T}$, PLR and the energy efficiency are plotted against the network load G, respectively. The energy efficiency is defined as the ratio of the number of repetitions (which is proportional to the total amount of energy used to transmit a packet) to the number of decoded packets (not counting repetitions). Two repetitions and a frame with $S = 10$ slots have been considered for all schemes. A Rayleigh block fading channel with 15 dB average SNR has been considered. The LDPC code of the WiMAX standard with parameters $N = 576$, $R = 1/2$, and BPSK modulation have been adopted. A maximum collision size of $K = 7$ has been set, i.e., collisions of more than 7 signals are discarded. The introduction of a maximum decodable collision size is justified by practical issues such as complexity and power saturation at the receiver. In Fig. 5 it can be

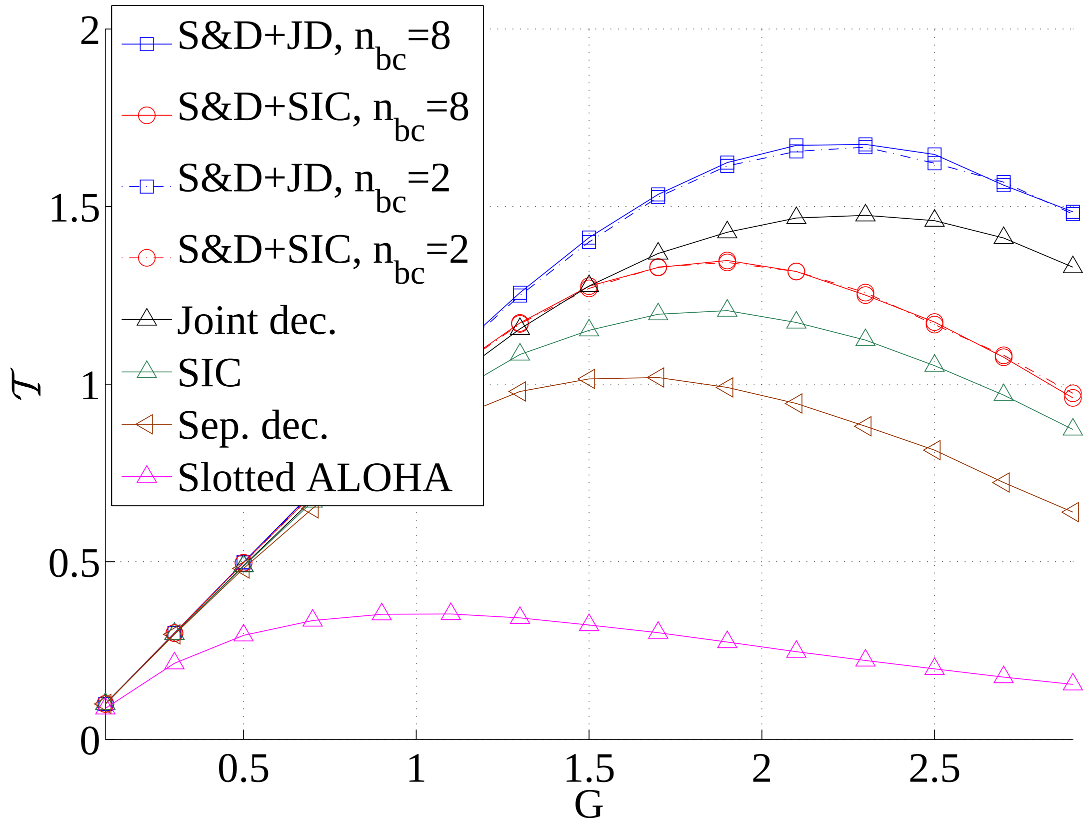

Fig. 5. Throughput in Rayleigh block fading channel, SNR=15 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

seen how S&D provides significant gains in terms of throughput with respect to the schemes that apply MUD only. The use of a larger field size in the pre-coding stage slightly increases the peak throughput and enhances the PLR performance at low network loads, as shown in Fig. 5 and 6, respectively. In order to quantify such enhancement, we evaluated through Monte Carlo simulations the probability that, once the iterative decoding stops, the rest of the packets can be decoded through matrix inversion. In correspondence to a load of $G = 2.1$ (for which the peak throughput with the configuration of Fig. 5 is achieved), such probability is around 4% for coefficients in $\mathbb{F}_{2^8}$ and 0.06% for coefficients in $\mathbb{F}_2$, i.e., the probability to decode the remaining packets is about sixty times larger when the field with higher cardinality is used. However, since both probabilities are relatively small, the overall improvement on the throughput is limited.

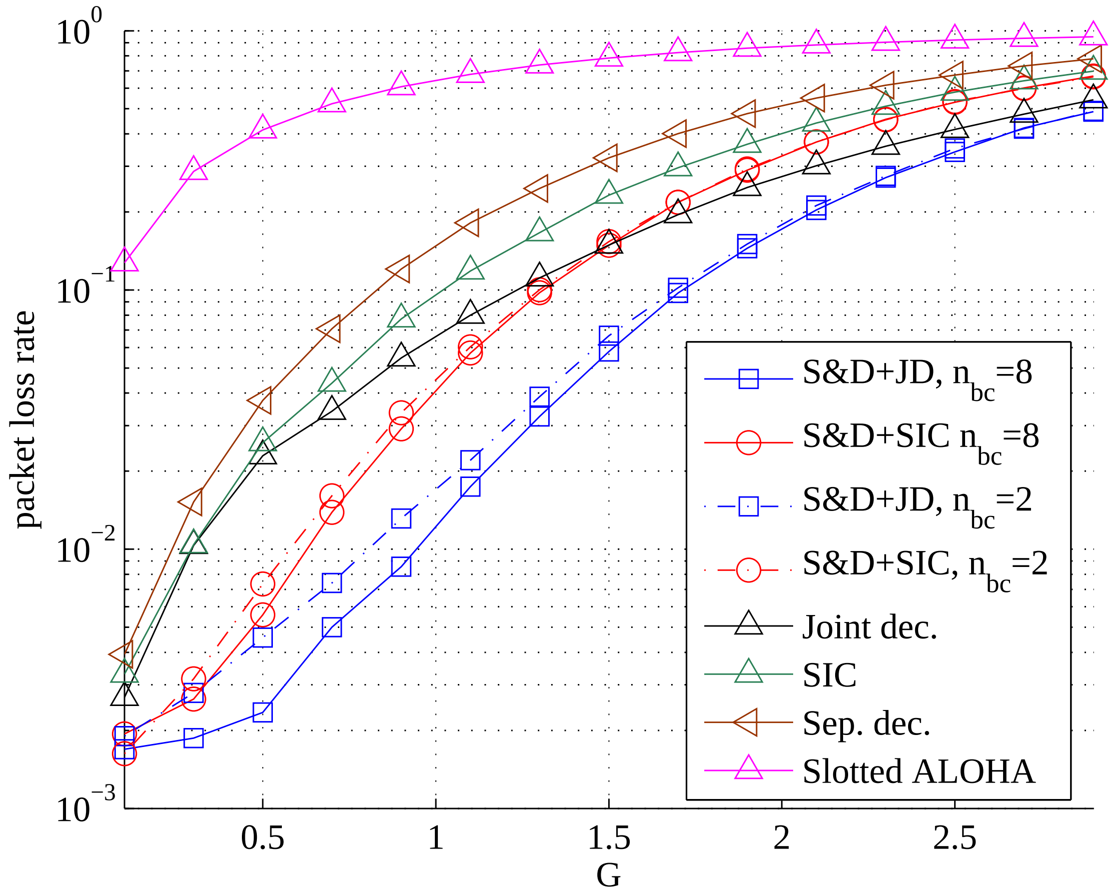

Fig. 6. Packet loss rate in Rayleigh block fading channel, SNR=15 dB. The channel code is the LDPC used in WiMAX standard with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

In figures 8, 9 and 10 the throughput, packet loss rate and energy efficiency for an average SNR of 10 dB are plotted, respectively. The rest of parameters are the same as in Fig. 5. By comparing the two sets of figures it can be seen how the channel SNR impacts the decoding at the PHY layer, which leads to a higher throughput and lower PLR when the SNR is higher, as expected. At both SNR values the JD scheme performs better than all others non-S&D schemes and at 10 dB closely approaches the S&D+SIC for lower network loads, outperforming it in the region $G > 1.5$. Such good performance is due to the fact that the decoding of all messages is done jointly rather than separately as in the SIC or the separate decoding schemes. The

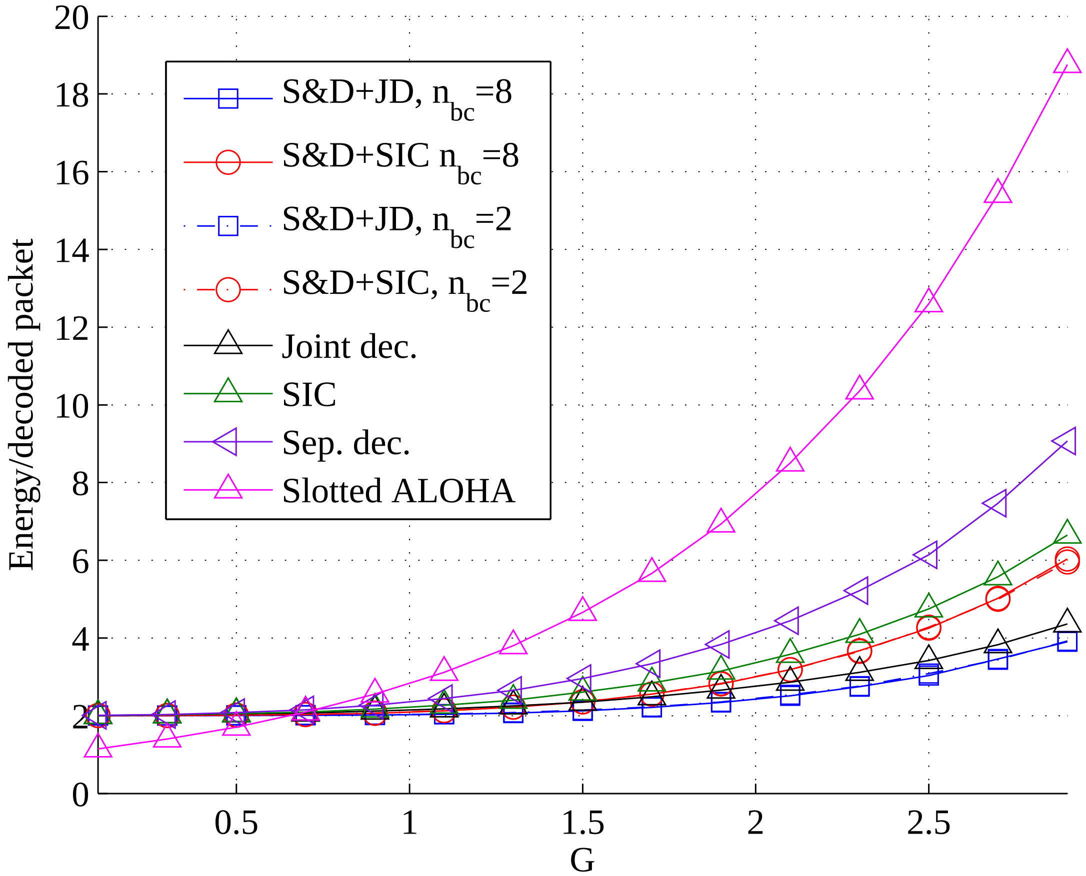

Fig. 7. Energy efficiency plotted against load in Rayleigh block fading channel, SNR=15 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

introduction of PLNC significantly increases the performance of the JD scheme of up to a 13 % at both SNR values, as can be seen in Fig. 5 and Fig. 8. In all figures the Slotted ALOHA scheme is also shown as a benchmark. In Slotted ALOHA all terminals transmit only one replica of their message, while in all other schemes two replicas are used, i.e., twice the energy is used. In order to compare the energy efficiency of the different schemes, in Fig. 7 and Fig. 10 we show the average energy consumption per decoded message plotted against the load G for an SNR of 10 dB and 15 dB, respectively. Slotted ALOHA shows a more efficient energy use at low network loads up to about 0.7. This is due mainly to the fact that in Slotted ALOHA each

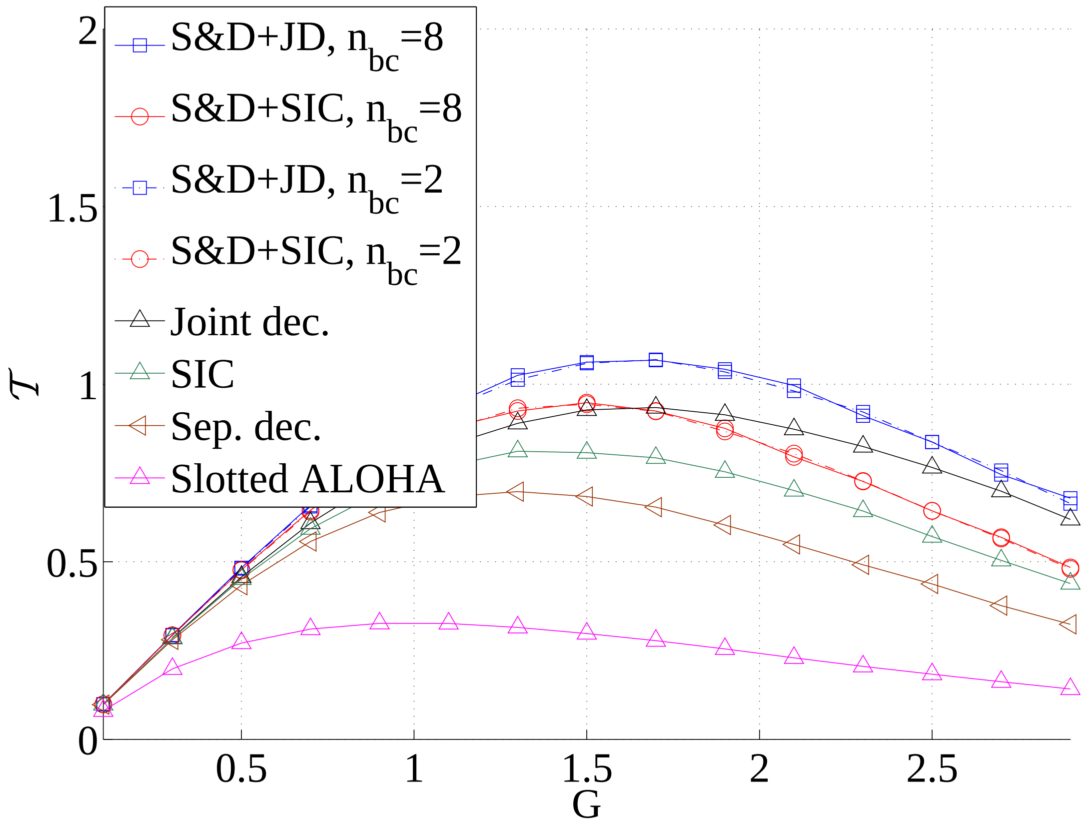

Fig. 8. Throughput in Rayleigh block fading channel, SNR=10 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

terminal transmits half of the power used in the other schemes. However, for $G > 0.7$ these, and most of all S&D+JD, perform significantly better than Slotted ALOHA in terms of energy efficiency, confirming the effectiveness of the proposed approach in situations characterized by a relatively high logical network load. The combined decoder is capable of extracting much more information from collisions than each of the two techniques taken individually. Furthermore, the solution presented here is robust against power unbalance (actually benefiting from it), which constitutes an issue if PLNC is applied with no MUD as in [18]. It is worth pointing out the fact that the joint decoding approach is optimal within each slot if this is treated as an isolated channel. If, instead, the slot is regarded as part of a frame and multiple replicas of the same packet are transmitted, using PLNC jointly with joint decoder brings a significant advantage.

This is not in contrast with the intuition that the joint decoder is optimal, since if it were applied over the whole frame at once, it would lead to the best possible performance. However, the huge increase in complexity makes such approach impractical. The advantage of our proposed approach is that it brings significant advantages with respect to the joint decoder applied at slot level with a limited increase in complexity, since the whole frame is processed only once at the physical layer, while the rest of operations are done over a finite field. Our approach is not an

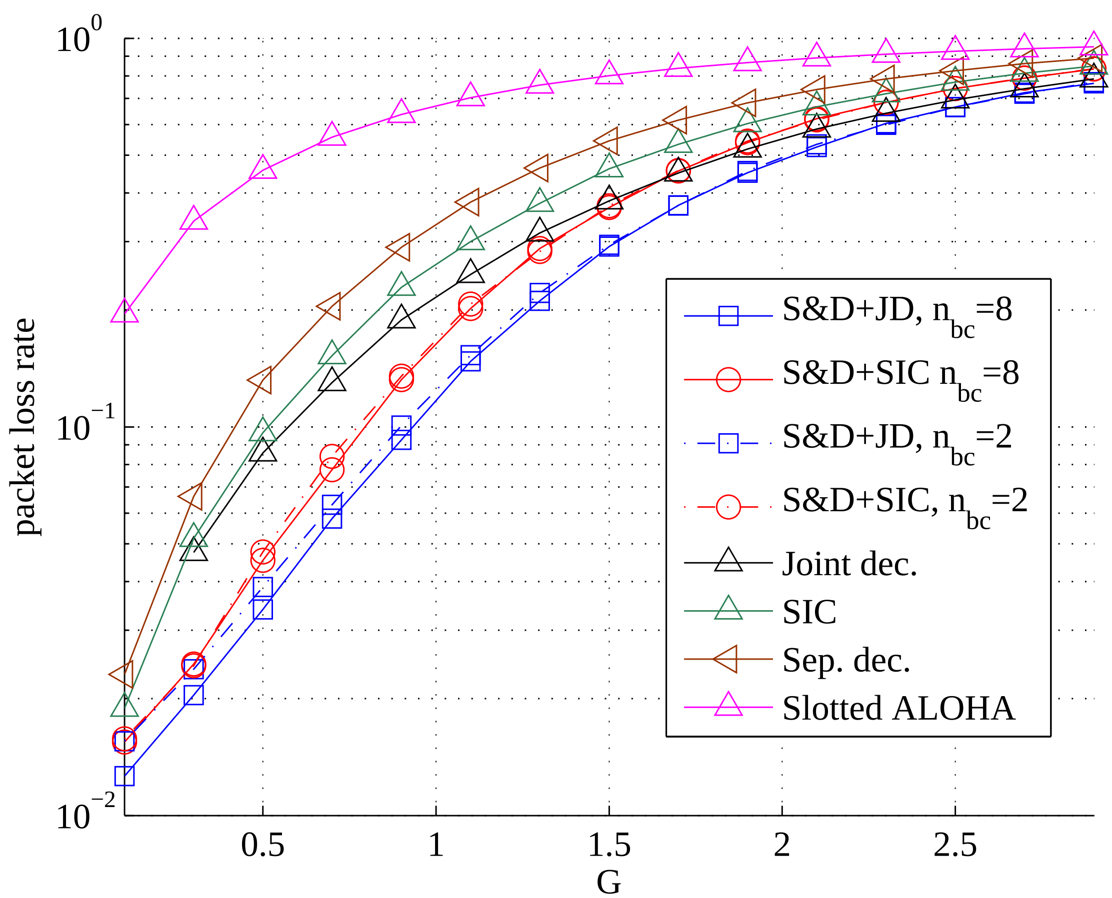

Fig. 9. Packet loss rate in Rayleigh block fading channel, SNR=10 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

alternative to other diversity schemes proposed for slotted ALOHA such as Irregular Repetition Slotted ALOHA (IRSA) [9]. As a matter of fact the S&D approach can be used on top of IRSA. The proposed scheme would allow either to increase the throughput for a given frame size or

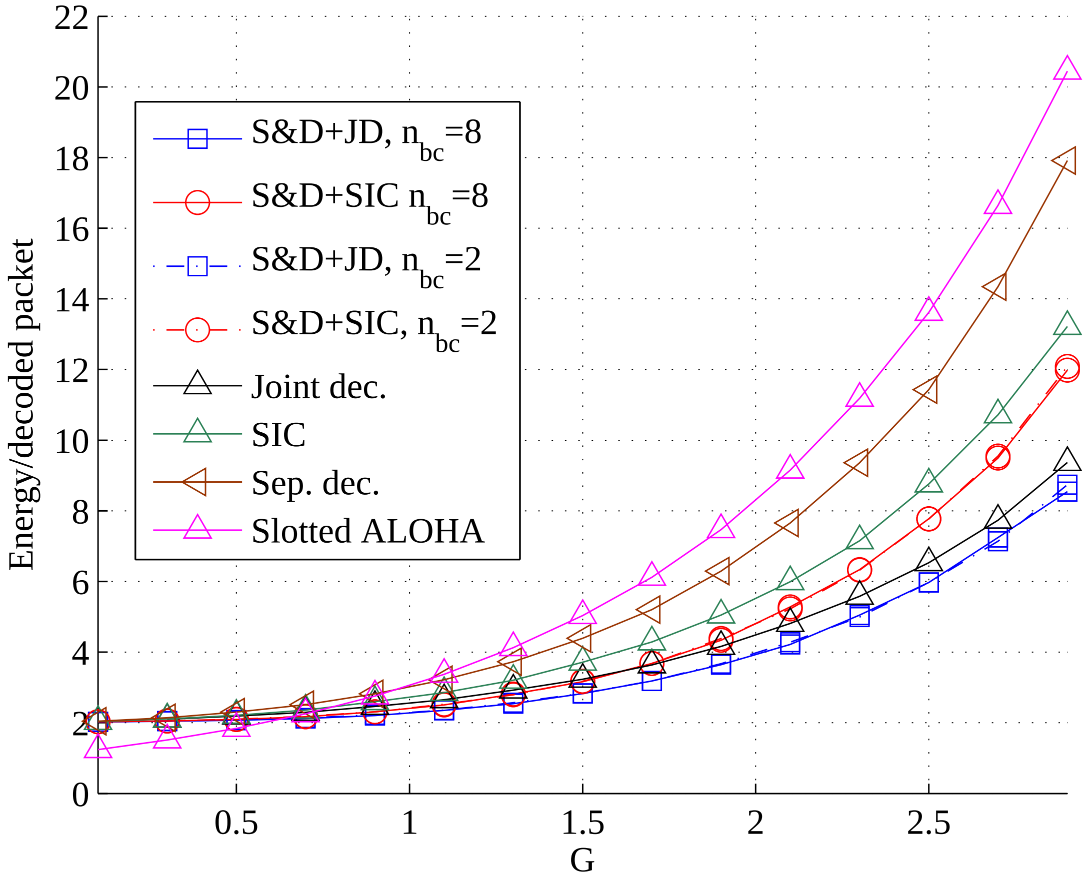

Fig. 10. Energy efficiency plotted against load in Rayleigh block fading channel, SNR=10 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. 2 replicas of the same packet are transmitted by each user. A maximum collision size of $K = 7$ has been set. Collisions of higher order are discarded. The frame size S has been set to $S = 10$ slots.

to reduce the frame size while guaranteeing the same throughput. Similar considerations have been presented in [43], where MUD is applied to IRSA. In order to show the gain deriving from applying S&D on top of IRSA, we compare the throughput and PLR curves of the two schemes for the case of a frame with 200 slots and Rayleigh fading channels with average SNR 15 dB. In the simulation the number of replicas transmitted by a given user is chosen according to the following degree distribution [9]:

$$\Lambda(x) = 0.5465x^2 + 0.1623x^3 + 0.2912x^6.$$

Unlike in [43], the results shown in figures 11 and 12 have been obtained applying a combination

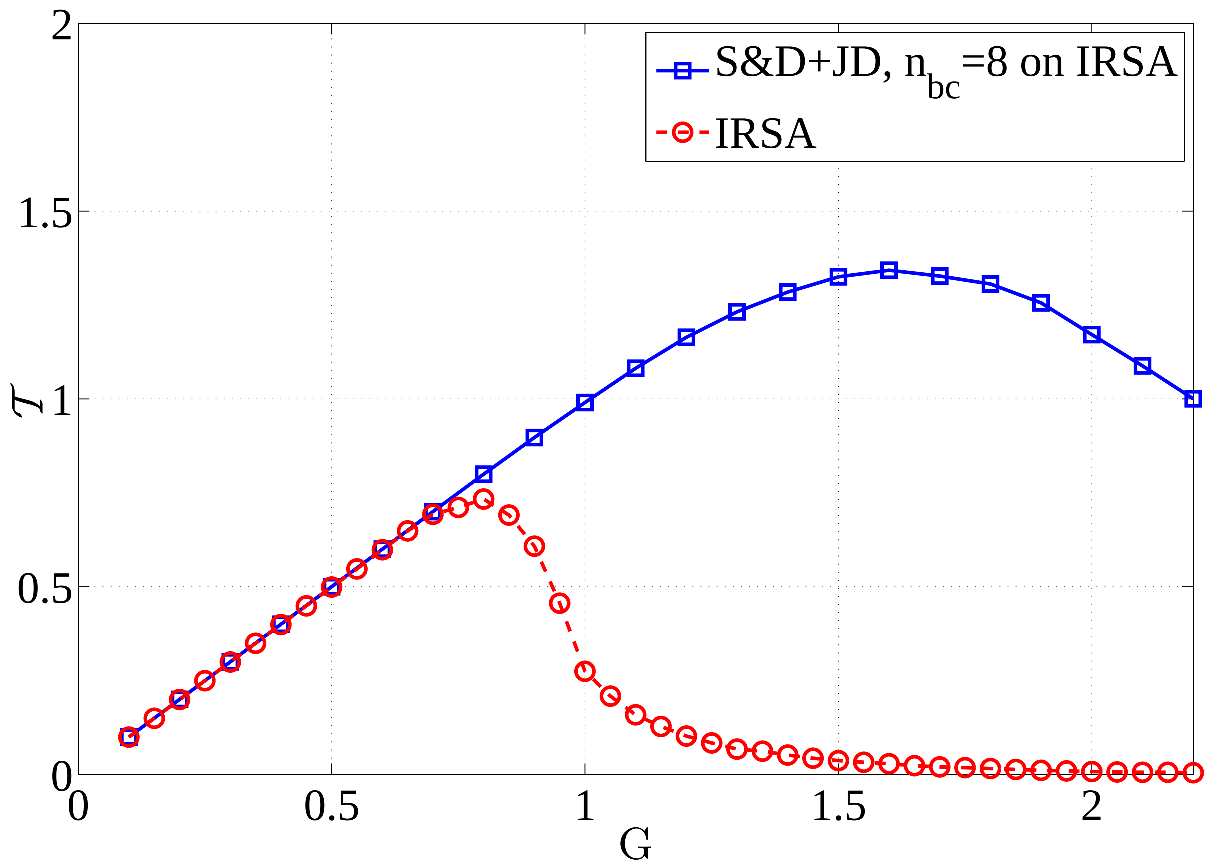

Fig. 11. Throughput in Rayleigh block fading channel, SNR=15 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. A maximum collision size of $K = 7$ has been set for S&D only. The frame size S has been set to $S = 200$ slots.

of PLNC and MUD to IRSA rather than MUD alone. Note that the results for S&D could be further enhanced by first running the IRSA cancellation in the analog domain and then applying S&D on the remaining collisions. Since the S&D would work on collisions that on average have a lower size, its performance would enhance.

We also point out that better performance can be obtained with an approach based on a frame-level joint detection and decoding of all of the packets rather than using a slot-based approach as we proposed in this paper. However, such approach would imply a considerable increase in the complexity of the decoder with respect to our method.

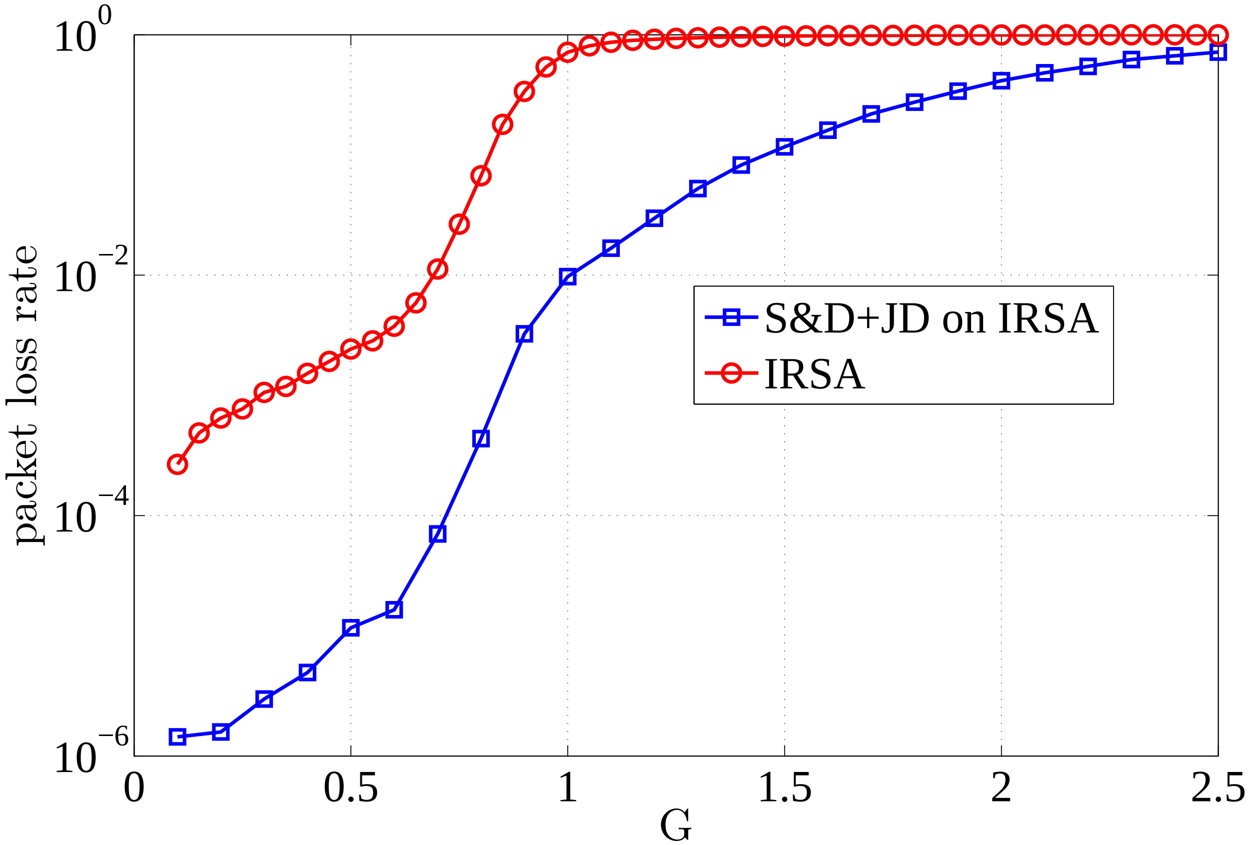

Fig. 12. Packet loss rate in Rayleigh block fading channel, SNR=15 dB. The channel code is the WiMAX LDPC with parameters N = 576, $R = 1/2$, BPSK modulation. A maximum collision size of $K = 7$ has been set for S&D only. The frame size S has been set to $S = 200$ slots.

C. Discussion

In order to have a complete picture of what are the performance limits of the proposed scheme as well as how the different parameters impact the behavior of the scheme it would be desirable to have an analytical expression for the throughput or, equivalently, for the packet loss rate. An approximate semi-analytical expression for the throughput has been derived in [1] under the assumption that each active user accesses the channel in each slot with probability $1 - 2^{-n_{bc}}$. Such expression is based on a bound on the probability to decode the sum of a subset of colliding messages. Deriving a formula for the general case is quite challenging, since it requires an analytical characterization of the packet error rate of finite length channel codes over fading channel with no channel state information at the transmitter and in the presence of interference. Furthermore, such characterization should also provide information on the behavior of the specific code when it is applied in a MUD or PLNC context. It is in general very difficult to model the performance of a specific error correcting code over a general discrete memoryless channel (unless the code is particularly short, or embeds a very strong structure, as in the case of convolutional and Reed Solomon codes). For LDPC and turbo codes, in general, rather than modeling the performance of a specific code, ensemble-based arguments are used, which nevertheless mostly assume maximum-likelihood (rather than iterative) decoding, and hence fail to provide a realistic model [44]. From the simulations we carried out and from the available bounds, we can say that larger n_{bc} lead to a higher probability of having a full rank matrix, although, as we showed previously in the present section, the gain when going from $n_{bc} = 2$ to $n_{bc} = 8$ is limited. The analysis presented in [19] can be regarded as a starting point, although the codes and the channel model are highly abstracted and thus particular care should be used when transposing such results to practical setups. As a final remark, we point out that the proposed method can be applied for channel codes of any packet length, although the performance of the S&D decoder in general depends, apart from the specific channel code that is consider, also on the codeword length.

VI. Conclusions

We proposed a novel cross-layer approach to random access systems that uses a hybrid PLNC-MUD decoder at the PHY layer and a frame level decoder based on matrix manipulation over extension fields. In the proposed scheme each terminal transmits several channel-coded replicas of the same message within a frame after a pre-multiplication by a random coefficient in an extension field $\mathbb{F}_q$. At the PHY layer the receiver decodes as many linear combination as possible in $\mathbb{F}_2$ of the signals colliding in each slot. In the second decoding stage, which is carried out at frame level, the set of combinations is treated by the receiver as a single system of equations in $\mathbb{F}_q$. We presented simulation results for throughput, packet loss rate and energy efficiency over a block fading channel. The whole decoding process at both PHY and frame level has been implemented in the simulations. Our results show that a significant enhancement in throughput and PLR can be achieved by combining PLNC and MUD. The combined decoder is capable of extracting much more information from collisions than each of the two techniques taken individually. In particular, we showed that the combination of PLNC and JD together with the frame-level decoding stage, considerably enhance the JD method, despite the fact that the latter is optimal when applied to slots in isolation. Furthermore, unlike in previously proposed schemes based on PLNC only, the approach presented in this paper is robust against block fading.

As future work we plan to optimize the multiple-access scheme taking into account the decoder performance, which is a function of the collision size and the specific linear combination within a collision, with the aim of maximizing the system throughput and minimizing the PLR also taking energy efficiency into account.

As a final remark, we would like to point out that evaluating the impact of the joint use of PLNC and MUD in random access systems is a challenging task and far from being concluded. The present work can be regarded as a further step towards a full exploitation of these two techniques in the Slotted ALOHA scenario.

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TABLE I

DECODING STRATEGIES AT PHY LAYER.

Method	**Description**	**Requires pre-coding**	**Requires joint decoding**
Separate dec.	joint detection, separate decoding	no	no
SIC	joint detection, separate decoding, then interference cancellation	no	no
S&D+SIC	as in SIC, then detect/decode combinations	yes	no
JD	joint detection, joint decoding	no	yes
S&D+JD	as in JD, then combine estimated messages	yes	yes