

# Distributed MIMO Radar Using Compressive Sampling

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**Abstract**—A distributed MIMO radar is considered, in which the transmit and receive antennas belong to nodes of a small scale wireless network. The transmit waveforms could be uncorrelated, or correlated in order to achieve a desirable beam pattern. The concept of compressive sampling is employed at the receive nodes in order to perform direction of arrival (DOA) estimation. According to the theory of compressive sampling, a signal that is sparse in some domain can be recovered based on far fewer samples than required by the Nyquist sampling theorem. The DOAs of targets form a sparse vector in the angle space, and therefore, compressive sampling can be applied for DOA estimation. The proposed approach achieves the superior resolution of MIMO radar with far fewer samples than other approaches. This is particularly useful in a distributed scenario, in which the results at each receive node need to be transmitted to a fusion center.

**Keywords:** Compressive sampling, MIMO Radar, DOA Estimation

## I. INTRODUCTION

A multiple-input multiple-output (MIMO) radar system, as originally proposed in [1]-[5] transmits multiple independent waveforms via its antennas. Consider a MIMO radar equipped with  $M_t$  transmit and  $M_r$  receive antennas that are close to each other relative to the target. The phase differences induced by transmit and receive antennas can be exploited to form a long virtual array with  $M_t M_r$  elements. This enables the MIMO radar system to achieve superior spatial resolution as compared to a traditional radar system. MIMO radar transmitting correlated signal waveforms in order to achieve a desired beam pattern has also been proposed [10]-[12]. This is useful in cases where the radar system wishes to avoid certain directions, because they either correspond to eavesdroppers, or are known to be of no interest.

Compressive sensing (CS) has received considerable attention recently, and has been applied successfully in diverse fields, e.g., image processing [6] and wireless communications [7]. The theory of CS states that a  $K$ -sparse signal  $\mathbf{x}$  of length  $N$  can be recovered exactly with high probability from  $\mathcal{O}(K \log N)$  measurements via linear programming. Let  $\Psi$  denote the basis matrix that spans this sparse space, and let  $\Phi$  denote a measurement matrix. The convex optimization problem arising from CS is formulated as follows:

$$\min \|\mathbf{s}\|_1, \quad \text{subject to } \mathbf{y} = \Phi \mathbf{x} = \Phi \Psi \mathbf{s} \quad (1)$$

where  $\mathbf{s}$  is a sparse vector with  $K$  principal elements and the remaining elements can be ignored;  $\Phi$  is an  $M \times N$  matrix incoherent with  $\Psi$  and  $M \ll N$ .

In this paper, we propose a distributed MIMO radar system, where transmit and receive antennas belong to nodes of a wireless network that are uniformly distributed on a disk or a certain radius, located wireless network nodes. The readings of the receive nodes are transmitted to a central node for DOA estimation. Energy efficiency is an important issue in such a wireless network as the nodes operate on battery. We employ the idea of compressive sampling in order to save in energy consumed during data transmission to the central node. Recently, the work of [8] considered DOA estimation of signal sources using CS. In [8], the basis matrix  $\Psi$  is formed by the discretization of the angle space. The source signals were assumed to be unknown, and an approximate version of the basis matrix was obtained based on the signal received by a reference vector. The signal at the reference sensor would have to be sampled at a very high rate in order to construct a good basis matrix. Here, we extend the idea of [8] to the problem of DOA estimation for MIMO radar. Since the number of targets is typically smaller than the number of snapshots that can be obtained, DOA estimation can be formulated as the recovery of a sparse vector using CS. Unlike the scenario considered in [8], in MIMO radar the transmitted waveforms are known at each receive antennas. This enables each receive antenna to construct the basis matrix locally, without knowledge of the received signal at a reference sensor or any other antenna. We consider the more general case of correlated signal waveforms. We provide analytical expressions for the average signal-to-jammer ratio (SJR) for the proposed approach. Simulation results show that the proposed approach can accomplish the super-resolution of MIMO radar systems while using far fewer samples than existing methods, such as Capon, amplitude and phase estimation (APES) and generalized likelihood ratio test (GLRT) [2]. In particular, the proposed approach can enable each node to obtain a good DOA estimate independently. Further, it results in much less information to be transmitted to a fusion center, thus enabling savings in terms of transmission energy.

## II. SIGNAL MODEL FOR MIMO RADAR

We consider a MIMO radar system with  $M_t$  transmit nodes and  $M_r$  receive nodes that are uniformly distributed on a disk of radius  $r$ . For simplicity, we assume that targets and nodes lie on the same plane. Further, we assume that each node in the network knows which are the nodes that serve as transmit and receive antennas and what their coordinates are relative to a fixed point in the network. This information can be provided by a higher network layer. Let us denote the locations in rectangular coordinates of the  $i$ -th transmit and receive antenna by  $(x_i^t, y_i^t)$  and  $(x_i^r, y_i^r)$ , respectively (see Fig.1).

The location of the  $k$ -th target is denoted by the polar coordinates  $(d_k, \theta_k)$ , where  $d_k$  is the distance between this target and the origin, and  $\theta_k$  is the azimuthal angle, which is the unknown parameter to be estimated in this paper. Under the far-field assumption  $d_k \gg \sqrt{(x_i^t)^2 + (y_i^t)^2}$  and  $d_k \gg \sqrt{(x_i^r)^2 + (y_i^r)^2}$ , the distance between the  $i$ th transmit/receive antenna and the  $k$ -th target  $d_{ik}^t/d_{ik}^r$  can be approximated as  $d_{ik}^{t/r} \approx d_k - \eta_i^{t/r}(\theta_k)$ , where  $\eta_i^{t/r}(\theta_k) = x_i^{t/r} \cos(\theta_k) + y_i^{t/r} \sin(\theta_k)$ .

Let  $x_i(n)$  denote the discrete-time waveform transmitted by the  $i$ -th transmit antenna. Assuming the transmitted waveforms are narrowband and the propagation is non-dispersive, the received baseband signal at the  $k$ -th target equals [4]

$$\begin{aligned} y_k(n) &= \beta_k \sum_{i=1}^{M_t} x_i(n) e^{-j \frac{2\pi}{\lambda} d_{ik}^t} \\ &= \beta_k e^{-j \frac{2\pi}{\lambda} d_k} \mathbf{x}^T(n) \mathbf{v}(\theta_k) \quad k = 1, \dots, K \end{aligned} \quad (2)$$

where  $\lambda$  is the transmitted signal wavelength,

$$\begin{aligned} \mathbf{v}(\theta_k) &= [e^{j \frac{2\pi}{\lambda} \eta_1^t(\theta_k)}, \dots, e^{j \frac{2\pi}{\lambda} \eta_{M_t}^t(\theta_k)}]^T \\ \mathbf{x}(n) &= [x_1(n), \dots, x_{M_t}(n)]^T. \end{aligned} \quad (3)$$

Due to reflection by the target, the  $l$ -th antenna element receives

$$z_l(n) = \sum_{k=1}^K e^{-j \frac{2\pi}{\lambda} d_{lk}^r} y_k(n) + \epsilon_l(n), \quad l = 1, \dots, M_r \quad (5)$$

where  $\epsilon_l(n)$  represents independent and identically distributed (i.i.d.) Gaussian noise with variance  $\sigma^2$ .

On letting  $L$  denote the number of snapshots, we have

$$\begin{aligned} \mathbf{z}_l &= \begin{bmatrix} z_l(0) \\ \vdots \\ z_l(L-1) \end{bmatrix} = \sum_{k=1}^K e^{-j \frac{2\pi}{\lambda} d_{lk}^r} \mathbf{y}_k + \mathbf{e}_l \\ &= \sum_{k=1}^K e^{-j \frac{2\pi}{\lambda} (2d_k - \eta_l^r(\theta_k))} \beta_k \mathbf{X} \mathbf{v}(\theta_k) + \mathbf{e}_l \end{aligned} \quad (6)$$

where  $\mathbf{y}_k = [y_k(0), \dots, y_k(L-1)]^T$ ,  $\mathbf{e}_l = [\epsilon_l(0), \dots, \epsilon_l(L-1)]^T$  and  $\mathbf{X} = [\mathbf{x}(0), \dots, \mathbf{x}(L-1)]^T$ .

By discretizing the angle space as  $\mathbf{a} = [\alpha_1, \dots, \alpha_N]$ , we can rewrite (7) as

$$\mathbf{z}_l = \sum_{n=1}^N e^{j \frac{2\pi}{\lambda} \eta_l^r(\alpha_n)} s_n \mathbf{X} \mathbf{v}(\alpha_n) + \mathbf{e}_l \quad (7)$$

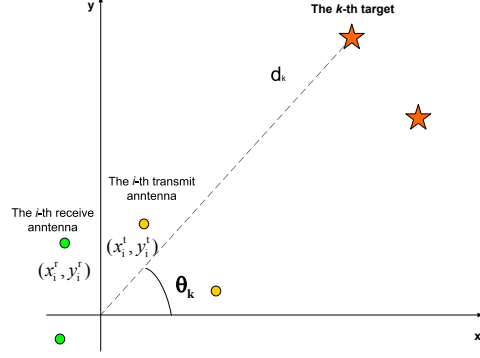


Fig. 1. MIMO Radar System

where

$$s_n = \begin{cases} e^{-j \frac{4\pi}{\lambda} d_k} \beta_k & \text{if there is target at } \alpha_n \\ 0 & \text{otherwise} \end{cases}.$$

## III. COMPRESSIVE SENSING FOR MIMO RADAR

Assuming that there exists a small number of targets, the DOAs are sparse in the angle space, i.e.,  $\mathbf{s} = [s_1, \dots, s_N]$  is a sparse vector. A non-zero element with index  $j$  in  $\mathbf{s}$  indicates that there is a target at the angle  $\alpha_j$ .

By CS theory, we can construct a basis matrix  $\Psi_l$  for the  $l$ -th antenna as

$$\Psi_l = [e^{j \frac{2\pi}{\lambda} \eta_l^r(\alpha_1)} \mathbf{X} \mathbf{v}(\alpha_1), \dots, e^{j \frac{2\pi}{\lambda} \eta_l^r(\alpha_N)} \mathbf{X} \mathbf{v}(\alpha_N)]. \quad (8)$$

Ignoring the noise, we have  $\mathbf{z}_l = \Psi_l \mathbf{s}$ . Then we measure linear projections of the received signal at the  $l$ -th antenna as

$$\mathbf{r}_l = \Phi_l \mathbf{z}_l = \Phi_l \Psi_l \mathbf{s}, \quad (9)$$

where  $\Phi_l$  is an  $M \times L$  random Gaussian matrix which has small correlation with  $\Psi_l$ . Combining the output of  $N_r$  receive antennas, we have

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_{N_r} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi_1 \Psi_1 \\ \vdots \\ \Phi_{N_r} \Psi_{N_r} \end{bmatrix}}_{\Theta} \mathbf{s}, \quad 1 \leq N_r \leq M_r. \quad (10)$$

Therefore, we can recover  $\mathbf{s}$  by applying the Dantzig selector to the convex problem in (10) as in [9]:

$$\hat{\mathbf{s}} = \min \|\mathbf{s}\|_1 \quad \text{s.t.} \|\Theta^H (\mathbf{r} - \Theta \mathbf{s})\|_\infty < \mu. \quad (11)$$

According to [9], we can recover the sparse vector  $\mathbf{s}$  with very high probability if we select  $\mu = (1+t^{-1})\sqrt{2 \log N \sigma^2}$ , where  $t$  is a positive scalar and  $\sigma^2$  is the noise power.

#### IV. PERFORMANCE ANALYSIS IN THE PRESENCE OF A JAMMER SIGNAL

In the presence of a jammer at location  $(d, \theta)$  the signal received at the  $l$ -th receive antenna can be represented as

$$\begin{aligned} \mathbf{r}_l &= \Phi_l \sum_{k=1}^K e^{-j\frac{2\pi}{\lambda}(2d_k - \eta_l^r(\theta_k))} \beta_k \mathbf{X} \mathbf{v}(\theta_k) \\ &+ \Phi_l e^{-j\frac{2\pi}{\lambda}(d - \eta_l^r(\theta))} \beta \mathbf{b} + \Phi_l \mathbf{e}_l. \end{aligned} \quad (12)$$

where  $\beta$ ,  $\mathbf{b}$  denote respectively the reflection amplitude and waveform of this jammer. Since  $\mathbf{b}$  is uncorrelated with the transmitted waveforms  $\mathbf{X}$ , the effect of the jammer signal is similar to that of additive noise. Let  $\mathbf{A}_l = \Phi_l^H \Phi_l$  and  $\mathbf{D}_l = \mathbf{X}^H \mathbf{A}_l \mathbf{X}$ , where  $\mathbf{D}(i, j)$  denotes the  $(i, j)$ th element of  $\mathbf{D}$ . We assume that the TX/RX nodes are uniformly distributed on a disk with the radius  $r$ . Thus, the average power of the desirable signal  $P_s(l)$  can be represented by

$$\begin{aligned} P_s(l) &= E\left\{ \sum_{k, k'=1}^K \underbrace{e^{j\frac{2\pi}{\lambda}[2(d_k - d_{k'}) - (\eta_l^r(\theta_k) - \eta_l^r(\theta_{k'}))]} \beta_k^* \beta_{k'}}_{\rho_l(k, k')} \right. \\ &\times \underbrace{\mathbf{v}^H(\theta_k) \mathbf{X}^H \mathbf{A}_l \mathbf{X} \mathbf{v}(\theta_{k'})}_{Q_{kk'}} \left. \right\} = E\left\{ \sum_{k=1}^K |\beta_k|^2 Q_{kk} \right\} \\ &+ E\left\{ \sum_{k \neq k'} \rho_l(k, k') \beta_k^* \beta_{k'} Q_{kk'} \right\} \end{aligned} \quad (13)$$

where  $Q_{kk'} = \sum_{i, j} \mathbf{D}_l(i, j) e^{j\frac{2\pi r}{\lambda}(\eta_j^t(\theta_{k'}) - \eta_i^t(\theta_k))}$ .

Following [3], we know that  $f_h(h) = \frac{2}{\sqrt{1-h^2}} \sqrt{1-h^2}$ ,  $-1 < h < 1$  if  $h = \frac{r}{\tilde{r}} \sin(\Psi)$ , where  $f_{\tilde{r}}(\tilde{r}) = \frac{2\tilde{r}}{r^2}$ ,  $0 < \tilde{r} < r$  and  $f_{\psi}(\psi) = \frac{1}{2\pi}$ ,  $-\pi < \psi < \pi$ . Then  $E\{e^{j\alpha h}\} = 2\frac{J_1(\alpha)}{\alpha}$ , where  $J_1(\cdot)$  is the first-order Bessel function of the first kind. Using this property and letting  $a_{ij} = \eta_j^{t/r}(\theta_{k'}) - \eta_i^{t/r}(\theta_k)$ , we have

$$E\{e^{j\frac{2\pi r}{\lambda} a_{ij}}\} = \begin{cases} \eta(4 \sin(\frac{\theta_{k'} - \theta_k}{2})) & i = j \\ \eta^2(2) & i \neq j \end{cases} \quad (14)$$

where  $\eta(x) = 2\frac{J_1(x\frac{\pi r}{\lambda})}{x\frac{\pi r}{\lambda}}$ .

Therefore, the average power of the desirable signal  $P_s(l)$  taken over the positions of TX/RX nodes can be found to be:

$$\begin{aligned} P_s(l) &= \sum_{k=1}^K |\beta_k|^2 \left[ \sum_i \mathbf{D}_l(i, i) + \sum_{i \neq j} \mathbf{D}_l(i, j) \eta^2(2) \right] \\ &+ \sum_{k \neq k'} \beta_k^* \beta_{k'} e^{j\frac{4\pi}{\lambda}(d_k - d_{k'})} \underbrace{\eta(4 \sin(\frac{\theta_{k'} - \theta_k}{2}))}_{\eta_{kk'}} \\ &\times \left[ \eta_{kk'} \sum_i \mathbf{D}_l(i, i) + \sum_{i \neq j} \mathbf{D}_l(i, j) \eta^2(2) \right]. \end{aligned} \quad (15)$$

Similarly, the power of the jammer signal is given by

$$\begin{aligned} P_j(l) &= (e^{-j\frac{2\pi}{\lambda}(d - \eta_l^r(\theta))} \beta) (e^{-j\frac{2\pi}{\lambda}(d - \eta_l^r(\theta))} \beta)^* \\ &\times \mathbf{b}^H \mathbf{A}_l \mathbf{b} = |\beta|^2 \mathbf{b}^H \mathbf{A}_l \mathbf{b}. \end{aligned} \quad (16)$$

The SJR given the node locations is the ratio of the power of the signal over the power of the jammer. Since the denominator

does not depend on node locations, the average SJR equals the ratio of (15) and (16).

Since the jammer signal is uncorrelated with the transmitted signal, the SJR can be improved by correlating the jammer signal with the transmitted signal. Combining this with CS, the measurement matrix in (9) is modified as

$$\tilde{\Phi}_l = \Phi_l \mathbf{X}^H. \quad (17)$$

Moreover, since  $\Phi_l$  is a Gaussian random matrix,  $\tilde{\Phi}_l$  is still Gaussian; therefore it satisfies the restricted isometry property (RIP) and is incoherent with  $\Psi_l$ , thus guaranteeing a stable solution to (11). Based on (17), the average power of the desirable signal  $P_s(l)$  is equal to (15) except  $\mathbf{D}_l = \mathbf{X}^H \mathbf{X} \mathbf{A}_l \mathbf{X}^H$ . The average power of the jammer signal using  $\tilde{\Phi}_l$  is rewritten as  $P_j(l) = |\beta|^2 \mathbf{b}^H \mathbf{X} \mathbf{A}_l \mathbf{X}^H \mathbf{b}$ .

Approximating  $\mathbf{X}^H \mathbf{X} \sim \mathbf{I}_{M_t}$  and using  $\mathbf{b}^H \mathbf{b} = 1$ , the SJRs based on  $\Phi_l$  and  $\tilde{\Phi}_l$  can be approximated as  $\frac{M_t \sum_{k=1}^K |\beta_k|^2}{|\beta|^2}$  and  $\frac{L \sum_{k=1}^K |\beta_k|^2}{|\beta|^2}$ , respectively. Therefore, the SJR using (17) can be generally improved by a factor of  $L/M_t$  since  $L \gg M_t$ . the DOA estimates can be improved by the increase in  $L$ . However, the time duration of the radar pulse might need to be longer as well.

As simulation results show (see Section V), the proposed method can yield good performance even using a single receive antenna. With a good initial estimate of DOA, the receive nodes can adaptively refine their estimates by constructing a higher resolution basis matrix  $\Psi_l$  around that DOA. Restricting the candidate angle space, may reduce the samples in the angle space that are required for constructing the basis matrix, thus reducing the complexity of the  $\ell_1$  minimization step. On the other hand, the resolution of target detection can be improved by taking the denser samples of the angle space around the intimal DOA estimate. Furthermore, the transmit node can design the correlated waveforms for transmit beamforming as well based on the good initial estimate.

#### V. SIMULATION RESULTS

In this section, we consider a MIMO radar system with the transmit/receive antennas uniformly distributed on a disk of radius 10m. The number of transmit nodes is fixed at  $M_t = 50$ . The carrier frequency is 8.62 GHz. A maximum of  $L = 512$  snapshots are considered at the receive node. The received signal is corrupted by zero mean Gaussian noise. The SNR is set to 20 dB.

There are two targets located at  $\theta_k = -1^\circ, 1^\circ$ , with reflection coefficients  $\beta_k = 1, k = 1, 2$ . A jammer is located at  $15^\circ$  and transmits an unknown Gaussian random waveform and with amplitude 20, i.e., 26 dB above the target reflection coefficients  $\beta_k$ . We sample the angle space by increments of  $0.5^\circ$  from  $-8^\circ$  to  $8^\circ$ , i.e.,  $\mathbf{a} = [-8^\circ, -7.5^\circ, \dots, 7.5^\circ, 8^\circ]$ .

First, we compare the performance of DOA estimation using the proposed method and three approaches [2], i.e., the Capon, APES and GLRT techniques. Fig. 2 and Fig. 3 show the moduli of the estimated reflection coefficients  $\beta_k$ , as functions of the azimuthal angle for  $N_r = 1$  and 10 receive antennas,

respectively. In Fig. 2, we use the uncorrelated QPSK waveforms; while in Fig. 3, we use correlated waveforms designed according to the desired beampattern  $P_d(\alpha_n)$  as

$$P_d(\alpha_n) = \begin{cases} 1 & -3^\circ \leq \alpha_n \leq 3^\circ \\ 0 & -8^\circ \leq \alpha_n < -3^\circ \text{ and } 3^\circ < \alpha_n \leq 8^\circ \end{cases} \quad (18)$$

Based on that beampattern, the method of [10] was followed to design  $\mathbf{R}$ . Then the transmitted waveforms can be constructed as  $\mathbf{x}(\mathbf{n}) = \mathbf{R}^{\frac{1}{2}}\mathbf{w}$ , where  $\mathbf{w}$  is a i.i.d random vector with zero mean and  $E\{\mathbf{w}\mathbf{w}^H\} = \mathbf{I}/L$ .

In both (a) and (b), the top three curves correspond to the azimuthal estimates obtained via Capon, APES and GLRT, using 512 snapshots. The bottom curve is the result of the proposed approach, obtained using 35 snapshots only. One can see that in the case of using only one receive node, the presence of the two targets is clearly evident via the proposed method based on 35 snapshots only using both independent and correlated waveforms. The other methods produce spurious peaks away from the target locations. When the measurements of multiple receive nodes are used at a fusion center, the proposed approach can yield similar performance to the other three methods. However, the comparison methods would have to transmit to the fusion center 512 received samples each, while in the proposed approach, each node would need to transmit 35 samples each.

The threshold  $\mu$  in (11) affects DOA estimation for the proposed method. The increase in  $\mu$  while keeping  $M_t$  and  $N_t$  constant can reduce the ripples of DOA estimates at the non-target azimuth angles at the expense of the accuracy of the target-reflection-coefficient estimates. The increase in  $\mu$  can also reduce the complexity of (11) because the constraint is looser than that of smaller  $\mu$ . If  $\mu$  is too large, however, the  $\ell_1$ -norm minimization does not work. In Fig.2 and 3, relatively large thresholds, i.e.,  $\mu = 12, 10$ , were used for the single receive node case. As a result, the CS method yielded less accurate estimates of the reflection coefficients magnitude than the Capon and APES, but with very few ripples.

Finally, we discuss the effect of  $L$ ,  $N_t$  and  $M_t$  on the performance of the Capon, APES, GLRT and CS. Fig. 4 compares the performance of these four approaches using independent waveforms for different combinations of  $N_r$  and  $L$ , whose product is fixed at 512. In order to quantify the performance of DOA estimation, we define the ratio of the square amplitude of the DOA estimate at the target azimuth angle to the sum of the square amplitude of DOA estimates at other angles as the peak-to-ripple ratio (PRR). Fig. 5 compares PRR as a function of  $L$  for these four approaches using uncorrelated signal waveforms. We consider the scenarios in which  $N_r = 1, 5, 10, 30$ . For fixed  $N_t$  and  $M_t$ , the increase in  $L$  can improve the performance of these four methods. In the presence of a moderate jammer, APES and CS can yield relatively accurate DOA estimates even with a small  $L$ . For Capon and GLRT,  $N_r$  must be greater than  $L$  in order to obtain a nonsingular sample covariance matrix of the received signal  $\hat{\mathbf{R}}$ . This is because Capon and GLRT need to calculate the inverse of  $\hat{\mathbf{R}}$ . On the other hand, an increase in both  $M_t$  and

$N_r$  can also improve the performance of the Capon, APES, GLRT and CS while  $L$  is fixed. If either  $M_t$  or  $N_r$  is too small, even a significant increase in the other parameter cannot improve performance of the first three approaches. However, CS can yield the desired DOA estimates even with a single receive antenna with a sufficient  $L$  and  $N_t$ . In the scenarios considered in our simulations where  $L = 2^9$ , for instance,  $M_t$  and  $N_r$  are required to be greater than 8 to yield the desired DOA estimates using the Capon, APES and GLRT. With a single receive node, CS requires at least 20 transmit nodes, while in the cases of multiple receive nodes, the requirement of  $M_t$  and  $N_r$  for CS is the same as in the other three methods.

## VI. CONCLUSION

We proposed a distributed MIMO Radar system implemented by a small size randomly dispersed wireless network. There are several advantages in using the proposed distributed approach as opposed to using a standard linear array. The radar system can be easily deployed; no pre-existing infrastructure is required. In a high density network there are many degrees of freedom to design the beampattern as desired around the look direction, which is important for clutter reduction or for reduction of scanning time. By randomizing the set of transmitters and receivers we can use the network power efficiently. By selecting well separated nodes we can increase spatial diversity. The resolution can be easily adjusted by employing more or less transmit nodes. The radar system is robust; should some nodes be deactivated the system performance will not be affected.

For the proposed MIMO radar system, a compressive sensing method has been exploited to estimate the DOAs of targets using both independent and correlated waveforms. The DOA of targets can construct a sparse vector in the angle space. Therefore, we can solve for this sparse vector by  $\ell_1$ -norm minimization with many fewer samples than conventional methods, i.e. the Capon, APES and GLRT techniques. The proposed method is superior to these conventional methods when one receive antenna is active. If multiple receive antennas are used, the proposed approach can yield similar performance to the other three methods, but by using far fewer samples.

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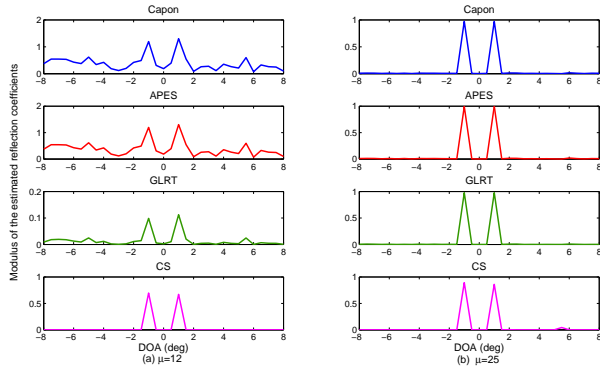


Fig. 2. DOA estimates of two targets with 1 (left column) and 10 (right column) receive antenna using independent waveforms. The top three curves were obtained using 512 snapshots. The bottom curve was obtained using 35 snapshots only.

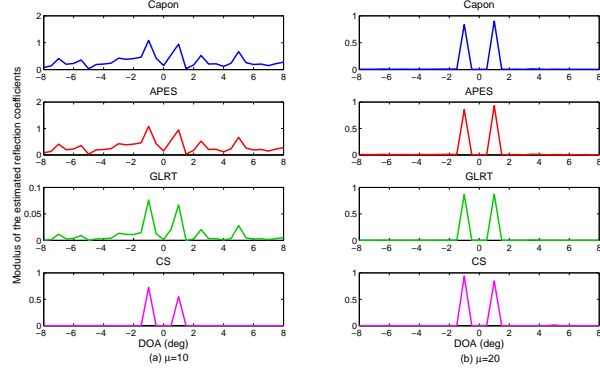


Fig. 3. DOA estimates of two targets with close azimuthal angles using 1 (left column) and 10 (right column) receive antenna using independent waveforms. The top three curves were obtained using 512 snapshots. The bottom curve was obtained using 35 snapshots only.

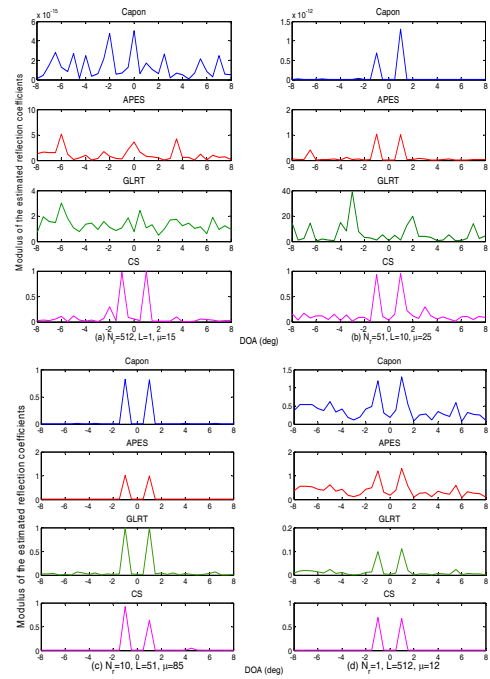


Fig. 4. DOA estimates with different sets of  $N_r$  and  $L$

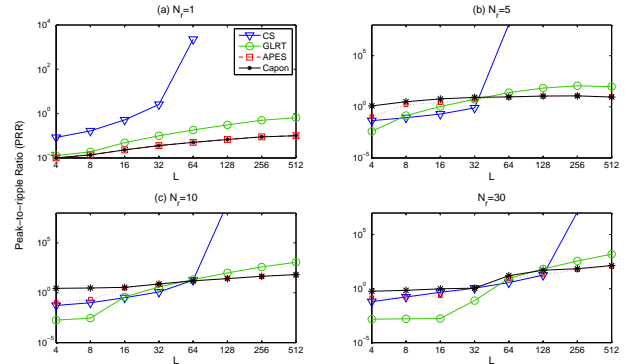


Fig. 5. Peak-to-ripple Ratio vs.  $L$  with different  $N_r$

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