A Succinct Representation Scheme for Cooperative Games under Uncertainty

Extended Abstract

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ABSTRACT

In this work we present a novel succinct representation for large partially observed cooperative games. The proposed representation exploits estimates over marginal contributions to form compact rules representing collaboration patterns with uncertain value. Specifically, given an initial set of MC-nets rules that use prior beliefs over values instead of the actual ones, we propose two types of merging that lead to a new set of even more compact rules.

KEYWORDS

Coalitional games representation; MC nets; Uncertainty

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1 INTRODUCTION

Multiagent systems model settings where many agents cooperate in order to resolve complicated tasks, due to resource or time limitations; or simply because a collaboration among many may yield better outcomes. Cooperative games [2] provide the formal framework for describing such systems. As a system grows, i.e., the number of agents rises, it becomes extremely harsh and tedious to fully represent the system. That is, in a large open environment with hundreds (or even thousands) of agents, fully representing all the possible collaborations among the different agents becomes exponential on the number of agents. Thus, in this paper we work towards a succinct representation of large coalitional games.

Moreover, taking into consideration the large number of agents that may participate in an open multiagent environment, it is unrealistic to assume that we have knowledge over all collaborations a priori. In other words, there are exponentially many different coalitions of agents, that in practice is impossible to know the exact worth of each one. Instead, we can assume that we have estimates (or beliefs) over the coalitions. As such, it is hard, and can be inefficient to represent large multiagent systems, let alone when the environment is not fully observable.

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Ieong and Shoham in [4] introduced the *Marginal Contribution Nets (MC-nets)* representation as a succinct way to describe characteristic function games (CFG) [2]. The MC-net for a CFG $G = \langle N, v\rangle$ is given by a set of rules of the form: $Pattern \rightarrow value$. A pattern consist of positive and negative literals, where each literal corresponds to some agent, while the 'positiveness' and the 'negativeness' of the literal indicates the presence or the absence of that agent in the pattern, respectively. A rule $r : p_1 \wedge p_2 \wedge \cdots \wedge p_x \wedge \neg n_1 \wedge \neg n_2 \wedge \cdots \wedge \neg n_y \rightarrow val_r$ is said to apply on a coalition $S \subseteq N$, denoted by $r \models S$, if and only if each positive literal p_i exists in S, i.e, $p_i \in S$ for $i = 1, \ldots, x$, and no negative literal n_j exists in S, i.e, $n_j \notin S$ for $j = 1, \ldots, y$. Given a coalition S, we can compute S's utility by summing up the values of all the rules that apply to S: $v(S) \equiv \sum_{r \models S} val_r$. Any characteristic function game can be represented by a set of such rules [4].

Michalak et al in [7] proposed an extension of the above representation scheme, the *Embedded MC-nets*; which are used to represent coalitional games with externalities. In the same direction [8] proposed the *Weighted MC-nets* also to represente coalitional games with externalities. In [6] Mamakos et al. proposed the *Relational Rules*, an MC-nets extension for representing overlapping coalitions. In that work, the rules are of the form $A \rightarrow \frac{\sum_{i \in A} \pi_{i,S}}{|A|} \cdot value$, where A is a subset of agents (corresponding to positive literals), S is a coalition such that $A \subseteq S$, and $\pi_{i,S}$ is the portion of the resource that agent i has invested in S. Against this background, in this work we focus on large open environments, build on MC-nets, and propose an even more compact representation scheme for such settings.

2 A COMPACT REPRESENTATION SCHEME

As we mentioned above, in a multiagent system with hundreds of agents, having a priori complete information about every possible collaboration is unrealistic. Lifting the assumption of complete information leads to a partially observed environment, i.e., an environment under uncertainty. The issue of uncertainty has gained a lot of attention in the game theory community, and there is a host of research papers tackling the problem. For instance, [1] propose, a class of cooperative games where agents are uncertain about their partners' *type*, and express beliefs over the type of other agents. The authors in [5] study a series of strategies and protocols for coalition formation under uncertainty; while [3] provides a definition of *Transferable Utility Games with Uncertainty (TUU).*

In this paper, we work with general TU games, and we assume uncertainty over coaltional utilities. Let N be a set of agents, and v be an underlying characteristic function of some TU game. Function

v is hidden to the whole system, however we have in our disposal a function that comprises beliefs over coalitions' utility.

Definition 2.1 (Believed Characteristic Function). Given a TU game $G = \langle N, v \rangle$ where v is an unknown characteristic function, a function $\tilde{v} : A \rightarrow \mathbb{R}$, where $A \subseteq 2^N$, constitutes a *believed characteristic function* estimating the underlying characteristic function v.

That is, $\tilde{v}(S)$ corresponds to an estimate of the utility of coalition S (e.g., inferred by past observations), rather than to the actual $v(S)$.

Let a $\tilde{v}$ of the game G be such that the estimate of a coalition S derives by the summation of the estimates over the collaborative pairs in S. That is, $\tilde{v}(S) = \sum_{i,j \in S} \tilde{v}(\{i, j\})$. Intuitively, pairwise collaborations can be considered as the basis for estimating the utility of collaboration with many agents. This concept suggests that larger collaboration patterns follow an additive behaviour. Following [4], we can represent the collaborative pairs by rules like $i \wedge j \rightarrow \tilde{v}(\{i, j\})$. We refer to such rules as "MC-net-like" rules.

Definition 2.2 (MC-net-like Rules). An MC-net-like rule r is of the form $Pattern \rightarrow val$, where $Pattern$ is restricted to pairs of agents, where an agent is represented by either a positive or a negative literal; and val is an estimate about the utility that portrays the collaboration between the agents indicated by the pattern, where val is provided by a believed characteristic function $\tilde{v}$.

A set of MC-net-like rules that describes the $\tilde{v}$ at hand, needs $\binom{2n}{2} - n$ rules (where n denotes the number of agents). Thus, even by restricting the representation to MC-net-like rules, in a setting with 10^3 agents we will need $\binom{2 \cdot 10^3}{2} - 10^3$ rules to fully describe the game. As such, it is essential to compress it even more, so that we can use it efficiently during decision making processes.

With an initial set of MC-net-like rules, we perform a series of merges on rules, and produce a more succinct set of rules. In order to do so, we allow a small error on the values of the initial rules. That is, two MC-net-like rules can be merged if they refer to a common agent (i.e., some agent i appears in both rules' pattern) and their values are at most a small ε apart, while the merged rule has a value equal to the average values on the initial rules. Given the fact that the values of the rules that can be merged differ at most this small ε, we can bound the value of the newly formed rule (the merged one) to $\max\{initial\ values\} - \varepsilon \leq new\_value \leq \min\{initial\ values\} + \varepsilon$. The newly formed rule now takes the form: $\{\text{agent } i\} \wedge \{\text{agent } j \text{ or agent } k\} \rightarrow Value$. That is, the new rule's pattern describes the collaboration of a single agent i with any agent in the merged set $\{j, k\}$.

Moreover, an initial MC-net-like rule can also be merged with a previously merged rule. That is, a newly formed rule by merging two initial MC-net-like rules, let us denoted as r_{merged}, can in turn be merged with a third MC-net-like rule. In order to allow this kind of merges, we require the value of the MC-net-like rule's value to lie at most ε away from the value of r_{merged}; *and* at the same time the value of the new rule (the one resulting by this merge, denoted as r_{new}) lies between the maximum value that has been already merged (in r_{merged}) minus ε, and the minimum value that has been already merged plus ε:

$$\max\{\text{already merged values in } r_{\text{merged}}\} - \varepsilon \leq new\ value \tag{1a}$$

$$new\ value \leq \min\{\text{already merged values in } r_{\text{merged}}\} + \varepsilon \tag{1b}$$

where the *new value* corresponds to the average value over all merged values to produce r_{new}. Requirements (1a) and (1b) ensure that initial rules that will be merged in a new rule will not alter the average value in an excessive way. Thus, we can reduce the initial set of rules by successive merges. Here we need to note that, in order to avoid any ambiguities we allow each initial rule to merged only once. That is, as soon as an initial MC-net-like rule is merged, we no longer consider this rule. When no more merges can be performed, we have reached a more compact set of rules, where each rule has the form $\{\text{some agent}\} \wedge \{\text{ any agent in the merged set}\} \rightarrow value$.

Example 2.3. Let $\varepsilon = 1$, and the following MC-net-like rules:

$$(r_1) : 1 \wedge 2 \rightarrow 5;\ (r_2) : 3 \wedge 4 \rightarrow 6;\ (r_3) : 1 \wedge 4 \rightarrow 7;$$
$$(r_4) : 3 \wedge 2 \rightarrow 16;\ (r_5) : 4 \wedge 5 \rightarrow 7.$$

Rule r_1 has a common agent with rules r_3 and r_4; however we cannot merge any of these rule with r_1 since their values lie apart by more than ε (2, and 11, respectively). Rule r_2 on the other hand, has a common agent with rule r_3, and the conditions are also satisfied: $|val_2 - val_3| = 1 \leq \varepsilon$. Thus a new compact rule is generated: (r_6) : $4 \wedge \{1, 3\} \rightarrow 6.5$; while rules r_2 and r_3 are no longer being considered. Finally, rule r_5 shares a common agent with the newly formed rule r_6; and the conditions are also satisfied: $|val_5 - val_6| = 0.5 \leq \varepsilon$, $\max\{6, 7\} - \varepsilon = 6 \leq \text{avg}(6, 7, 7)$, and $\text{avg}(6, 7, 7) \leq \min\{6, 7\} + \varepsilon = 7$. As such, a new compact rule is generated: $(r_7) : 4 \wedge \{1, 3, 5\} \rightarrow 6.67$, while rules r_5 and r_6 are no longer being considered. The final set of compact rules is:

$$(r_1) : 1 \wedge 2 \rightarrow 5;\ (r_4) : 3 \wedge 2 \rightarrow 16;\ (r_7) : 4 \wedge \{1, 3, 5\} \rightarrow 6.677.$$

3 ONGOING AND FUTURE WORK

As part of this work, we have also produced an algorithm that compresses an initial set of MC-net-like rules into a set of rules having the (new) form of ({agent} $\wedge$ {set of agents}), and have studied its computational complexity. Moreover, we have provided theoretical bounds on the additional error (loss of information) that is placed upon our initial estimates during the compression process. Moreover, we are performing a systematic evaluation to confirm the effectiveness of the algorithm in large environments with a large number of agents, and a large number of initial MC-net-like rules. Finally, working in partially observed environments, an interesting research direction we already have results in, is that of adopting the notion of equivalent classes of agents; and of exploiting such a notion to *(a)* compress the representation even more, and *(b)* discover new, previously unknown collaborations.

4 CONCLUSIONS

In this work we presented a merging technique that can lead to a compact representation for cooperative games. In particular, we build on the work of [4], and extend it to have rules that include sets of agents, instead of just individuals. First, we discussed the primary assumptions that usually exist in cooperative games, and the non-realistic nature of certain such assumptions in practice. Next we formally defined the key elements of our proposed representations, namely the prior estimates, the MC-net-like rules and the types of merges. Finally we discussed the conditions that needs to be satisfied for a merge to occur, and we showed how to employ such merges on the initial set of rules to reach new compact rules.

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