Optimal Redundancy Management in Reconfigurable Control Systems Based on Normalized Nonspecificity

N. Eva Wu and George J. Klir
Center for Intelligent Systems
Binghamton University, Binghamton, NY 13902-6000
Telephone: (607)777-4375, FAX: (607)777-4464, Email: evawu@binghamton.edu

Abstract

In this paper, the notion of normalized nonspecificity is introduced. The nonspecificity measures the uncertainty of the estimated parameters that reflect impairment in a controlled system. Based on this notion, a quantity called a reconfiguration coverage is calculated. It represents the likelihood of success of a control reconfiguration action. This coverage links the overall system reliability to the achievable and required control, as well as diagnostic performance. The coverage, when calculated on-line, is used for managing the redundancy in the system.

1. Introduction

Reliability has always been a subjective issue in the analysis and design of fault-tolerant control systems. It is rarely associated with an objective criterion that guides the design. This predicament is due to the fact that standard reliability assessment techniques are not geared toward systems with the type of redundancy that is involved in reconfigurable control. Therefore, it is difficult to establish a functional linkage between reliability and diagnostic/control performance. This paper is intended to establish such a linkage.

One way to achieve fault-tolerance in a controlled system is to reconfigure its control law when the system fails. The method of control reconfiguration becomes feasible and effective in a system if adequate redundancy exists for possible accommodation of few critical but foreseeable failures. The reader is referred to a recent survey paper by Patton (1997) for an outline of the state of the art in the field of fault-tolerant control. Some causes of difficulty, common to all fault-tolerant control designs, are the vagueness in the definition of a failure in the context of control performance, the uncertainties in the system and in the exogenous signal models, the limited processing/memory capabilities in carrying out diagnosis, and, above all, the lack of reliable means of managing redundancy, especially analytic redundancy. For example, the ailerons

of an aircraft are primarily for controlling the roll movement when used differentially. But they have also a secondary function of aiding elevons for controlling the pitch movement, when used collectively. Therefore, by analytically reconfiguring the control actions of the surfaces, redundancy could be effectively provided without added hardware. Unlike the hardware redundancy, analytical redundancy is inherent in the static and dynamic relations among the system variables, and is more difficult to manage.

The design of reconfigurable control systems is commonly perceived to involve designs of three separate subsystem modules (Jacobson and Nett 1997): a control subsystem module, a diagnostic subsystem module, and a reconfiguration subsystem module that links the former two. A schematic diagram of a reconfigurable control system is shown in figure 1. The control module contains a finite number of pre-designed or online-designed control settings. Each control setting, when properly selected under a given impairment condition, is to provide the controlled system with a certain prescribed performance level. The diagnostic module processes the measurements to estimate the current impairment condition. The reconfiguration module decides which control setting should be switched on to accommodate the condition. Note that control reconfiguration need not take place whenever an impairment condition occurs. It is only needed when control performance falls below a prescribed threshold. In that case, a failure is said to have occurred.

An attempt was made by Wu (1997) to link the reconfiguration coverage with a diagnostic resolution and a control performance threshold. The reconfiguration coverage measures the likelihood of success of a reconfiguration action that enters the assessment of the overall system reliability as one of the dominating parameters. The diagnostic resolution is introduced on the basis of a relevant nonspecificity measure.

A measure of nonspecificity, as one type of uncertainty, was first conceived in terms of finite crisp sets by Hartley (1928); it is usually called a Hartley measure. A measure of nonspecificity of convex sets in the n-dimensional Euclidean space was proposed by Klir and Yuan (1995); they call it a Hartley-like measure.

One of the main contributions in this paper is a refinement of the nonspecificity measure, which can be applied to measure and compare uncertainties of various physical quantities of different physical dimensions in a meaningful manner. This is achieved by a normalization process. As a result, nonspecificity is dimensionless, and has a range between zero and one. Two important measures of reconfigurable control systems are then derived from the normalized nonspecificity. One measures the performance of the diagnostic module in terms of diagnostic resolution, and one measures the performance of the reconfiguration module in terms of coverage. This coverage allows us to depict in a precise manner how the overall system reliability is related to the performance of the control subsystem module and to the performance of the diagnostic subsystem module. Therefore, it is suitable as a criterion for the management of analytic redundancy.

The paper is organized as follows. Section 2 discusses some background material, including the modeling of a plant in a way suitable for control reconfiguration, and the proper assessment of the control performance. Section 3 introduces the notion of normalized nonspecificity upon which the diagnostic resolution is defined. Section 4 introduces the reconfiguration performance measure in the form of a coverage. The coverage serves not only as a means for incorporating the likelihood of a successful/failed reconfiguration action into the reliability assessment, but also as a device for managing the analytic redundancy. In this regard it is used as a ranking criterion for selecting the most reliable control law. An example is discussed in which the relationship between the level of reconfiguration coverage and the levels of diagnostic resolution and control performance threshold are graphed. The graphs offer a clear view of the alternatives by which the design objectives can be either accomplished or compromised.

2. A fuzzy set description of control performance

The purpose of this section is to eliminate the vagueness in the definition of a failure in the context of control performance, and to create a reference framework for redundancy management.

Since the design of both the control module and the diagnostic module in a reconfigurable control system is based on the knowledge of the plant, it is important that a model suitable for the purpose of control reconfiguration be established. By suitable we mean that impairment of critical nature enter the model in an appropriate manner, and available redundancy is fully reflected in the model. In the following discussion, it is assumed that all impairments under consideration enter the plant model in the form of appropriate parameters. Some enter as physical parameters when the model is formed based on the laws of physics. Some enter as coefficients when the model of a prescribed structure is identified through some experimental means. Others enter simply as dimensionless scaling factors to indicate the degree of abnormality of some particular components. Suppose that at a given operating point, the state-space linear model

$$\dot{x}(t) = Ax(t) + Bu(t),\quad y(t) = Cx(t),\quad x(0) = x_0, \tag{1}$$

simulates the input-output characteristics of a plant, where x, y and u are n-, m-, and p-dimensional state, input, and output vectors, respectively, and A, B and C are known matrices (which may be time dependent) of appropriate dimensions. As an example, the impairments

representing the loss of sensor/actuator effectiveness can be characterized by sensor and actuator effectiveness factors. The factors enter the model in the form

$$B = B_0 - B_0\Delta_a,\quad C = C_0 - \Delta_s C_0,$$

where $\Delta_a = diag\{\delta_1, \ldots, \delta_m\}$, and $\Delta_s = diag\{\delta_{m+1}, \ldots, \delta_{m+p}\}$ (Wu 1996). Each δ_l ranges between 0, representing no loss of effectiveness, and 1, representing complete loss of the effectiveness in the lth effector (sensor or actuator). In general, we define an impairment parameter space as the Euclidean space of all parameters that change their values as the result of some impairment. The prescribed range of variation of such parameters form a set in the impairment parameter space. Let θ denote a vector in the impairment parameter space of dimension N and Ω denote the set over which θ resides when impairments occur. Without loss of generality, Ω can be regarded as a hyper-rectangle

$$\Omega = \{\theta \mid \theta_{l,min} \leq \theta_l \leq \theta_{l,max},\ l = 1, \ldots, N\}.$$

Denote by Ω_N the normalized impairment parameter domain. It is obtained by scaling each axis of the impairment parameter space by the respective Lebesgue measures of the projections of impairment domain Ω onto that axis.

$$\Omega_N = \{\frac{\theta_l - \theta_{l,min}}{\theta_{l,max} - \theta_{l,min}}, l = 1, \ldots, N \mid \theta \in \Omega\}. \tag{2}$$

Obviously, Ω_N is a unit hypercube which will be used to define the normalized nonspecificity. In the above example of sensor/actuator impairment, both Ω and Ω_N are $m + p$th dimensional hyper-rectangles in $m + p$th dimensional Euclidean spaces. Next, control performance will be defined over the universal set of the impairment domain.

In the schematic diagram shown in figure 1, $G(\theta)$ represents a model for the input to output mapping of the plant, including models of actuators and the sensors. The argument θ is made explicit to indicate that the model is dependent on the impairment parameter vector. Vector w contains all external signals, including disturbances, sensor noises and reference signals. Controlled output z is an error vector, capturing the design specifications on the system; y is the vector of measured variables; u is the vector of control inputs.

Let $T_{wz}(\theta)$ represent the closed-loop input-to-output mapping from w to z. The control design problem under a given impairment condition can be formulated as follows. Select a control setting that maps y to u so that

$$\sup_{\|w\|_{in} \leq 1} \|T_{wz}(\theta^N)\|_{out} \leq \gamma, \tag{3}$$

where the subscripts *in* and *out* indicate the norms used for measuring the sizes of the input and the output space signals, respectively $\gamma > \gamma_{opt}$ is a positive real number representing a prescribed performance level, and γ_{opt} is the optimally achievable performance level. Depending on what the input and output spaces are, design procedures vary and the resulting controllers are different. Several software packages are available, such as some MATLAB Toolboxes (Balas, et al. 1991), which contain routines for synthesizing such controllers when G is a linear and time-invariant input-output mapping and both input and output signal spaces are Hilbert spaces of energy bounded signals $L_2(0, \infty)$). No matter how well a model represents the plant to be controlled, uncertainties due to modeling errors and in the exogenous signals are always present. Such uncertainties can be formalized in plant model G of figure 1 as weighting factors (Doyle, et al. 1989). A good control design achieves the required performance in the face of these uncertainties. In this case, the controller is said to provide a robust performance. It should be pointed out that the purpose of this paper is not to discuss *fuzzy logic control* of some *model free* plant (Mendel 1995), nor to discuss any human-intervened *fuzzy supervision* (Frank, et al. 1993). The plant to be controlled is reasonably well modeled, and a complete automation is required for control, diagnosis, and reconfiguration.

Suppose that various impairment scenarios require that M different controllers be designed, each guaranteeing that performance level γ be attained under a specific set of impairment conditions (a subset of Ω). Suppose a set of such M controllers has been obtained. Let these controllers be denoted by $C_1, C_2, \ldots, C_M$, and let us address the issue of control performance measure. Define an alternative control performance measure

$$\mu_i(\theta) = \frac{1}{\sup_{\|w\|_{in} \le 1} \|T^i_{wz}(\theta)\|_{out}} \tag{4}$$

as a function of θ, where the superscript i indicates that controller C_i has been used as the feedback mapping in the performance evaluation. Some software packages (Balas, et al. 1991) can be useful in calculating the pointwise measure for each $\theta \in \Omega$. As impairment parameter vector θ moves away from the nominal value at which the design of C_i is carried out, the value of $\mu_i(\theta)$ generally decreases. Naturally, M fuzzy sets

$$C_i = \{(\theta, \mu_i(\theta)), i = 1, \ldots, M\} \tag{5}$$

on universal set Ω are formed. Each fuzzy set represents linguistically *control performance achieved by using controller* C_i. Figure 2 illustrates such a situation where three controllers

have been designed to cover a one dimensional impairment parameter domain. Without loss of generality, it is assumed that $\theta_{max} - \theta_{min} = 1$. Note that fault tolerance can be achieved only if sufficient redundant control authority exists in the system, which is the case in figure 2. This point will be further elaborated shortly.

Let μ_T denote a prescribed control performance threshold to distinguish the normal from a failed operation for the controlled system, i.e., a failure is declared if

$$\mu_i(\theta) < \mu_T. \tag{6}$$

Whenever this becomes the case, a control reconfiguration is necessary. The essence of control reconfiguration is the management of the control relevant redundancy. The subsequent section will discuss the criteria for making control reconfiguration decisions, and the risk associated with a particular decision.

Referring to figure 2 again, it can be seen that some constraints must be imposed on the control module when the set $\{C_i\}_{i=1}^M$ is constructed.

First, in order to guarantee that the control performance is always kept above the threshold by at least one controller anywhere in the impairment parameter domain, a sufficient overlap must exist among fuzzy sets (4) or (5). In mathematical terms, this condition can be stated as

$$\max_i \mu_i(\theta) > \mu_T, \quad \forall \theta \in \Omega. \tag{7}$$

Instead of using the maximum operator in (7), we may use a particular t-conorm associated with the union of fuzzy sets $\{C_i\}_{i=1}^M$ (Klir and Wierman 1998). This condition implies that adequate redundancy must exist in a system in order to make fault tolerance possible.

Secondly, since complexity is detrimental to the reliability of a system, the number of controllers (M) in the control module ought to be kept to the minimum. This condition implies that each controller ought to be designed to achieve the maximal robustness with respect to the variation in the impairment parameter vector. Suppose $\theta^i \in \Omega$ is the nominal impairment parameter value at which the design of controller C_i is carried out. Let $|B_r(\theta^i)|$ denote the Lebesgue measure of a ball centered at θ^i with radius r in the N dimensional impairment parameter space. A robust design problem could be formulated as follows.

$$\max_{C_i} r,$$

for which

$$\inf_{\theta \in B_r} \frac{1}{\sup_{\|w\|_{in} \le 1} \|T^i_{wz}(\theta)\|_{out}} \ge \mu_T.$$

Although the past two decades have marked some major development in systematic approaches to robust designs (Green and Limebeer 1995), a focused effort is still very much needed for the development of an iterative search procedure that leads to a *set* of interactive robust controllers with a well defined overall optimality. Our effort along this direction is under way, but will not be further discussed in this paper. On the other hand, for any existing design of the control module, regardless of the approach and the criterion by which a design is carried out, a control performance evaluation in the form of a set of fuzzy sets can always be obtained. However, if the result of the control performance evaluation violates one or both constraints mentioned above, some modification must be made in the control module design and/or in the control performance specifications.

Knowing that a successful control reconfiguration action depends on the accurate knowledge of impairment parameter θ, the challenge facing us is to acquire and to represent this knowledge in the presence of uncertainties.

3. Normalized nonspecificity

This section introduces the notion of normalized nonspecificity which is then applied to provide a measure of the uncertainty in the impairment parameter estimate. The section also discusses how existing deterministic and probabilistic based diagnostic schemes can be retrofitted into the possibilistic formalization under the uncertainty invariance principle (Klir and Wierman 1998).

The past two decades have witnessed much progress in the techniques of fault diagnosis (Frank 1996). Within the category of model-based diagnostic techniques, there are both deterministic and probabilistic approaches, both crisp and fuzzy model-based approaches (Dexter 1995), both analytical and fuzzy-logic approaches (Aubrun, et al. 1993). The emphasis of these developments has been on the prompt and accurate identification of the system condition, in the face of uncertainties. In a control reconfigurable system, the role of the diagnostic module is to provide information to the reconfiguration module on the current condition of the controlled system so that the existing redundancy can be best utilized through control reconfiguration. However, due to limited processing/memory capability and the presence of model/signal uncertainties, conclusions on the system conditions are always based on insufficient information. One is tempted to directly utilize the available diagnosis techniques and become preoccupied with the concerns regarding such issues as false alarm, missed detection, and false identification. As a consequence, conclusions are drawn prematurely at the output of the diagnostic module, which do not take into consideration the control module involved.

A remedy is to entrust the decision to the reconfiguration module that can combine the characteristics of the control and the diagnostic modules. What is needed from the diagnostic subsystem, in addition to the estimate of the impairment parameter vector, is a description of the uncertainty associated with each estimate in terms of a possibility function. The rationale for the use of possibility functions is that regardless of the diagnostic scheme used, impairment parameter estimates can always be described by fuzzy numbers (more discussion at the end of this section) and these can be given a natural possibilistic interpretation (Klir 1999). In addition, fuzzy sets have been utilized in describing the control performance. The interaction of these two consistent types of fuzzy sets should offer a natural steerage toward making reconfiguration decisions. Effort toward the possibilistic diagnosis, though limited at the time, already exists (Kang, et al. 1991). It was observed that a more prudent treatment of uncertainty can result in an improved reliability, and maximum/minimum operations in possibility theory can increase the computational efficiency.

Suppose the identified impairment parameter has been represented by a normal fuzzy set

$$I = \{(\theta, \mu_I(\theta)) \mid \theta \in \Omega\},$$

as examplified by a triangular-shape membership function in figure 3. The quantity $/R$ in the figure will be discussed when resolutio R is formally defined later in the section. (See equation (12).) Set $I = \{\theta \in \Omega \mid \mu_I(\theta) \geq \alpha\}$ for some value $\alpha \in [0, 1]$ is called the α-cut of F. The fuzzy set bears a possibilistic interpretation (Klir 1999). Since F is normal, the associated possibility distribution, r_I, is given in this case by the formula

$$r_I(\theta) = \mu_I(\theta)$$

for all $\theta \in \Omega$. Given now an arbitrary fuzzy set $F = \{(\theta, \mu_F(\theta)) \mid \theta \in \Omega\}$, the possibility measure of F based on possibility distribution r_F is given by the formula

$$Pos_I(F) = \sup_{\theta \in} \min \{\ldots\}. \tag{8}$$

The notion of normalized nonspecificity is now introduced for a fuzzy set defined on an N-dimensional Euclidean space (impairment parameter space). Nonspecificity is not the only known uncertainty measure. Within the domain of possibility theory, however, nonspecificity has been shown to dominate the total measure of uncertainty (Klir and Wierman 1998).

The Hartley-like measure of a convex se A in th N dimensional Euclidean space has been shown to take the form

$$HL(A) = \min_{t \in T} \{ \log_2 [\prod_{i=1} (1 + |A_{it}|) + |A| - \prod_{i=1} |A_{it}|] \} \quad (9)$$

under some standard uncertainty measure axioms (Klir and Yuan 1995), where T is the set of all unitary transformations on the N-dimensional Euclidean space, and $|A_{it}|$ is the Lebesgue measure of the projection of se A on to the th axis of the unitary transformed coordinate system under transformation . In principle, the logarithm in (9) can be of any base. Base 2 is chosen for the purpose of simplifying a normalization process to be introduced shortly. When the experimental framework is confined to the normalized impairment domain Ω_N as defined in (2), the original Euclidean space is effectively re-scaled along each axis by the Lebesgue measure of the projection of hypercube Ω. Using the Hartley-like measure given in (9), the nonspecificity $U(F)$ of normal fuzzy se F is calculated by the formula

$$U(F) = \int_0^1 HL(F) \, d\alpha, \quad (10)$$

as explained in Klir and Wierman (1998). In the next theorem, the normalized impairment domain Ω_N is used

Theorem 1. $< U(F) < 1$ $U(F) = 0$ if and only i F is a singleton.

Proof. $HL(A)$ is co[illegible]ted to satisfy the axiomatic requirements (Klir and Wierman 1998) that [illegible] wher $HL(A) = 0$ if A is a singleton in its universal se X, and tha $HL(A) \leq HL(B)$ wheneve $A \subseteq B$ wher $A, B \subseteq X$. Therefor $HL(F) > 0$, where $HL(F) = 0$ iff F is a singleton in Ω_N. In combination with the definition o $U(F)$ given in (10), this implies tha $U(F) > 0$, and tha $U(F) = 0$ iff F is a singleton for any give α. This implies, in turn, tha F is a singleton. On the other hand, $F \subseteq \Omega$. By the monotonicity of the Hartley-like measure $HL(F) \cdot HL(\Omega_N)$. Consequently $U(F) < U(\Omega)$. Bu $HL(\Omega_N) = 1$ by the definition given in (9) because Ω_N is a hypercube. Therefore $U(F) \leq 1$.

□

When Ω_N is one-dimensional, the calculation of the normalized nonspecificity is much simpler. In this case,

$$U(F) = \int_0^1 \log_2(1 + |F|) \, d\alpha, \quad (11)$$

where $F \subseteq \Omega_N$, and $|F|$ is the Lebesgue measure of F.

We define now a diagnostic resolution $R(F)$ for a specific diagnostic outcome described by fuzzy se $F \subseteq \Omega_N$ by the formula

$$R(F) = \frac{1}{2^{U(F)} - 1}. \quad (12)$$

Theorem 2. In a one dimensional impairment parameter situation, diagnostic resolution $R(F)$ satisfies the inequalities

$$1 \leq R(F) \leq \infty. \quad (13)$$

In addition, $R(F)$ is equal to the inverse of the core of a crisp set that has the same nonspecificity as fuzzy set F. (See figure 3).

Proof. Since $0 \leq U(F) \leq 1$ by Theorem 1, the definition of R by (12) yields immediately (13). Suppose there is a crisp set $C \subseteq \Omega_N$ such that $|C_\alpha| = 1/R(F)$ for all $\alpha \in [0, 1]$. Then its nonspecificity is given by the formula

$$U(C) = \int_0^1 \log_2(1 + \frac{1}{R(F)}) d\alpha.$$

On the other hand, $U(C)$ and $U(F)$ are the same by assumption. With $U(C)$ replaced by $U(F)$, $R(F)$ of the form (12) is obtained by solving the above equation.

□

The geometric interpretation of $R(F)$ as it relates to the nonspecificity of F is depicted in the figure 3 when $\theta_{max} - \theta_{min} = 1$ is satisfied. Before normalization, on the other hand, the nonspecificity appears as

$$U(F) = \int_0^1 \log_2(1 + |{}^\alpha F|/|\Omega|) d\alpha. \quad (14)$$

In this case, the Lebesgue mesure of the nonspecificity-equivalent crisp set C relates to $R(F)$ through the equation

$$|C| = \frac{|\Omega|}{R(F)}.$$

It is important to point out that normalization is absolutely necessary when nonspecificity is to be used in a real world problem. Normalization had not been considered for the definition of nonspecificity prior to this work. By using the one-dimensional case as an example, let us analyze the consequence of employing the non-normalized nonspecificity. This is the case when $|{}^\alpha F| \subseteq \Omega$ in (14) is not divided by the Lebesgue measure of impairment domain $|\Omega|$. It is noted that the numerical value of $|{}^\alpha F|$ can be made entirely arbitrary, because the unit used for the impairment parameter can be arbitrarily selected. As a result, the numerical value of nonspecificity $U(F)$ is arbitrary. When the unit of impairment parameter θ is fixed, the non-normalized nonspecificity can be useful but only in a relative sense. In addition, adding a dimensionless quantity 1 with a quantity with a definite physical dimension (F) is a fundamentally incorrect mathematical operation. When the impairment parameter space is of multiple dimensional, without normalization, one is indeed comparing apples with oranges.

Since most existing diagnostic schemes are either deterministic or probabilistic, the retrofit issue, i.e., the transformation of the diagnostic outcome from its original representation to the possibilistic representation, needs to be discussed.

An estimate from a deterministic diagnostic scheme is represented by a point p in the impairment parameter space with an error bound r. This error bound is typically the radius of a hypersphere surrounding the point. Let $I(p, r)$ denote the set enclosed by the hypersphere. This set can be defined by the characteristic function

$$(\cdot) = \begin{cases} \; \in I(p, r) \\ \; \notin I(p, r) \end{cases}$$

Hence, it may be viewed as a special fuzzy set F. The corresponding possibility measure of this fuzzy set is readily obtained by (8).

On the other hand, an estimate from a probabilistic diagnostic scheme is represented by a probability distribution function. The transformation from the probability to a possibility formalization is however more involved. The reader is referred to recent papers by Klir (1998) and Harmanec and Klir (1997) for more information. The following example shows how an uncertain-invariant probability-to-possibility transformation is made for an impairment parameter estimate called elevon effectiveness factor that enters an aircraft model. Please see Balas et al (1991) and Wu and Chen (1996) for a description of the 4th order linearized state space model and its modification.

The elevon effectiveness is estimated using an adaptive Kalman estimator (Wu, et al. 1998), and at each given time is described by a computed probability density function. This density function is then integrated to a discrete probability distribution function, a snap shot of which at $= 4$sec is shown in the upper plot of figure 4. Although treatments for both continuous and discrete universal sets are available, our continuous problem is to be treated in the discrete domain for computational efficiency. The lower plot in figure 4 shows the possibility distribution at $= 4$sec. The following result is used for carrying out the probability to possibility transformation for the example.

Theorem 3 (Harmanec and Klir 1997) Let H denote the number of distinct values of in the k-tuple $< p_1, p_2, \cdots, p_k >$, representing a probability distribution arranged in descending order. Then there exist H integers, $i_1, \ldots, i_H \in \{1, \ldots, k\}$, such that $p_1 = \cdots = p_{i_1} > p_{i_1+1} = \cdots = p_{i_2} > \cdots > p_{i_{H-1}+1} = \cdots = p_{i_H}$. All possibility distribution $< r_1, \cdots, r_k >$ consistent with the given probability distribution and containing the same amount of uncertainty are all those possibility distributions that satisfy

$$r_{i_q+1} = \sum_{l=i_q+1} p_l$$

for all $q = 0, \ldots, H-1$ (with $i_0 = 0$) and

$$r_j \geq r_{\ldots} - \sum_{l=\ldots} p_l$$

for $j \in \{\ldots, \cdots, \} - \ldots + 1$, $= 0, 1, \cdots, H-1\}$.

The transformation expressed by Theorem 3 remains the same for any multidimensional case, provided that the probabilities and the possibilities involved are ordered as specified. When there are equal probabilities in the distribution, an additional criterion is needed to uniquely determine the corresponding possibility measure. The criterion of maximal nonspecificity (Harmanec and Klir 1997) is adopted, for one wishes to be constrained only as much as necessary in making decisions based on the possibility distribution.

Figure 5 shows the plots of the nonspecificity and the corresponding diagnostic resolution as functions of time. These plots provide a basis for comparing various diagnostic schemes despite the method used for diagnosis. The volatile behavior in these plots is caused by a sudden drop of the elevon control effectiveness at $= 5$sec. The impairment parameter estimator recognizes the large uncertainty in the estimate during the transient process.

4. Optimal redundancy management

The focus of our discussion in this section is shifted from the diagnostic module to the reconfiguration module where the decision is made on whether and how a control reconfiguration should take place in order to accommodate a failure. In this regard, the reconfiguration module carries out the task of redundancy management. Optimal redundancy management amounts to selecting in a prescribed class of controllers one and only one that offers the highest reliability.

It is possible to estimate the likelihood of success and the likelihood of unsuccess in a reconfiguration action, which are respectively named a coverage, c, and a complementary coverage, $\bar{c}$, in the following discussion. More specifically, the coverage indicates the likelihood of fuzzy set (controller C_i being selected while the actual impairment parameter lies within the interval over which the prescribed control performance level is achieved $\mu(\cdot) \geq \mu_T$). The complementary coverage indicates the likelihood of fuzzy set C_i being selected while the actual impairment parameter lies outside of the interval.

Suppose the reconfiguration module has selected C_i by using some ranking method. The coverage can be defined by using the ratio of two weighted nonlinear intervals. Recall that fuzzy set F represent the outcome of the impairment parameter estimation. Express F as the union of two sets $F^u \cup F^l$, where $F^u = \{(\theta, F(\theta)) \mid \mu(\theta) \geq \mu_T\}$, and $F^l = \{(\theta, F(\theta)) \mid \mu(\theta) < \mu_T\}$. Then

$$c = \frac{\int_0^1 HL({}^{\alpha}F^u)d\alpha}{\int_0^1 HL({}^{\alpha}F)d\alpha} = \frac{U(F^u)}{U(F)}, \tag{15}$$

and, similarly,

$$\bar{c} = \frac{\int_0^1 HL({}^{\alpha}F^l)d\alpha}{\int_0^1 HL({}^{\alpha}F)d\alpha} = \frac{U(F^l)}{U(F)}, \tag{16}$$

where $HL(\cdot)$ denotes the Hartley-like measure defined by (9), and $U(.)$ denotes the nonspecificity defined by (10). Fuzzy set F is subdivided according to whether a particular *possible* impairment condition is accommodated by controller C_i. The following one-dimensional expressions for the coverage and the complementary coverage may reveal more explicitly their dependence on the partitioned fuzzy sets.

$$c = \frac{\int_0^1 log_2(1 + |{}^{\alpha}F^u|)d\alpha}{\int_0^1 log_2(1 + |{}^{\alpha}F|)d\alpha}, \tag{17}$$

and

$$\bar{c} = \frac{\int_0^1 log_2(1 + |{}^{\alpha}F^l|)d\alpha}{\int_0^1 log_2(1 + |{}^{\alpha}F|)d\alpha} \tag{18}$$

These expressions show that the coverage and the complementary coverage depend on the diagnostic performance, characterized by nonspecificity $U(F)$ which uniquely determines the resolution, as well as the control performance, characterized by threshold μ_T, which uniquely determines subdivisions F^u and F^l. Therefore it links the performance of individual modules to overall system reliability.

Theorem 4. $0 \leq c \leq 1$ and $0 \leq \bar{c} \leq 1$. In addition, $1 - \bar{c} \leq c$ whenever the subadditivity of the Hartley-like measure with respect to set union (19) holds for ${}^{\alpha}F^u$ and ${}^{\alpha}F^l$. The subadditivity always holds when $N = 1$.

Proof. By the monotonicity of the Hartley-like measure (Klir and Wierman 1998), $0 \leq HL({}^{\alpha}F^u) \leq HL({}^{\alpha}F)$. Integrating over $\alpha \in [0, 1]$ yields $0 \leq U(F^u) \leq U(F)$. Therefore, $0 \leq c \leq 1$ from (15).

Similarly, $0 \leq \bar{c} \leq 1$, or $0 \leq 1 - \bar{c} \leq 1$, by using (16). Assume now that the subadditivity of the Hartley-like measure holds for all α, i.e.,

$$HL({}^{\alpha}F^u \cup {}^{\alpha}F^l) \leq HL({}^{\alpha}F^u) + HL({}^{\alpha}F^l), \quad \forall \alpha. \tag{19}$$

It follows from (15) and (16) that

$$c + \bar{c} = \frac{\int_0^1 (HL({}^{\alpha}F^u) + HL({}^{\alpha}F^l))d\alpha}{\int_0^1 HL({}^{\alpha}F)d\alpha} \geq 1.$$

Although the subadditivity may not hold in general, due to the minimum operator in (9), it certainly holds when $N = 1$ because in this case

$$HL({}^{\alpha}F^u) + HL({}^{\alpha}F^l) = log_2(1 + |{}^{\alpha}F^u|)(1 + |{}^{\alpha}F^l|) \geq log_2(1 + |{}^{\alpha}F^u| + |{}^{\alpha}F^l|) \geq log_2(1 + |{}^{\alpha}F^u \cup {}^{\alpha}F^l|).$$

Therefore, $1 - \bar{c} \leq c$.

□

It is seen that coverage $1 - \bar{c}$ is bounded above by coverage c. $1 - \bar{c}$ and c can be regarded as lower and upper bounds for an interval-valued coverage. Their difference reflects the amount of information deficiency in the diagnostic outcome.

Suppose there are M possible control settings, each of which is designed to accommodate a particular set of system conditions. In determining which control setting is most suitable, the coverage defined by (15) can be used as a criterion in the following manner. If, for a diagnostic outcome F, $c_1, \cdots, c_M$ are calculated, and

$$c_k = \max_{i \in \{1, \cdots, M\}} \{c_i\}, \tag{20}$$

then control setting C_k is selected. This is the essence of optimal redundancy management. As a result, the highest coverage, and, hence, also the highest reliability at the overall system level are achieved. Apparently, a real time computation of coverage values for every candidate control setting is required.

The next example will shed some light on how the computation of the coverage is carried out. It also shows some quantitative relations of the coverage to the control performance threshold, as well as to the diagnostic resolution. For ease of visualization, a one-dimensional impairment space is considered, as shown in figure 6. Suppose controller C_i is under consideration to see what level of coverage it can provide if it is chosen by the reconfiguration module. Suppose the membership function that represents the performance of this controller has the shape of a triangle. Its pea M_c represents the level of nominal performance achieved at some value of impairment parameter, and ϵ, the Lebesgue measure of its support, gives an indication on performance robustness provided by controlle C. The estimated impairment parameter is represented by the normal fuzzy se F, also with a triangular membership function. $\epsilon = a\epsilon$, the Lebesgue measure of the support o F, is to be varied between 0 and ϵ_0 via parameter $a \in [0, 1]$. Furthermore, suppose that sufficient analytical redundancy exists so that figure 6 describes the extreme case scenario

$$ar \quad max \quad \epsilon \quad 2$$

under which control setting C would still be selected. This means that a different controller would have been chosen il $ar \quad max \quad < \epsilon \quad 2$. In addition, suppose the diagnostic module provides a sufficiently high resolution that

$$/\epsilon > M /\epsilon,$$

wher $M > 1$ is assumed without loss of generality. Note that most of the assumptions above are for simplifying purposes, and can be relaxed.

For the scenario in figure 6, formulae for computing coverage c and complementary $\bar{c}$ introduced in the previous section take the following forms

$$c = \frac{U(F^u)}{U(F)} =$$

$$\begin{cases} 1, & b \leq \epsilon_0/2 - \epsilon_F/2 \\ \frac{2[1-b-\epsilon_0(1+a)/2]ln[1-b+\epsilon_0(1-a)/2]-(1+\epsilon_0-2b)ln(1+\epsilon_0-2b)-\epsilon_0 a}{(1+\epsilon_0 a)ln(1+\epsilon_0 a)-\epsilon_0 a}, & \epsilon_0/2 - \epsilon_F/2 < b \leq \epsilon_0/2 \\ \frac{2[1-b-\epsilon_0(1+a)/2]ln[1-b+\epsilon_0(1-a)/2]+2b-\epsilon_0(1+a)}{(1+\epsilon_0 a)ln(1+\epsilon_0 a)-\epsilon_0 a}, & \epsilon_0/2 < b \leq \epsilon_0/2 + \epsilon_F/2 \\ 0, & b > \epsilon_0/2 + \epsilon_F/2 \end{cases}$$

$$\bar{c} = \frac{U(F^l)}{U(F)} =$$

$$\begin{cases} 0, & b \leq \epsilon_0/2 - \epsilon_F/2 \\ \frac{2[1+b-\epsilon_0(1-a)/2]ln[1+b-\epsilon_0(1-a)/2]-2b+\epsilon_0(1-a)}{(1+\epsilon_0 a)ln(1+\epsilon_0 a)-\epsilon_0 a}, & \epsilon_0/2 - \epsilon_F/2 < b \leq \epsilon_0/2 \\ \frac{2[1+b-\epsilon_0(1-a)/2]ln[1+b-\epsilon_0(1-a)/2]-(1+2b-\epsilon_0)ln(1+2b-\epsilon_0)-\epsilon_0 a}{(1+\epsilon_0 a)ln(1+\epsilon_0 a)-\epsilon_0 a}, & \epsilon_0/2 < b \leq \epsilon_0/2 + \epsilon_F/2 \\ 1, & b > \epsilon_0/2 + \epsilon_F/2. \end{cases}$$

Note that the subdivision of F into F^u and F^l occurs at the point where C_i and μ_T intersect. This point is marked by $b = \frac{\mu_T \epsilon_c}{2M_c}$, measured from $\theta = 0$. Since b is proportional to the control

performance threshol μ_T, it can also be used as an indication o μ_T. $a = \epsilon /\epsilon_0$ is a fraction indicating the support of fuzzy se F relative to its worst case support ϵ_0.

The nonspecificity and the resolution for this example are

$$U(F) = \frac{(1 + \epsilon \quad b\,(1 + \epsilon \quad - \epsilon\, a}{ln(2)\epsilon_0 a},$$

and

$$R(F) = \frac{e}{(\epsilon_0 a + 1)^{(\epsilon_0 a \quad 1)/(\epsilon_0 a)} - e},$$

by equations (11) and (12), where $e = 2.7183$. Plots of $U(F)$ and R versus ϵ_F/ϵ_0 are shown in figure 7. It can easily be derived that $U(F) \approx \epsilon_F$ when $\epsilon_F \ll 1$.

c and $\bar{c}$ are now computed as functions of control performance threshold μ_T, with diagnostic resolution R as a parameter.

The fourth plot in figure 8 shows the interval coverage between boundaries c and $1 - c$ as a function b (μ_T) with resolution $R = 4.13$ when $a = \epsilon_F/\epsilon_0 = 1$, calculated using the above given formulae. It is assumed that the most *possible* value for θ is at $\theta = \epsilon_0/2 = 0.25$ when $\epsilon_0 = 0.5$, and the peak location of the membership function for F remains as a changes. The change in coverage corresponding to a gradual decrease in the value of a are shown in the third ($a = 0.65$, $R = 6.30$), the second ($a = 0.35$, $R = 11.58$), and the first ($a = 0.05$, $R = 80.17$) plots, respectively.

By observing the graphs obtained, the following conclusions can be drawn. The gap between the two bounds on coverage increases with decreasing R (diagnostic resolution), and with increasing μ_T (control performance) at the high coverage end. The value of coverage itself increases with increasing R at high coverage end, and with decreasing μ_T. All the conclusions are within our expectations. Their significance lies in that a guideline for design iteration is provided in a quantitative manner for meeting a prescribed reliability requirement.

Suppose that coverage $c_0 = 0.99$ is required for achieving a certain prescribed reliability. With a relatively high resolution such as shown in the first plot of figure 8 ($R = 80$), the control performance threshold can be set at $b = 0.22$ or lower. Suppose the diagnostic module designer can afford a resolution only at $R = 4.13$ as shown in the last plot of figure 8. In this case, one must be content with a much lowered control performance at around $b = 0.022$ (10% of the previous case). If this performance level is not acceptable, one can attempt the following: increasing the processing/memory capability of the diagnostic module, or increasing the performance level of the control module at around $\theta = 0.5$ without sacrificing elsewhere. The first attempt is aimed at enhancing resolution I within the given decision time, say from $I = 4$ to $I = 80$, and the second attempt is aimed at raising the allowable level for μ_{γ}, say from $= 0.02$ to $= 0.2$, so that the required coverage level c_t can be achieved.

5. Conclusions

A method of redundancy management in control reconfigurable systems is presented. Redundancy management in such systems amounts to making control reconfiguration decisions. It is assumed that adverse conditions that may cause the failure of the systems are parameterized in the system models in terms of impairment parameters. Our method of decision making is based on the interaction of two classes of fuzzy sets defined on the bounded universal set in an impairment parameter space. One class of fuzzy sets represents the set of measures of control performance for all control settings. The other class represents the outcome of the impairment parameter estimation. By introducing the notion of normalized nonspecificity measure for the latter class, we are able to meaningfully quantify the performance of the reconfigurable control systems using coverage, which is defined as the likelihood of successful control law reconfiguration. On the one hand, the coverage affects the reliability of the overall systems directly. On the other hand, our definition of the coverage is functionally related to the control performance, as well as to the diagnostic performance. It sets the criterion and provides a means for the integrated design of control reconfigurable fault-tolerant systems.

A major theoretic issue that still remains unresolved is the subadditivity of the Hartley-like measure of two bounded sets on an multi-dimensional Euclidean space without imposing the convexity condition on the sets. This has restricted the application of our solution to problems of sequential single-fault scenarios, instead of sequential simultaneous-faults scenarios. Overcoming this restriction may require that a new type of Hartley-like measure be defined.

Acknowledgments

This project was supported in part by the NSF under Grant # ECS-9615956, in part by the Xerox Corporation under Grant #HE 1321-98, and in part by the NASA under Cooperative Agreement # NCC-1-336. The authors would like to acknowledge the contribution by Dr. Youmin Zhang in helping obtain the computational results presented in figure 4 and figure 5 of this paper. The authors also would like to thank the anonymous reviewers of the paper for their careful review and helpful comments.

References

Aubrun, C., Sauter, D., Noura, H., and Robert, M., 1993, Fault diagnosis and reconfiguration of systems using fuzzy logic: application to thermal plant, *International Journal of Systems Science* **2[illegible]**, 1945-1954.

Balas, Doyle, Glover, Packard, and Smith, 1991 *[illegible]-Analysis and Synthesis Toolbox* (for use with MATLAB), The MATH WORKS.

Dexter, A.L., 1995, Fuzzy model based fault diagnosis, IEE Proc. Control Theory Appl. **14[illegible]**, 545-550.

Doyle, J [illegible], Khargonekar, P. P., and Francis, B.A., 1989, State-space solutions to standard H_2 and H_∞ control problems, *IEEE Trans. Automatic Control* **3[illegible]**, 831-847.

Frank, P., 1996, Analytical and qual[illegible] model-based fault diagnosis-a survey and some new results *European Journal of Control* **[illegible]**, 6-28.

Frank, P. and Kiupel, N., 1993, Fuzzy supervision and application to lean production, *International Journal of Systems Science* **2[illegible]**, 1935-1944.

Green, M., and Limebeer, D.J.N., 1995 *Linear Robust Control* (Prentice Hall).

Harmanec, D., and Klir, G.J. 1997, On Information-Preserving Transformations, *International Journal of General Systems* **2[illegible]**, 265-290.

Hartley, R.V.L., 1928, Transmission of information *The Bell Systems Technical Journal* **[illegible]**, 535-563.

Jacobson, C.A., and Nett, C.N., 1991, An integrated approach to controls and diagnostics using the four parameter controller, *IEEE Control Systems Magazine* **11**, 22-28.

Kang, H., Cheng, J., Kim, I., and Vachtsevanos, G., 1991, An application of fuzzy logic and Dempster-Shafer theory to failure detection and identification *Proc. of the 30th IEEE Conference on Decision and Control*, pp.1555-1560.

Klir, G.J., and Wierman, M. J., 1998 *Uncertainty-Based Information Elements of Generalized Information Theory* (Physica-Verlag/Springer/Verlag)

Klir, G.J, and Yuan B., 1995, On nonspecificity of fuzzy sets with continuous membership functions *Proc. 1995 International Conference on Systems, Man, and Cybernetics*, pp.627-630.

Klir, G.J., 1998. On fuzzy set interpretation of possibility theory *Fuzzy Sets and Systems* **10[illegible]**.(in press)

Mendel, J.M., 1995, Fuzzy Logic Systems for Engineering: A tutorial *Proceedings of the IEEE*, 8 , 345-377.

Patton, R.J., 1997, fault-tolerant control: the 1997 situation *Proc. IFAC SAFEPROCESS'9* , 1033-1054.

Wu, N.E., and Chen, T.J., 1996, Feedback de in reconfigurable control systems, *International Journal of Robust and Nonlinear Contro* , 561-570.

Wu, N. Eva, 1997, Reliability of reconfigurable control systems: a fuzzy set theoretic perspective, *Proc. 1997 IEEE Conference on Decision and Contro* , pp.3352-3357.

Wu, N.E., Y. Zhang, and K. Zhou, 1998, Control effectiveness estimation using an adaptive Kalman estimator *Proc. 13th IEEE International Symposium on Intelligent Contro* , pp.181-186.

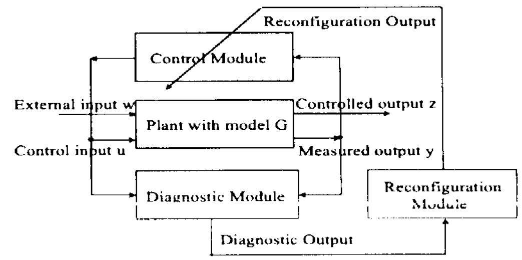

Figure 1 A reconfigurable control system

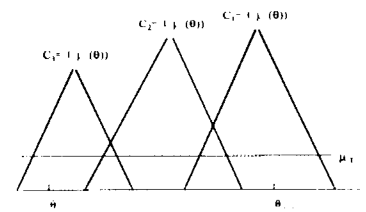

Figure 2 Control performance measures as fuzzy sets

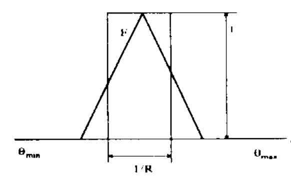

Figure 3 Impairment parameter estimate as fuzzy set

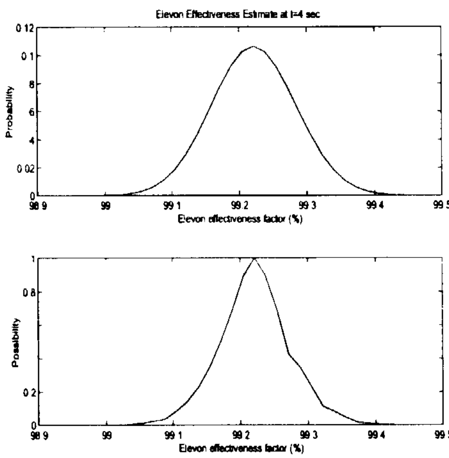

Figure 4 Example of probability to possibility transformation

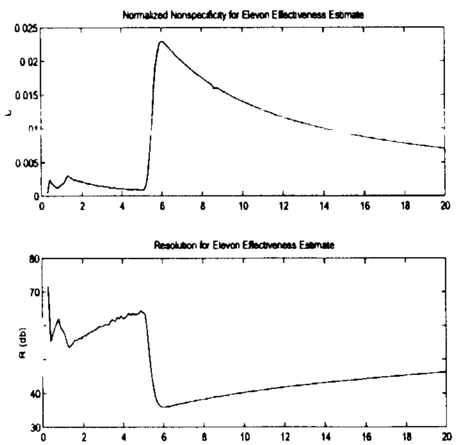

Figure 5 Example of nonspecificity and resolution calculation

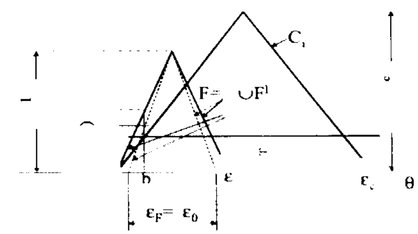

Figure 6 A one-dimensional coverage computation example

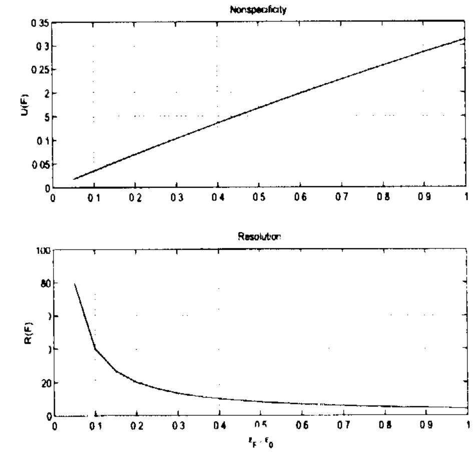

Figure 7 Nonspecificity and resolution versus ϵ_F

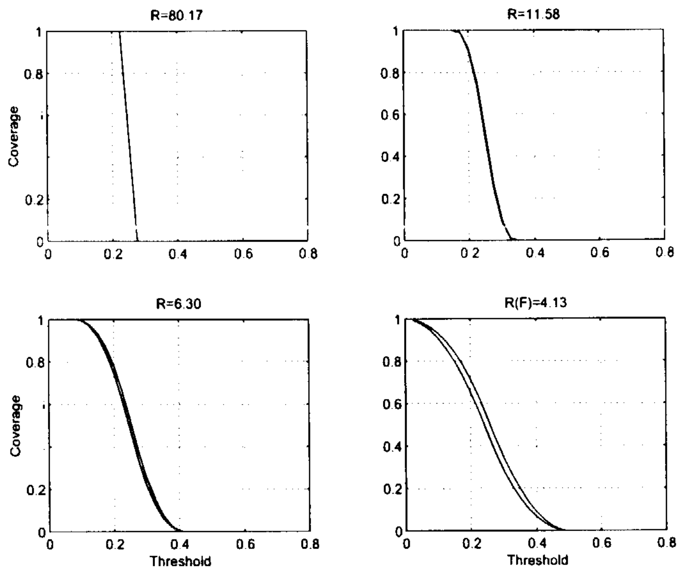

Figure 8 Coverage computation