Actuator Fault estimation with pole placement constraints for Takagi-Sugeno fuzzy systems with interval time varying delay: An LMI approach

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Abstract: This paper adresses the problem of fault estimation for continuous-time Takagi-Sugeno (T-S) fuzzy systems with interval time varying delay. The approach is based on a fuzzy Adaptive Fault Diagnostic Observer (AFDO). By considering an appropriate Lyapunov function, less conservative linear-matrix-inequality (LMI) conditions for the existence of the fault estimator are proposed. Furthermore, a pole placement is introduced in order to guarantee a faster convergence of estimation errors on the states and the faults. An efficient example is provided to illustrate the effectiveness of the proposed result.

Keywords: Linear Matrix Inequalities, Takagi-Sugeno fuzzy systems, Fault estimation, Adaptive fuzzy observer

1. INTRODUCTION

As is well known, Takagi-Sugeno(TS) Fuzzy observer design is become an active research field owing to its particular importance in observer-based control, fault diagnosis, and Fault Tolerant Control(FTC)of the nonlinear systems (Zhang and al. (2002), Chadli and El Hajjaji (2006), Kim and Lee (2000), Boukas and El Hajjaji (2006), Oudghiri M. and al (2008)). Different design techniques of TS fuzzy observer are been developed in the literature (Akhenak (2004), D. Ichalal and al. (2010), D. Ichalal and al. (2009), Chadli M. and al (2009), Bouattour M. and al (2010)). Among these techniques, we can find the Adaptive Fault Diagnostic Observer (AFDO) technique which allow to estimate the state vector and actuator fault simultaneously (Jiang and al. (2006), Zhang and al. (2008), Zhang and al. (2009)).
On the other hand, time delay is one of the instability sources in dynamical processus. For this reason, T-S fuzzy model has been extended to deal with nonlinear systems with time delay. Different delay-independent methodologies have been proposed for analysis and synthesis for T-S fuzzy systems with time delay (Chen and Liu (2005), Lee and al. (2000), Xu and Lam (2005)). It is generally recognized that delay-dependent results are usually less conservative than delay-independent ones. That's why, delay-dependent techniques have been reported in (Li and al. (2004), Guan and Chen (2004), Chen and Liu (2005)). Considering the AFDO design, the problem of fault estimation for linear system are treated in Zhang and al. (2008). This idea is extended to deal with T-S fuzzy models with constant time delay in Zhang and al. (2009). The obtained result is delay-independent. By adopting the free weighting matrix technique, a delay-dependent results are obtained for linear system with bounded time delay Jiang and al. (2009). However, the use of too many free weighting matrices makes the design method more complicate . To the best of our knowledge, so far, the problem of fault estimation of T-S fuzzy systems with interval time varying delay has not been addressed in the literature.
Motivated by the aforementioned observation, in this paper, we study the fault estimation for Takagi-Sugeno fuzzy systems with interval time varying delay based on AFDO. A pole placement is introduced in order to deliver sufficiently fast and well-responses of fault estimation.

Notations $W + W^T$ is denoted as $W + (*)$ for simplicity.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider an T-S fuzzy system with time-varying delay. The i^{th} rule of the system is expressed in the following IF-THEN rules.
Plant Rule $i (i = 1, 2, \cdots, r)$: If θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i u(t) \\ &\quad + E_i f(t) \\ y(t) &= C x(t) \end{aligned} \tag{1}$$

where $\theta_j(x(t))$ and $\mu_{ij} (i = 1, \cdots, r, j = 1, \cdots, p)$ are respectively the premise variable and the fuzzy sets; $\psi(t)$ is the initial conditions; $x(t) \in \Re^{n_x}$ is the state; $u(t) \in \Re^{n_u}$

is the control input; $f(t) \in \Re^{n_f}$ represents the actuator fault vector. It is assumed that the derivative of $f(t)$ with respect to time is norm bounded, i.e. $\|\dot{f}(t)\| \leq f_1$ and $0 \leq f_1 < \infty$. r is the number of IF-THEN rules.

the interval time varying delay satisfies

$$0 < \tau_m \leq \tau(t) \leq \tau_M, \dot{\tau}(t) \leq \beta \tag{2}$$

Denote

$$\tau_1 = \frac{\tau_m + \tau_M}{2}; \tau_2 = \frac{\tau_M - \tau_m}{2}$$

The AFDO is constructed as
Observer Rule $i (i = 1, 2, \cdots, r)$: If θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_i\hat{x}(t) + A_{\tau i}\hat{x}(t-\tau(t)) + B_i u(t)\\ &\quad + E_i\hat{f}(t) - L_i(\hat{y}(t) - y(t))\\ \hat{y}(t) &= C\hat{x}(t)\end{aligned} \tag{3}$$

Denote $e_x(t) = \hat{x}(t) - x(t)$, $e_y(t) = \hat{y}(t) - y(t)$, $e_f(t) = \hat{f}(t) - f(t)$

then the error dynamic is given by

$$\begin{aligned}\dot{e}_x(t) &= [(A(t) - L(t)C)e_x(t)\\ &\quad + A_\tau(t)e_x(t-\tau(t)) + E(t)e_f(t)]\\ e_y(t) &= Ce_x(t)\end{aligned}$$

where

$$A(t) = \sum_{i=1}^{r} h_i A_i; A_\tau(t) = \sum_{i=1}^{r} h_i A_{\tau i}; E(t) = \sum_{i=1}^{r} h_i E_i$$

3. RESULT

In this section, we first present useful lemma and then present the main result.

Lemma 1. For a symmetric positive definite matrix P, the following inequality holds

$$2x^T y \leq x^T P x + y^T P^{-1} y$$

Lemma 2. Consider a negative definite matrix $\Pi < 0$. Given a symmetric matrix X of appropriate dimension such that $X^T \Pi X < 0$, then, $\exists \lambda \in \Re^+$ such that

$$X^T \Pi X \leq -2\lambda X - \lambda^2 \Pi^{-1}$$

Lemma 3. Given matrices $M, E, F(t)$ with compatible dimensions and $F(t)$ satisfying $F(t)^T F(t) \leq I$.
Then, the following inequalities hold for any $\epsilon > 0$
$MF(t)E + E^T F(t)^T M^T \leq \epsilon M M^T + \epsilon^{-1} E^T E$

Definition 1. A subset D of the complex plane is called an LMI region if there exist a symmetric matrix α and a matrix β such that

$$D = \{z \in \mathcal{C},\ f_D(z) < 0\}$$

with

$$f_D(z) = \alpha + \beta z + \beta^T \bar{z}$$

A dynamical system is called D stable if all its poles lie in D (that is, all eigenvalues of the matrix A lie in D).

Lemma 4. Chilali and Gahinet (1996) A is D stable if and only if there exists a symmetric matrix $P > 0$ such that

$$\alpha \otimes P + \beta \otimes AP + \beta^T \otimes PA^T < 0$$

where $\otimes$ denotes the Kronecker product of matrices

Theorem 1. For given positif scalar λ, if there exist symmetric positive definite matrices P, $Q_l (l = 1, 2, 3, 4, 5)$, $R_m (m = 1, 2, 3, 4)$, M and matrices Yi and F_i such that the following LMI hold

$$\begin{aligned}&\alpha \otimes P + \beta \otimes A_i^T P - \beta \otimes C^T Y_i^T\\ &\quad + \beta^T \otimes PA_i - \beta^T \otimes Y_i C < 0,\ i = 1, 2, \cdots, r\end{aligned} \tag{4}$$

$$\Xi_{ij} + \Xi_{ji} < 0,\ i, j = 1, 2, \cdots, r, i \leq j \tag{5}$$

$$E_i^T P = F_i C,\ i = 1, 2, \cdots, r \tag{6}$$

then $A(t) - L(t)C$ is D stable and the fuzzy Fast Adaptive Fault Estimation (FAFE) algorithm

$$\dot{\hat{f}}(t) = -\Gamma \sum_{i=1}^{r} h_i F_i(\dot{e}_y(t) + e_y(t)) \tag{7}$$

can realize that $e_x(t)$ and $e_f(t)$ are uniformly ultimately bounded.

In this case, the L_i are given by

$$L_i = P^{-1} Y_i \tag{8}$$

$$\Xi_{ij} = \begin{bmatrix} \xi_{ij}^{11} & R_4 & PA_{\tau i} & 0 & 0 & \xi_{ij}^{16} & \xi_{ij}^{17}\\ * & \xi_{22} & R_3 & Q_2 + R_1 & 0 & 0 & 0\\ * & * & \xi_{33} & 0 & R_3 & -A_{\tau i}^T PE_j & A_{\tau i}^T P\\ * & * & * & \xi_{44} & -Q_2 + R_2 & 0 & 0\\ * & * & * & * & \xi_{55} & 0 & 0\\ * & * & * & * & * & \xi_{ij}^{66} & E_i^T P\\ * & * & * & * & * & * & \xi_{ij}^{77} \end{bmatrix}$$

where

$$\begin{aligned}\xi_{ij}^{11} &= PA_i - Y_i C + A_i^T P - C^T Y_i^T + Q_4 + Q_5 - R_4;\\ \xi_{ij}^{16} &= -A_i^T PE_j + C^T Y_i^T E_j;\\ \xi_{ij}^{17} &= A_i^T P - C^T Y_i^T\\ \xi^{22} &= Q_1 - Q_5 - R_1 - R_3 - R_4;\\ \xi^{33} &= -(1-\beta)Q_4 - 2R_3;\\ \xi^{44} &= Q_3 - Q_1 - R_1 - R_2;\\ \xi^{55} &= -Q_3 - R_2 - R_3;\end{aligned}$$

$$\xi_{ij}^{66} = -E_i^T P E_j - E_j^T P E_i + M;$$
$$\xi_{ij}^{77} = -2\lambda P + \lambda^2((\tau_2)^2 R_1 + (\tau_2)^2 R_2 + (2\tau_2)^2 R_3 + (\tau_m)^2 R_4)$$

Proof:
Constraints (4): By using lemma (4), $A(t) - L(t)C$ is D stable if and only if there exist a symmetric matrix P such that

$$\alpha \otimes P + \beta \otimes (A(t) - L(t)C)P + \beta^T \otimes P(A(t) - L(t)C)^T < 0 \quad (9)$$

which can be rewritten as

$$\sum_{i=1}^{r} h_i[\alpha \otimes P + \beta \otimes (A_i - L_iC)P + \beta^T \otimes P(A_i - L_iC)^T] < 0 \quad (10)$$

Since the solution of $det(A_i - L_iC) = 0$ is the same as that $det((A_i - L_iC)^T) = 0$, as long as D stability is the only concern, (10) is equivalent to

$$\sum_{i=1}^{r} h_i[\alpha \otimes P + \beta \otimes (A_i - L_iC)^T P + \beta^T \otimes P(A_i - L_iC)] < 0 \quad (11)$$

Therefore, if (4) hold, then $(A(t) - L(t)C)$ is D stable.

For constraints (5) and (6), Consider the following Lyapunov function

$$V(t) = \sum_{i=1}^{6} V_i(t) \quad (12)$$

where

$$V_1(t) = e_x(t)^T P e_x(t) + \int_{t-\tau_1}^{t-\tau_m} \{e_x(s)^T Q_1 e_x(s) + 2e_x(s)^T Q_2 e_x(s-\tau_2) + e_x(s-\tau_2)^T Q_3 e_x(s-\tau_2)\} ds + \int_{t-\tau(t)}^{t} e_x(s)^T Q_4 e_x(s) ds + \int_{t-\tau_m}^{t} e_x(s)^T Q_5 e_x(s) ds$$

$$V_2(t) = \tau_2 \int_{-\tau_1}^{-\tau_m} \int_{t+\sigma}^{t} \dot{e}_x(s)^T R_1 \dot{e}_x(s) ds d\sigma$$
$$V_3(t) = \tau_2 \int_{-\tau_M}^{-\tau_1} \int_{t+\sigma}^{t} \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds d\sigma$$
$$V_4(t) = 2\tau_2 \int_{-\tau_M}^{-\tau_m} \int_{t+\sigma}^{t} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds d\sigma$$
$$V_5(t) = \tau_m \int_{-\tau_m}^{0} \int_{t+\sigma}^{t} \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds d\sigma$$
$$V_6(t) = e_f(t)^T \Gamma^{-1} e_f(t)$$

Taking the derivation of $V(t)$, we have

$$\dot{V}_1(t) = 2e_x(t)^T P \dot{e}_x(t) + \Delta$$

where

$$\Delta = e_x(t-\tau_m)^T Q_1 e_x(t-\tau_m) + 2e_x(t-\tau_m) Q_2 e_x(t-\tau_1) + e_x(t-\tau_1) Q_3 e_x(t-\tau_1) - e_x(t-\tau_1)^T Q_1 e_x(t-\tau_1) - 2e_x(t-\tau_1)^T Q_2 e_x(t-\tau_M) - e_x(t-\tau_M)^T Q_3 x(t-\tau_M) + e_x(t)^T Q_4 e_x(t) - (1-\dot{\tau}(t)) e_x(t-\tau(t)) Q_4 e_x(t-\tau(t)) + e_x(t)^T Q_5 e_x(t) - e_x(t-\tau_m)^T Q_5 e_x(t-\tau_m)$$

$$\dot{V}_2(t) = \tau_2\{\tau_2 \dot{e}_x(t)^T R_1 \dot{e}_x(t) - \int_{t-\tau_1}^{t-\tau_m} \dot{e}_x(s)^T R_1 \dot{e}_(s) ds\} \quad (13)$$

Similarly, we obtain

$$\dot{V}_3(t) = \tau_2\{\tau_2 \dot{e}_x(t)^T R_2 \dot{e}_x(t) - \int_{t-\tau_M}^{t-\tau_1} \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds\}$$
$$\dot{V}_4(t) = 2\tau_2\{2\tau_2 \dot{e}_x(t)^T R_3 \dot{e}_x(t) - \int_{t-\tau_M}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds\}$$
$$\dot{V}_5(t) = \tau_m\{\tau_m \dot{e}_x(t)^T R_4 \dot{e}_x(t) - \int_{t-\tau_m}^{t} \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds\}$$
$$\dot{V}_6(t) = -2e_f(t)^T F(t) C(\dot{e}_x(t) + e_x(t)) - 2e_f(t)^T \Gamma^{-1} \dot{f}(t)$$

By considering equality (6), we obtain:

$$\dot{V}_1(t) + \dot{V}_6(t) = 2e_x(t)^T P[(A(t) - L(t)C)e_x(t) + A_\tau(t) e_x(t-\tau(t))] + \Delta - 2e_f(t)^T F(t) C \dot{e}_x(t) - 2e_f(t)^T \Gamma^{-1} \dot{f}(t) \quad (14)$$

From lemma (1), we can obtain that

$$-2e_f(t)^T \Gamma^{-1} \dot{f}(t) \leq e_f(t)^T M e_f(t) + \dot{f}(t)^T \Gamma^{-1} M^{-1} \Gamma^{-1} \dot{f}(t) \leq e_f(t)^T M e_f(t) + \delta \quad (15)$$

where

$$\delta = f_1^2 \lambda_{max}(\Gamma^{-1} M^{-1} \Gamma^{-1})$$

Applying jessen's inequality Gu and al. (2003) to deal with the cross product items, we have

$$-\tau_2 \int_{t-\tau_1}^{t-\tau_m} \dot{e}_x(s)^T R_1 \dot{e}_x(s) ds$$

$$\leq \begin{bmatrix} e_x(t-\tau_m) \\ e_x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ * & -R_1 \end{bmatrix} \begin{bmatrix} e_x(t-\tau_m) \\ e_x(t-\tau_1) \end{bmatrix}$$

$$-\tau_2 \int_{t-\tau_M}^{t-\tau_1} \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds$$

$$\leq \begin{bmatrix} e_x(t-\tau_1) \\ e_x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ * & -R_2 \end{bmatrix} \begin{bmatrix} e_x(t-\tau_1) \\ e_x(t-\tau_M) \end{bmatrix}$$

$$-2\tau_2 \int_{t-\tau_M}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds$$

$$\leq -(\tau(t)-\tau_m) \int_{t-\tau(t)}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds$$

$$-(\tau_M-\tau(t)) \int_{t-\tau_M}^{t-\tau(t)} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds$$

$$\leq \begin{bmatrix} e_x(t-\tau_m) \\ e_x(t-\tau(t)) \\ e_x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & 0 \\ * & -2R_3 & R_3 \\ * & * & -R_3 \end{bmatrix} \begin{bmatrix} e_x(t-\tau_m) \\ e_x(t-\tau(t)) \\ e_x(t-\tau_M) \end{bmatrix}$$

$$-\tau_m \int_{t-\tau_m}^{t} \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds$$

$$\leq \begin{bmatrix} e_x(t) \\ e_x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_4 & R_4 \\ * & -R_4 \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_x(t-\tau_m) \end{bmatrix}$$

Let $\eta(t) = [e_x(t)^T, e_x(t-\tau_m)^T, e_x(t-\tau(t))^T, e_x(t-\tau_1)^T, e_x(t-\tau_M)^T, e_f(t)^T]^T$.

$$\dot{V}(t) \leq \eta(t)^T \Omega(t) \eta(t) \tag{16}$$
$$+\dot{e}_x(t)^T((\tau_2)^2 R_1 + (\tau_2)^2 R_2 + (2\tau_2)^2 R_3 + (\tau_m)^2 R_4)\dot{e}_x(t) + \delta$$

where

$$\Omega(t) = \begin{bmatrix} \omega_{11}(t) & R_4 & PA_\tau(t) & 0 & 0 & \omega_{16}(t) \\ * & \xi_{22} & R_3 & Q_2+R_1 & 0 & 0 \\ * & * & \xi_{33} & 0 & R_3 & A_\tau(t)^T PE(t) \\ * & * & * & \xi_{44} & -Q_2+R_2 & 0 \\ * & * & * & * & \xi_{55} & 0 \\ * & * & * & * & * & \omega_{66}(t) \end{bmatrix}$$

in which

$$\begin{aligned} \omega_{11}(t) &= P(A(t)-L(t)C) + (A(t)-L(t)C)^T P \\ &\quad +Q_4+Q_5-R_4; \\ \omega_{16}(t) &= -(A(t)-L(t)C)^T PE(t); \\ \omega_{66}(t) &= -2E(t)^T PE(t) + M \end{aligned}$$

By the Schur complement, we obtain

$$\Phi(t) = \begin{bmatrix} \Omega(t) & \phi_{12}(t) \\ * & \begin{matrix} -P((\tau_2)^2 R_1 + (\tau_2)^2 R_2 \\ +(2\tau_2)^2 R_3 + (\tau_m)^2 R_4)^{-1} P \end{matrix} \end{bmatrix}$$

where

$$\phi_{12}(t)^T = [\, P(A(t)-L(t)C) \ 0 \ PA_\tau(t) \ 0 \ 0 \ PE(t) \,]^T$$

Applying lemma (2), if $\Xi_{ij} + \Xi_{ji} < 0, i, j = 1, 2, \cdots, r, i \leq j$, then there exist a scalar $\epsilon > 0$ such that $\dot{V}(t) < -\epsilon\|\eta(t)\|^2 + \delta$. It follows that $\dot{V}(t) < 0$ for $\epsilon\|\eta(t)\|^2 > \delta$, which means that $\eta(t)$ converges to a small set $S = \{\eta(t) | \|\eta(t)\|^2 \leq \frac{\delta}{\epsilon}\}$ according to Lyapunov stability theory. Therefore, estimation errors of both the state and the fault are uniformly ultimately bounded.

Remark 1. The purpose of introducing the pole placement constraints is to improve the transient performance of fault estimation.

Remark 2. Zhang and al. (2008) It is easy to solve the inequalities (4)-(5) by using LMI Toolbox. For equation (6), we can make a transform into the following optimization problem
Minimize $\rho > 0$
Subject to

$$\begin{bmatrix} \rho I & E_i^T P - F_i C \\ * & \rho I \end{bmatrix} > 0, i = 1, \cdots, r \tag{17}$$

Remark 3. Examples of LMI region :

• **Half-plane** $Re(s) < -a$. In this case, (4) can be rewritten as

$$2aP + A_i^T X - C^T Y_i^T + XA_i - Y_i C < 0 \tag{18}$$

• **Disk centred at** $(-q, 0)$ **with radius** r. In this case, (4) can be rewritten as

$$\begin{bmatrix} -rP & -rP + A_i^T P - C^T Y_i^T \\ & -rP \end{bmatrix} < 0 \tag{19}$$

• **Conic sector with apex at the origin and inner angle** 2θ. In this case, (4) can be rewritten as

$$\begin{bmatrix} \sin(\theta)(A_i^T P - C^T Y_i^T + P A_i - Y_i C) & \cos(\theta)(A_i^T P - C^T Y_i^T - P A_i + Y_i C) \\ -\cos(\theta)(A_i^T P - C^T Y_i^T - P A_i + Y_i C) & \sin(\theta)(A_i^T P - C^T Y_i^T + P A_i - Y_i C) \end{bmatrix} < 0 \qquad (20)$$

4. SIMULATION RESULT

Consider the following T-S fuzzy model

$$\dot{x}(t) = \sum_{i=1}^{2} h_i[A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i u(t) + E_i f(t)] \quad (21)$$

The membership functions for rules 1 and 2 are:

$$h_1(x_2(t)) = 1 - \frac{x_2(t)^2}{2.25},\ h_2(x_2(t)) = 1 - h_1(x_2(t)) \quad (22)$$

where

$$A_1 = \begin{bmatrix} -0.1125 & -0.0200 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1125 & -1.5270 \\ 1 & \end{bmatrix}$$

$$A_{\tau 1} = \begin{bmatrix} -0.0125 & -0.0050 \\ 0 & 0 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -0.0125 & -0.2300 \\ 0 & 0 \end{bmatrix}$$

$$E_1 = E_2 = B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [\,0\ \ 1\,]$$

By solving the conditions in theorem 1, without pole placement, we obtain

$$F = 10^3 \times 1.8555$$

$$L_1 = \begin{bmatrix} 1.7056 \\ 2.1580 \end{bmatrix}, L_2 = \begin{bmatrix} 0.1081 \\ 2.1555 \end{bmatrix}$$

First, it assumed that a constant fault $f_1(t)$ is created as

$$\begin{aligned} f_1(t) &= 0, \quad 0 \le t < 5 \\ &= 20, \quad 5 \le t \le 20 \end{aligned} \quad (23)$$

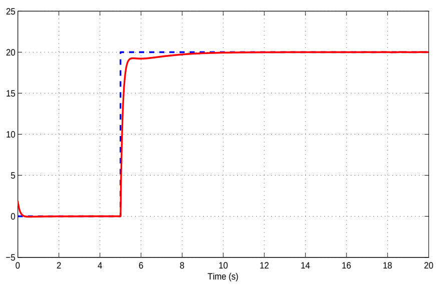

Fig. 1. Fault $f_1(t)$ (Dotted) and its estimated $\hat{f}_1(t)$ (solid) without pole placement

Then, a time-varying fault is simulated

$$\begin{aligned} f_2(t) &= 0, \quad 0 \le t < 5 \\ &= 10\sin(t-5),\ 5 \le t \le 20 \end{aligned} \quad (24)$$

By solving the conditions in theorem 1, with pole clustering in the region $Re(s) < -1$, we obtain

$$F = 10^3 \times 4.0686$$

$$L_1 = \begin{bmatrix} 2.2594 \\ 2.3831 \end{bmatrix}, L_2 = \begin{bmatrix} 0.6692 \\ 2.3631 \end{bmatrix}$$

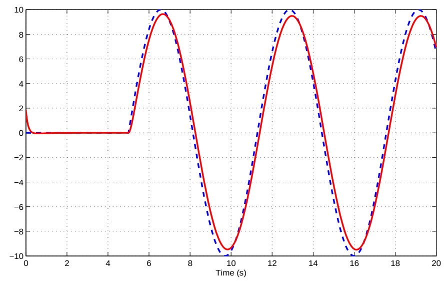

Fig. 2. Fault $f_2(t)$ (Dotted) and its estimated $\hat{f}_2(t)$ (solid) without pole placement

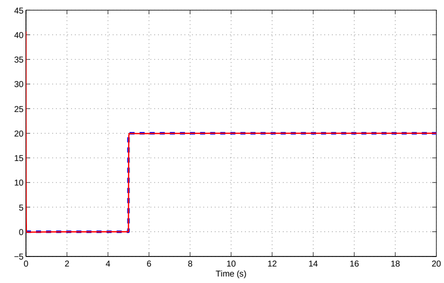

Fig. 3. Fault $f_1(t)$ (Dotted) and its estimated $\hat{f}_1(t)$ (solid) with pole placement

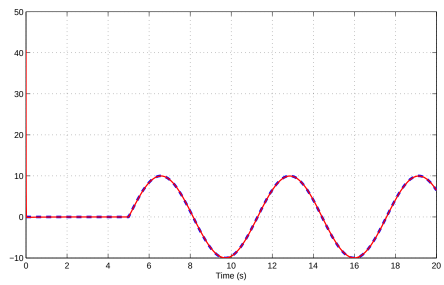

Fig. 4. Fault $f_2(t)$ (Dotted) and its estimated $\hat{f}_2(t)$ (solid) with pole placement

5. CONCLUSION

The problem of delay dependent AFDO design to a class of T-S fuzzy systems with interval time varying delay has been investigated. Based in improved Lyapunov function, a delay dependent conditions for the existence of the fault estimator are given in terms of linear matrix inequalities. This result overcomes the drawbacks of delay independent results. An illustrative example has been presented to demonstrate the potential of the method.

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