ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

Cellular Neural Networks: from Chaos generation to Complexity modelling

Paolo Arena and Luigi Fortuna
Dipartimento Elettrico, Elettronico e Sistemistico,
Universita' di Catania,
V.le A.Doria 6, 95125, Catania, Italy
e-mail:parena@dees.unict.it.

Abstract. In this paper some recent results of the authors' research on Cellular Neural Networks (CNNs) are reported and some novel applications are introduced. In particular the CNN paradigm is shown to be able to represent chaotic and hyperchaotic motions generated by some canonical circuits. Moreover thoroughly different dynamics often met in living systems and represented by non linear partially differential equations have been shown to be suitably reproduced in CNN arrays. Finally some interesting applications to motion control, currently under investigation, are briefly introduced.

1. Introduction

In the last few years, a broad spectrum of activities in the circuits and systems community has been devoted to discovering how the nonlinearity and the interactions of simple circuits lead to complex dynamics [1]. The initial impetus for this appealing activity was provided by the introduction of Chua's circuit [2]. Recently, the active use of chaos in real electrical engineering applications [3] has further stimulated the activity of presenting new circuits with strange nonlinear behaviour. However, the invention of the Cellular Neural Network (CNN) [4]- [5] has led to a charming possibility of research among the scientific community. Moreover, current CNN systems are assuming a fundamental role as an appropriate platform for studying complexity [6].
The approach discussed in the first part of this paper allows exact generation of the dynamics (chaotic and hyperchaotic) of a wide class of nonlinear dynamic systems by a suitable connection of State-Controlled CNN (SC-CNNs) cells. In this way, the SC-CNN can be considered a simple analog computing element for complex nonlinear system simulation. The approach proposed has also some interesting consequences with regard to the circuit implementation of these systems. Among these, the use of inductors is avoided and the nonlinearities are obtained by the natural saturation of amplifiers.
Theoretical propositions are presented to fix the templates of the SC-CNNs in such a way as to exactly match the dynamic behaviour of many well-known nonlinear circuit dynamics [11]- [15]. In each case the corresponding SC-CNN

ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

based implementation is presented referring to its easy and cheap implementation and the experimental results are reported.
In the second part of the paper it is shown that, due to their locally distributed way of exchanging signals, CNNs can be used as powerful devices to simulate and to reproduce, in an analog fashion and low cost, complex behaviours, i.e. dynamics commonly encountered in living systems, such as autonomous wave formation and propagation [20] as well as morphogenetical pattern development [23], [24]. In fact it is proven that both of these behaviours can be simulated with CNNs with the same cell structure, and the thoroughly different dynamics can arise only suitably modulating the CNN cell parameters [16], [21]. However, in this paper only the aspects referring to autowaves formation are discussed, in view of their use for a novel type of applications in the motion modelling field.

2. State Controlled Cellular Neural Networks for non linear dynamic generation

The first issue of this Section is to introduce the *State Controlled CNN model* (SC-CNN). In the original definition of Chua and Yang [4] the state variable of a cell directly depends on the outputs and inputs of the cells belonging to its neighbour set; so the dependence on the state variables of the neighbours is indirect because it comes from the neighbour cells outputs. In the SC-CNN, on the other hand, the dependence on state variables is explicitly obtained by adding a new template, named state template.

Definition 2..1 *The State Controlled Cellular Neural Network (SC-CNN) is a two-dimensional array of nonlinear dynamic systems, described by the following state equations:*

$$\dot{x}_{ij}(t) = -x_{ij}(t) + \sum_{C(k,l)\in N_r(i,j)} \{A_{ij;kl}y_{kl}(t) + B_{ij;kl}u_{kl}(t) + C_{ij;kl}x_{kl}(t)\} + I_{ij} \quad (1)$$

$$y_{ij} = f(x_{ij}) = 0.5 \cdot (| x_{ij} + 1 | - | x_{ij} - 1 |) 1 \leq i \leq M \quad 1 \leq j \leq N \quad (2)$$

where N and M are the array dimensions; $N_r(i,j)$ is the neighbourhood set of the cell $C(i,j)$; $x_{ij}(t)$, $y_{ij}(t)$ and $u_{ij}(t)$ are the state variable, the output and the input of the cell respectively; $A_{ij;kl}$, $B_{ij;kl}$ and $C_{ij;kl}$ are known as templates (here assumed as constants) called feedback, control and state templates respectively, while $f : \Re \longrightarrow \Re$ is the output nonlinearity (saturation function).

The definition, given for a two-dimensional array of cells, can be generalized to an n-dimensional array of cells.
The procedure establishing the SC-CNN templates for some assigned nonlinear circuit state equations consists of comparing the full set of SC-CNN equations with the known circuit state equations. In the particular case of linear SC-CNNs, where the output nonlinearity is defined by equation (2), there are

ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

some restrictions on the nonlinear terms that can actually be present. In fact only nonlinearities of the type:

$$y = f(x) = a \cdot x + b \cdot (| x + 1 | - | x - 1 |) \tag{3}$$

can be included into the circuit equations (in the equation above a and b are real constants, x is a state variable and y is its corresponding nonlinear term). However, changing the state representation of the given system via a suitable non-singular linear transformation can be useful to represent circuit equations containing a nonlinear term that is a function of two state variables [22]. However, such a strategy often leads to SC-CNNs with space-variant templates. In Fig.1, some well known chaotic and hyperchaotic attractors, reproduced by using the SC-CNN paradigm are reported as examples of a much wider gallery of complex dynamics already obtained.

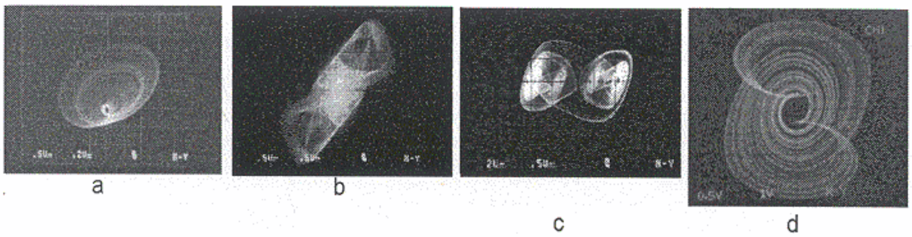

Figure 1: *SC-CNN generation of some well-known chaotic attractors: a:Chua's circuit [2]; b: hyperchaotic attractor [25]; c:N-Double scroll [26]; d:Multimode Chaos [27]*

3. CNNs to generate complex dynamics

In this section some studies regarding the generation of reaction-diffusion type dynamics by using CNNs are presented. A reaction-diffusion systems can be considered as an ensemble of a large number of identical subsystems, coupled one another by diffusion [8]. These systems are often met in living structures, physiological systems, as well as in chemical reactions or in combustion [7], [9]. They have been traditionally been modelled by means of non linear partial differential equations. In this framework, CNNs represent a powerful tool for their real-time simulation [17]. In particular the traditional CNN scheme with constant templates and without considering the state template C in equation (1) has been taken into consideration. Each cell in the CNN array is a second order non linear system characterised by a limit cicle regime showing a slow-fast dynamics. The connection of each cell to the neighboring ones by means of templates representing the discretized version of the laplacian operator gives rise to an autonomous two-layer CNN called Reaction-Diffusion CNN [10]. Suitable circuit parameters and suitable initial conditions lead to the onsed of the so-called auto waves [20], which consist in particular waves,

ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

which differ from traditional waves and take place in non linear active media. A theoretical study of a CNN cell able to generate autowaves was performed in [16]. Moreover several simulations were performed in order to verify the suitability of the designed CNN. A study revealed also the robustness of such phenomena against parameter uncertainties and noise. Therefore the hardware realization of the autonomous CNN has been carried on [29]. In particular a main board, able to include a CNN matrix $5x5$ has been built and autowaves have been observed and measured (Fig.2). Among the various applications of

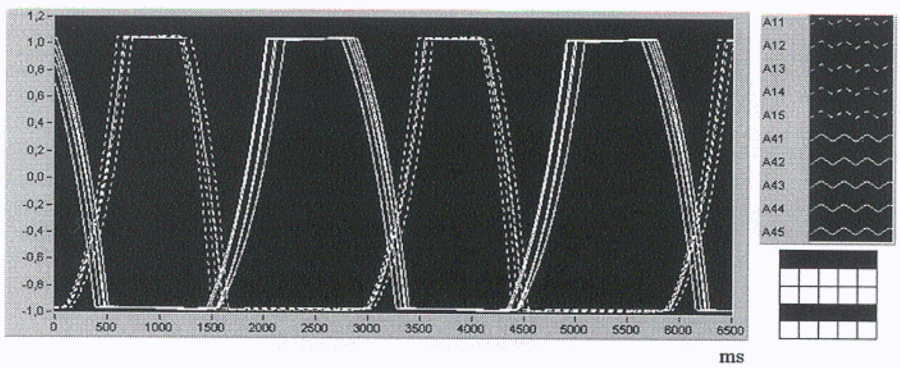

Figure 2: *Autowave propagating through the CNN matrix rows. The CNN matrix is schematically shown in the bottom right part of the figure*

autowaves reported in literature [18], [19], here attention is focused on typical examples taken from mathematical biology which refer to non linear reaction-diffusion PDEs in order to model the excitation and propagation of impulses in nerve membranes. From such considerations the idea has ariasen to use autowaves for motion control purpouses. In fact, since autowaves, as solutions of nonlinear PDEs, represent active spatiotemporal phenomena, they can be used to control motion, which is a classical spatio-temporal phenomenon. If the cell outputs of, for example, one column of the $5x5$ CNN array are used to drive some motion actuators, it is possible to observe the autowave shape directly on some moving objects. The experiment taken into consideration regards a worm-like walking mechatronic system, whose motion is controlled directly from the values of the first output variables of each of five cells of the CNN array (Fig. 3). This is the first example of how complex phenomena, typical of living systems, can be suitably reproduced in CNN arrays and used to emulate living motions.

4. Conclusions

In this paper an overview on some appealing results obtained in the study of Cellular Neural Networks is presented. In the first part of the paper it is shown that CNNs are able to exactly match a wide variety of chaotic and hyperchaotic

ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

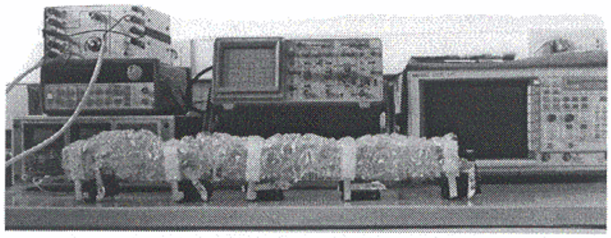

Figure 3: *Walking cellular robot driven by autowaves reproduced by CNNs*

motions, while the second part focuses attention on the application of CNNs to suitably reproduce reaction-diffusion phenomena, often met in living cells motions. Referring to a hardware setup already built by the authors, in which autowaves have been observed and measured, an interesting new application to motion control is breafly discussed. The brief overview reported in the paper on the various theoretical and practical aspects of CNNs gives an idea on the powerful possibilities offered by such devices to model and reproduce a wide variety of complex motions, which range from chaos to self-organization phenomena.

References

[1] L.O.Chua (editor), Special Issue on Nonlinear Waves, Patterns and Spatio-Temporal Chaos, *IEEE Trans. on Circuits and Systems - Part I*, vol. 42, no.10, 1995.

[2] L.O.Chua, "Global Unfolding of Chua's circuit", *IEICE Trans. Fundamentals*, vol.E76-A, pp.704-734, 1993.

[3] T.L. Carroll, "Communicating with use of filtered, synchronized, chaotic signals", IEEE Trans. Circ. and Syst., Part I, Vol.42 No.3, pp.105-110, 1995.

[4] L.O.Chua and L.Yang, "Cellular Neural Networks: Theory", *IEEE Trans. on Circuits and Systems*, vol.35, pp.1257-1272, 1988.

[5] L.O.Chua and T.Roska, "The CNN Paradigm", *IEEE Trans. on Circuits and Systems - part I*, vol.40, pp.147-156, 1993.

[6] L.O.Chua, "Cellular Neural Networks: A Paradigm for Complexity", *Workshop on Organization lines within complexity*, Florence, December 1994.

[7] S.Kondo, R.Asai, "A reaction-diffusion wave on the skin of the marine Angelfish Pomacanthus", *Nature*, vol.376, pp.765-768, 1995.

[8] G.Nicolis, I.Prigogine, *Exploring Complexity: An Introduction*, W.H.Freeman, New York, 1989.

[9] V.I.Krinsky, "Autowaves: Results, Problems, Outlooks", in *Self-Organization: Autowaves and Structures Far from Equilibrium*, Springer-Verlag, Berlin, 1984.

[10] L.O.Chua, M.Hasler, G.S.Moschytz, J.Neirynck, "Autonomous Cellular Neural Networks: A Unified Paradigm for Pattern Formation and Active Wave Propagation", *IEEE Trans. on Circuits and Systems - Part I*, vol. 42, no.10, 1995.

ESANN'1998 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 22-23-24 April 1998, D-Facto public., ISBN 2-9600049-8-1, pp. 309-314

[11] P. Arena, S. Baglio, L. Fortuna, G. Manganaro, "Chua's circuit can be generated by CNN cells", IEEE Trans on Circuits and Systems - Part I, Feb.1995.

[12] P.Arena, S.Baglio, L.Fortuna and G.Manganaro, "Hyperchaos from Cellular Neural Networks", *IEE Electronics Letters*, vol.31, pp.250-251, 1995.

[13] P.Arena, S.Baglio, L.Fortuna and G.Manganaro, "A Simplified Scheme for the Realisation of the Chua's Oscillator by Using SC-CNN Cells", IEE Electronic Letters, vol.31, N21, pp.1794-1795, Oct.1995.

[14] Arena, P., S.Baglio, L.Fortuna, G.Manganaro "Generation of N-Double Scrolls via Cellular Neural Networks" ", International Journal on Circuit Theory and Applications, Special issue on CNN-II , Part II, No.3, Vol. 24, 1996.

[15] P. Arena, S. Baglio, L. Fortuna, G. Manganaro, "State Controlled CNN: A new strategy for generating high complex dynamics", IEICE Trans. on Fundamental, Vol. E79-A N10, Oct. 96.

[16] P. Arena, S. baglio, L. Fortuna, G. Manganaro, "Self-Organization in a two-layer CNN", *IEEE Trans. on Circuits and Systems - Part I*, to appear.

[17] Arena, P., S.Baglio, L.Fortuna, G.Manganaro "Air quality modeling with CNNs", ECCTD'95.

[18] V.Perez-Munuzuri, V.Perez-Villar, L.O.Chua, "Autowaves for Image Processing on a Two-Dimensional CNN Array of Excitable Nonlinear Circuits: Flat and Wrinkled Labyrinth", *IEEE Trans. on Circ. and Syst. - Part I*, vol. 40, 1993.

[19] A.Perez-Munuzuri, V.Perez-Munuzuri, V.Perez-Villar, L.O.Chua, "Spiral Waves on a 2-D Array of Nonlinear Circuits", *IEEE Trans. on Circuits and Systems - Part I*, vol. 40, pp.872-877, 1993.

[20] V.I.Krinsky (editor), *Self-Organization: Autowaves and Structures Far from Equilibrium*, Springer-Verlag, Berlin, 1984

[21] P. Arena, S. baglio, L. Fortuna, G. Manganaro, "A CNN Cell for pattern formation and active wave propagation", European Conference on Circuit Theory and Design, ECCTD 97, Budapest 1997.

[22] P. Arena, S. Baglio, L. Fortuna, G. Manganaro, "SC-CNN based systems to realize a class of autonomous and coupled chaotic circuits", ISCAS 97, Hong Kong, 1997.

[23] A. M. Turing, "The chemical basis of morphogenesis", Philos. Trans. R. Soc. London B327, pp.37-72, 1952.

[24] J. D. Murray, *Mathematical Biology*, Springer-Verlag, Berlin, 1989.

[25] T.Saito, "An Approach Toward Higher Dimensional Hysteresis Chaos Generators", *IEEE Trans. on Circuits and Systems*, vol. 37, pp.399-409, 1990.

[26] J.A.K.Suykens and J.Vandewalle, "Generation of n-Double Scrolls (n=1,2,3,4,...)", *IEEE Trans. on Circuits and Systems - Part I*, vol. 40, pp.861-867, 1993.

[27] Y.Nishio and A.Ushida, "Multimode Chaos in Two Coupled Chaotic Oscillators with Hard Nonlinearities", *IEEE Proc.of ISCAS'94*, vol.6,pp.109-112, 1994.

[28] P. Arena, S. Baglio, L. Fortuna, G. Manganaro "Complexity in a two-layer CNN" Proc. CNNA '96, Seville, Spain, June 1996.

[29] P.Arena, M. Branciforte, L.Fortuna, "A CNN based experimental frame for patterns and autowaves", sub. to the Int. J. on Circuit theory and Applications.