Max-sum with Quadtrees for Continuous DCOPs with Application to Lane-Free Autonomous Driving

Extended Abstract

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ABSTRACT

In this paper we put forward a novel extension of the classic Max-Sum algorithm to the framework of Continuous Distributed Constrained Optimization Problems (Continuous DCOPs), in which we model the exchanged messages by means of a popular geometric algorithm, Quadtrees. As such, the discretization process is dynamic and embedded in the internal Max-Sum operations (addition and marginal maximization). We apply our *Max-Sum with Quadtrees* approach to *Lane-Free Autonomous Driving* in a highway populated with vehicles. Our experimental evaluation verifies the efficiency of our approach in this challenging dynamic coordination domain, demonstrating its superior performance with respect to the standard Max-Sum algorithm.

KEYWORDS

Distributed Problem Solving; Max-Sum Algorithm; Quadtrees; Factor Graphs; Autonomous Driving; Lane-Free Traffic

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1 INTRODUCTION

The framework of Distributed Constraint Optimization Problems (DCOPs) [11] is associated with a broad range of methodologies for distributed problem solving and coordination, such as Factor Graphs [8] and the Max-Sum message passing algorithm [3]. Continuous DCOPs [6, 14, 15], in which the control variables' domain is continuous, are of particular interest corresponding as they do to a multitude of real-world settings, and posing several issues and challenges originating from the continuity of the problem domains. A host of methodologies for tackling Continuous DCOPs has been proposed in the literature, such as Local Search [14], Distributed Pseudo-tree Optimization [6]; Particle Swarm Optimization algorithms [2]; Bayesian methods [5]; and also the Max-Sum algorithm.

Existing work on Max-Sum in continuous domains makes certain assumptions regarding the factors and messages. For instance, [18] assumes access to the factors' gradients; while [15] models factors and messages as Continuous Piecewise Linear Functions (CPLFs), which are quite restrictive, both in terms of construction (with simplices) and management. By contrast, in our approach, we model the exchanged messages with Quadtrees [4], which are among the simplest geometric algorithms for data representation of a given set of points, but exhibit in general high efficiency in many applications.

Specifically, we *embed* Quadtrees in the Max-Sum algorithm, to effectively achieve a fine-grained discretization of the continuous domain—the degree of which is determined by the Quadtree function approximation process itself, in an "online", dynamic fashion. The Quadtrees' embedding in Max-Sum, is achieved via properly redefining the necessary core Max-Sum operations (*addition* and *marginal maximization*), that are applied on the messages exchanged.

Now, the need for coordination is intense in the field of autonomous driving [13], particularly when *Lane-Free vehicle movement* [12] is assumed. In Lane-Free traffic, vehicles are no longer restricted by specific lane positioning; thus they do not perform any lane-changing operation, but rather adjust their lateral placement smoothly. This inherently continuous domain enables much higher utilization of the available road capacity, and gives rise to the design of novel techniques and methodologies, as evident by the results of [7, 10, 12, 17, 19]. A Factor Graph representation with vehicles controlling variables relating to longitudinal and lateral acceleration is natural in this domain, and allows the use of DCOPs optimization algorithms.

2 MAX-SUM WITH QUADTREES FOR CONTINUOUS DOMAINS

Messages in Max-Sum [3], i.e., the functions $q_{i\to j}(x_i), r_{j\to i}(x_i)$, are 1-D, since they are associated with a single scalar variable x_i (which is discrete) [3]. However, in most multiagent settings of interest, agents have more than one variables under their control—and in many cases the variables are continuous. For instance, consider the application domain of interest in our work here—that is, Lane-Free autonomous driving [12]—where vehicles need to coordinate their movement. The use of Quadtrees for calculating these messages adds much desired flexibility regarding the dimensionality

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of variables and messages. To this end, given a Factor Graph representation, we now model each agent i controlling a set of variables $\mathbf{x}_i \subseteq \mathbf{x}$, and thus all functions corresponding to the messages are now sent between factors j and agents i with dimension $X_i = |\mathbf{x}_i|$.

Utilizing Max-Sum in continuous domains with Quadtrees means essentially that we use them to represent a continuous function of multiple variables; and then define appropriately how the addition process is carried out, along with the marginal maximization process—as was done in [15], but in our case such operations may involve messages of higher dimensionality, represented by Quadtrees. We focus on the equations of Max-Sum, now using a vector input $\mathbf{x}_i$ for all messages. Consider a factor $F_j(\mathbf{s}_j)$ that depends on $|M_j|$ variables. For messages $r^{(Qt)}_{j\to i}(\mathbf{x}_i)$, sent *from factor j to agent i*:

$$r^{(Qt)}_{j\to i}(\mathbf{x}_i) \quad = \quad \max_{\mathbf{s}_j \setminus \mathbf{x}_i} \left[F_j(\mathbf{s}_j) + \sum_{k \in N_j \setminus i} q^{(Qt)}_{k\to j}(\mathbf{x}_k) \right] \qquad (1)$$

where (Qt) superscript indicates that these functions now have the form of a Quadtree representation of a real multivariate function $f : \mathbb{R}^N \to \mathbb{R}$. Now, the process initiates from factor $F_j(\mathbf{s}_j)$, which is a real multivariate function $F_j : \mathbb{R}^n \to \mathbb{R}$, with n being the size of vector $\mathbf{s}_j$. The addition and marginal maximization operations, as imposed by Eq. 1, are properly designed for Quadtree messages. Likewise, for messages $q^{(Qt)}_{i\to j}(\mathbf{x}_i)$, sent *from agent i to factor j*:

$$q^{(Qt)}_{i\to j}(\mathbf{x}_i) \quad = \quad a_{ij} + \sum_{k \in M_i \setminus j} r^{(Qt)}_{k\to i}(\mathbf{x}_i) \qquad (2)$$

Here, we perform a summation involving many Quadtree representations, $r^{(Qt)}_{k\to i}, \forall k \in M_i \setminus j$, but they all depend on the same variables $\mathbf{x}_i$. The summation here results also in a Quadtree representation, and we obtain the final message by adding the normalization constant a_{ij}, so that $\sum_{\forall \mathbf{x}_i} q^{(Qt)}_{i\to j}(\mathbf{x}_i) = 0$, the calculation of which is properly embedded within the Quadtree representation of $q^{(Qt)}_{i\to j}(\mathbf{x}_i)$ messages. Finally, by retaining the maximum point observed on the Quadtree messages, it is quite straightforward to select the maximizing variable configuration $\mathbf{x}^*_i = \arg\max \sum_{k\in M_i} r^{(Qt)}_{j\to i}(\mathbf{x}_i)$.

3 EXPERIMENTAL EVALUATION

We consider a highway populated with many vehicles entering from an origin point, where each vehicle possesses a desired speed parameter v_d, chosen at random within a specified speed region $[v_{d,low}, v_{d,high}]$. The induced speed deviations among nearby vehicles, along with heavier traffic in a Lane-Free environment, constitute a complex multi-agent environment appropriate for the application of the proposed method. Experimental evaluation is conducted using the tool of [16], an extension of the SUMO platform [9].

3.1 Experimental Setup and Results

While the discretization utilized in [17] is sufficient with respect to the traffic environment settings and the safety parameters utilized in that work, we wish to highlight the superior performance of the proposed approach in intensified traffic conditions. To do so, we make the traffic environment more challenging, by adjusting the safety parameters of the factors so that vehicles employ a much more aggressive driving style (i.e., the parameters now effectively induce smaller reaction time windows and smaller safety distances).

For experimental evaluation, we used four different inflow rates $\{5400, 7200, 9000, 12000\}$ veh/hr at the origin point of the highway. We compare three different approaches, specifically the "discrete Max-Sum" method for Lane-Free driving proposed in [17] (which scales to up to 9000 veh/hr); the same "discrete Max-Sum" model, but with the adjusted "aggressive" parameterization discussed above; and the proposed Quadtrees-employing approach, again with the more aggressive safety scheme.

Our experiments demonstrate that Quadtrees allow us to strike a balance between efficiency and safety: the vehicles are typically able to reach their desired speeds, while exhibiting *no collisions*, even for the 12000} veh/hr inflow rate—an unrealistically high rate for *lane-based* traffic, but now easily tackled by our approach. Of course, the induced aggressive behaviour of vehicles, also substantially improves the performance of the "discrete" approach with respect to the average speed and delay times measured, compared to the more cautious choices of [17]. Specifically, for the range of desired speeds $[25, 35]$ m/s, the use of such parameters results in an improvement on average speed within the range of $[1.0, 1.5]$ m/s for the inflow rates examined, which is not negligible at all, considering the range of speeds among vehicles. The increased efficiency is better depicted in the measurements of *delay*.[1] Specifically, we observe $\sim$ 2.7 *sec* delays for the "aggressive" discrete variant across all inflow rates, and delays ranging from 3.9 to 6.5 *sec* (depending on the inflow rate examined) for the Quadtree-employing approach. This is a more than substantial improvement when compared to the 10 to 14 *sec* delays for the "safe" discrete Max-Sum version of [17].

Yet, for the "discrete Max-Sum" cases, this improvement comes at the cost of a significant increase in collisions. Specifically, collisions for "discrete Max-Sum" increase dramatically with increasing inflow rates, ranging from 56 to 3104 collisions for the inflow rates examined. By contrast, as mentioned, vehicles employing Max-Sum with Quadtrees did not exhibit *any collisions*, while exhibiting the much improved delay times, showcasing the need for a dynamic discretization in demanding traffic conditions.

4 CONCLUSIONS AND FUTURE WORK

In this work, we presented an extension of the well-known Max-Sum algorithm for Continuous DCOPs, with application in the novel Lane-Free autonomous driving domain. Our results confirm that embedding Quadtrees in the Max-Sum algorithm, render the latter appropriate for demanding Continuous DCOP domains.

In future work, alternative geometric algorithms could be considered for dynamic discretization—for instance, ones that share similar characteristics with Quadtrees, such as different variants of k-d trees [1]. Another direction is to focus more on the convergence properties of the Max-Sum with Quadtrees approach.

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[1] Delay is the difference between the actual time the vehicle spent inside the motorway, and the ideal time it would have spent with a constant speed equal to its desired speed.

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