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Ali Dehghanbanadaki

A.Dehghanbanadaki@damavandiau.ac.ir

Islamic Azad University Damavand Branch Kamarudin Ahmad Universiti Teknologi Malaysia Ahmad Safuan A. Rashid UTM Skudai: Universiti Teknologi Malaysia Nor Zurairahetty Mohd Yunus UTM Skudai: Universiti Teknologi Malaysia

# **Research Article**

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# Novel approach to predicting the ultimate bearing capacity of footings located on soft soil improved by DCM columns using Gaussian process regression models – A practical example

# Ali Dehghanbanadaki<sup>1</sup>,<sup>2</sup>\*, Kamarudin Ahmad<sup>3</sup>, Ahmad Safuan A. Rashid<sup>4</sup>, Nor Zurairahetty Mohd Yunus<sup>5</sup>

<sup>1</sup>Assistant professor, Department of Civil Engineering, Islamic Azad University, Damavand Branch, Iran

<sup>2</sup>Research Center of Concrete and Asphalt, Damavand Branch, Islamic Azad University, Damavand, Iran

Email: A.Dehghanbanadaki@damavandiau.ac.ir

\*Corresponding author

<sup>2</sup>Associated professor, Department of Geotechnics & Transportation, School of Civil Engineering, Faculty of Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia Email: <u>kamaraudin@utm.my</u>

<sup>3</sup>Associated professor, Department of Geotechnics & Transportation, School of Civil Engineering, Faculty of Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia Email: <u>ahmadsafuan@utm.my</u>

<sup>4</sup>Associated professor, Department of Geotechnics & Transportation, School of Civil Engineering, Faculty of Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia Email: <u>nzurairahetty@utm.my</u>

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- Code availability: The code used in the paper will be available for the request.

### Abstract

The use of deep cement mixing (DCM) columns is an effective and affordable technique for ground stabilization. However, designing this method can be complex due to uncertainties in the geotechnical properties of the soil and DCM columns, area improvement ratio, column arrangement, and required cement content. This study aimed to address this issue by using Gaussian process regression (GPR) models to estimate the ultimate bearing capacity (UBC) of soft soil improved with DCM columns.To create and train the GPR models, the study utilized a database of 46 physical modeling tests under end-bearing and floating conditions. The researchers used different kernel functions, including rational quadratic, squared exponential, Matern 5/2, and exponential, for the GPR models. The models were then optimized through Bayesian optimization and compared to other predictive techniques such as multilayer perceptron (MLP), radial basis function (RBF), and neuro-fuzzy inference systems (ANFIS) using test data.As a case study, the researchers evaluated a decision-making model for designing the geotechnical properties of DCM columns. The results showed that the optimized GPR model's accuracy in terms of performance indices was satisfactory for both end-bearing and floating DCM column conditions. The optimized GPR model outperformed MLP, RBF, and ANFIS performance indices using test data. Overall, the study demonstrated that optimized GPR models are a promising method for early prediction of stabilized ground UBC.

Keywords Gaussian process regression models, DCM columns, ANFIS, MLP, RBF

#### **1** Introduction

Deep soil mixing (DSM) is a commonly used technique for soil stabilization that enhances the overall loadbearing capacity of weak soil (Broms and Boman, 1979; Bergado, 1994; Bouassida and Porbaha, 2004; Dehghanbanadaki et al. 2016; Liu et al. 2017; Rashid et al. 2017; Bunawan et al. 2018; Yi et al. 2018; Dehghanbanadaki et al. 2022; Yin et al., 2023). This technique also can decrease the total settlement of a structure (Porbaha, 1998; Bouassida et al. 2009; Chai and Pongsivasathit, 2010; Yao et al. 2016; Alipour et al. 2016; Dehghanbanadaki et al. 2020). The major advantages of this method include its environmental benefits, low noise, application in a limited workspace, and low vibration (Frikha et al., 2017). Binders such as cement, gypsum, lime, fly ash, and combinations of these are used to construct DSM columns (EuroSoilStab, 2002); however, practical projects using DSM columns have shown that cement performs better than other binders (CDIT, 2011). This has led to the development of deep cement mixing (DCM) in place of DSM. In DCM, the installation method, revolution speed, curing time, mixing time, and the types and amount of binder have significant effects on the improvement of a site (Arulrajah et al. 2018). These parameters can directly affect the degree of improvement and are dependent upon on-site specifications. Fig. 1 shows an example of DCM columns in soft clay (Topolnicki, 2014).

Numerous studies have explored the vertical ultimate bearing capacity (UBC) of soft soil that has been improved through the use of end-bearing or floating DCM columns. Researchers including Terashi and Tanaka (1981), Kitazume et al. (1999), Omine et al. (1999), Yin and Fang (2010), Rashid et al. (2011, 2017), Wonglert et al. (2018), Dehghanbanadaki et al. (2020), Dehghanbanadaki (2021), and Zhao et al. (2023) have all contributed to this body of knowledge. The choice between end-bearing or floating DCM columns is dependent on the specific site conditions. Factors such as the area improvement ratio (Eq. (1)), undrained shear strength of the soil ( $C_{us}$ ), undrained shear strength of the DCM columns ( $C_{uc}$ ), skin interaction between the soil and DCM columns, and DCM column arrangement and loading rate (EuroSoilStab, 2002) all impact the UBC of the stabilized ground. Additionally, a model of a foundation over soft soil that has been improved with end-bearing DCMs can be seen in Fig. 1.

$$\alpha = \frac{A_c}{A_t} \tag{1}$$

where  $A_c$  is the area of the DCM columns and  $A_t$  is the total loaded area.

Soft computing methods, such as artificial neural networks (ANNs), fuzzy modeling, and adaptive neurofuzzy inference systems (ANFIS), have become increasingly popular for addressing civil engineering problems. Several studies, including those conducted by Khari et al. (2018), Jokar et al. (2018), Dehghanbanadaki et al. (2019), Jelušič and Žlender (2020), Amiri et al. (2020), Chaudhuri and Maity (2020), Kashani et al. (2020), and Liu et al. (2020), have utilized these methods. For instance, Tinoco et al. (2019) estimated the ultimate bearing capacity (UBC) of soil samples that had been stabilized with cement, using ANNs and support vector machine (SVM) models. They created the models based on the results of 444 unconfined compressive strength (UCS) tests, considering ten input parameters, including clay, sand, silt, organic matter, water, cement contents, water/cement ratio, curing time, and two coefficients related to binder type. Their results indicated that the best ANN model accurately estimated the UCS, with an average regression index of R = 0.95.

In sensitivity tests, it was discovered that the water/cement ratio, cement content, and organic matter content had the most significant impact on estimating the unconfined compressive strength (UCS) of stabilized samples. For DSM projects, Ghorbani and Hasanzadehshooiili (2018) estimated the UCS and California bearing ratio (CBR) of silty sulfate sand treated with lime and micro-silica. They conducted 90 experimental tests and developed computational artificial neural network (ANN) and evolutionary polynomial regression (EPR) models. These models were trained and tested with the aim of using them in DSM methods (QA/QC). As determined by the controlling error criteria, both targets were estimated to have high accuracy. Validation of the proposed models was done using practical examples.

The ultimate shear resistance of silty sand stabilized by a group of geotextile-encased stone columns as a column-like element was estimated by Ardakani et al. (2019) using an ANN model. The training performance of the ANN model was improved by an imperialist competitive algorithm (ICA). A total of 39 large-scale tests were used for training. Input parameters were area improvement, normal stress, and the fines content of the soil. The results showed that the ANN trained by ICA performed better than the conventional ANN that was trained using a back-propagation algorithm. A correlation coefficient of 0.9913 for the test data confirmed the high accuracy of the model for the estimation of the target.

Das and Dey (2018) developed an artificial neural network (ANN) model to predict the ultimate bearing capacity (UBC) of a group of stone columns in soft clay. They collected data from 90 experimental tests to determine the input parameters, including undrained cohesion of the soft clay, friction angle of the stone column, ratio of spacing to the diameter of the stone columns, length of the stone columns, and number of stone columns. The estimated UBC values were compared to the results of numerical modeling using FEM Plaxis 2D, and the ANN model showed satisfactory results when compared with established theories. Sensitivity analysis indicated that the friction angle of the stone column had the greatest impact on the UBC.

Ornek et al. (2012) investigated the estimation of UBC of circular footings placed on soft clay improved with granular soil. They used data from 28 field tests as large-scale experiments to train ANNs and multi-linear regression (MLR) models. The granular fill used as a column-like element was obtained by passing natural granular material through a 4.75-mm sieve. The granular columns were arranged in a rectangle. The UBC of the treated ground was calculated as a load-settlement relationship. To develop the computational models, the diameter and vertical displacement of the foundation and the height of the granular columns were considered as the input parameters and the UBC was the target. A comparison of the performance indices indicated that the ANN model better estimated the UBC than MLR.

Studies have shown that both MLP and ANFIS approaches have limitations such as overfitting, slow convergence speed, and poor generalization performance (Park and Rilett, 1999; Kecman 2001; Huang, 2009; Lee et al. 2019; Jin, 2020; Lima et al. 2020). In addition, determining the topology of the model, including the number of hidden layers and neurons, is still a challenge, and the usual method is through trial-and-error (Pal et al. 2010; Liu et al. 2016). Therefore, there is a need for other computational estimation methods. Gaussian process regression (GPR) models have been increasingly used in solving engineering problems (Samui, 2019; Suthar, 2019; Akbari, 2019; Liu et al. 2020, Momeni et al. 2020). For instance, Suthar (2019) estimated the UCS of stabilized pond ash using soft computing methods such as M5 model tree, random forest, ANN, SVM, and GPR. Akbari (2019) used soft computing methods, including MLP, SVM, GRNN, and GPR models, to estimate the discharge coefficient of a gated piano-key weir. The GPR models showed superior performance compared to other methods (RMSE = 0.011,  $R^2 = 0.992$  and MAPE = 1.167%).

Pal and Deswal (2010) compared the performance indices of SVMs and GPRs for the estimation of driven pile UBC in cohesionless soil. The radial basis function and Pearson VII function kernels were used for both the

GPR and SVM models. It was found that the GPRs performed better than SVMs for the estimation of the UBC of piles. Momeni et al. (2020) used 296 dynamic pile load tests to predict the UBC of piles. They utilized GPR and MLP models, which were trained using a genetic algorithm. The results showed that the GPR model outperformed the GA-based MLP model with performance indices of VAF = 86.41% and R2 = 0.84. Previous studies have also reported the superiority of GP over MLPs for predicting the friction capacity of piles in clay soil and estimating stress parameters of unsaturated soils (Samui, 2019)

#### 1.1 Related Works

An ANFIS-PSO hybrid model was developed by Dehghanbanadaki (2021) to estimate the UBC of soft soil stabilized with floating stone columns (SC) and floating DCM columns. The study used a mixed database of 86 physical modeling tests, and input parameters such as undrained shear strength, area improvement ratio, and length-to-diameter ratio of the columns were considered. The proposed model's accuracy was validated using testing data, and a decision-making model for designing geotechnical properties was proposed. The results indicated satisfactory performance indices with an R<sup>2</sup> value greater than 0.9. In the next study, Dehghanbanadaki et al. (2022) conducted a review of 1-g physical modeling tests on soft soil stabilized with DCM columns in clay and peat. They proposed four computational estimation functions using CFTOOL in Matlab software to estimate the UBC, but these functions were limited in terms of the number of inputs. As a result, there is a need for a comprehensive machine learning model that can consider all influencing parameters using limited datasets to accurately estimate the UBC.

1.2 Novelty of this study:

The present study offers several novel contributions to the field of geotechnical engineering, including:

- The utilization of GPR models to predict the UBC of soft soil improved with DCM columns is a topic that has not been extensively researched.
- The optimization of the GPR models using Bayesian optimization to accurately predict UBC performance indices in both end-bearing and floating DCM column conditions provides a more robust and reliable method for early prediction of stabilized ground UBC.
- The study introduces a decision-making model for designing the geotechnical properties of DCM columns as a case study to demonstrate the accuracy of the optimized GPR model in predicting the UBC of stabilized ground.

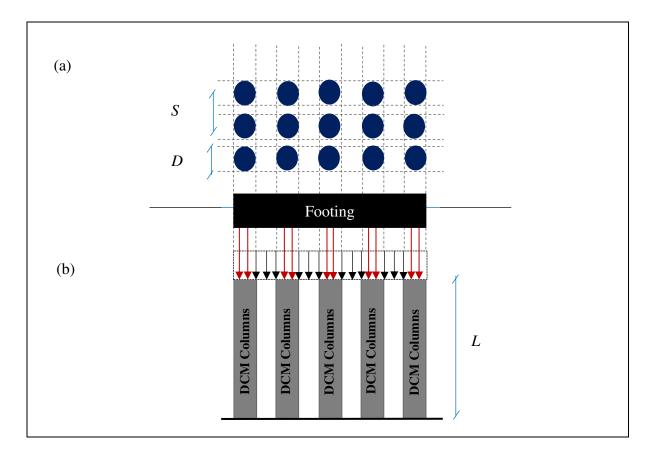


Fig. 1 Model of a foundation over soft soil improved with end-bearing DCMs: (a) plan; (b) transverse cut (D = Diameter of columns; L = length; S = distance between columns).

#### 2 Databases

The authors collected and developed datasets for soft soil improved with both end-bearing and floating DCM columns, as summarized in Tables 1 and 2. The datasets consist of 28 1g physical modelling tests conducted for end-bearing conditions and 17 1g tests conducted for floating conditions. The tables include information on the size of the tank, undrained shear strength of the soil and DCM column, and the area improvement ratio for each test. For the floating columns, the ratio of DCM column length to soil height is also provided. The tests aimed to determine the UBC in undrained conditions after subjecting the stabilized soil to failure under stress or strain conditions. The authors tested soft clay and soft peat with a  $C_{us}$  of less than 10 kPa in most cases.

To examine the effect of DCM columns on the behavior of stabilized soil, two separate computational models were developed, one for end-bearing conditions (Model A) and one for floating conditions (Model B). Both models were designed to predict the UBC of the stabilized soil, with input parameters including  $C_{us}$  and  $C_{uc}$  for both models, and  $L_r$  also considered in Model B. To assess the performance of the models, approximately 10% of the data was reserved for testing. The UBC of the soil stabilized with DCM columns can be represented as:

 $UBC_{(Model-A)} = f(C_{us}, C_{uc}, \alpha)$ (3 Inputs and 1 target) (4)  $UBC_{(Model-B)} = f(C_{us}, C_{uc}, \alpha, L_r)$ (4 Inputs and 1 target) (5)

where  $C_{us}$  is the undrained shear strength of the basic soil,  $C_{uc}$  is the undrained shear strength of the DCM columns,  $\alpha$  is the area improvement ratio,  $L_r$  is the DCM column length-to-soil height, UBC (*model A*) is the UBC end-bearing condition, and UBC (*Model-B*) is the UBC in the floating condition.

Test	Box dimensions		$C_{us}$	$C_{uc}$	α(%)	UBC	BCF	Reference
number	(mm) & soil type	$L_r$	(kPa)	(kPa)	u (%)	(kPa)	DCF	Reference
TS-1	300×200×200 - Peat	1	8.8	79.6	13.1	77.7	8.82	(Dehghanbanadaki et al. 2016)
TS-2	300×200×200 - Peat	1	9.4	89.3	19.6	87.6	9.31	(Dehghanbanadaki et al. 2016
TS-3	300×200×200 - Peat	1	9.8	83.4	26.2	91.1	9.3	(Dehghanbanadaki et al. 2016)
TS-4	400×150×430 - Clay	1	6.9	86.75	17.3	80.33	11.64	(Rashid, 2011)
TS-5	400×150×430 - Clay	1	6.4	83.71	26.2	90.11	14.08	(Rashid, 2011)
TS-6	400×150×430 - Clay	1	6.4	87.31	26.2	96.17	15.03	(Rashid, 2011)
TS-7	400×150×430 - Clay	1	6.3	90.28	34.7	107.24	17.02	(Rashid, 2011)
TS-8	400×150×430 - Clay	1	6.9	137.5	34.7	128.8	18.66	(Rashid, 2011)
TS-9	400×150×430 - Clay	1	6.9	125	34.7	120.1	17.4	(Rashid, 2011)
TS-10	400×150×430 - Clay	1	6.9	112.5	34.7	111.4	16.14	(Rashid, 2011)
TS-11	400×150×430 - Clay	1	6.9	100	34.7	102.6	14.8	(Rashid, 2011)
TS-12	400×150×430 - Clay	1	6.9	137.5	26	106.1	15.37	(Rashid, 2011)
TS-13	400×150×430 - Clay	1	6.9	125	26	99.47	14.41	(Rashid, 2011)
TS-14	400×150×430 - Clay	1	6.9	112.5	26	92.91	13.46	(Rashid, 2011)
TS-15	400×150×430 - Clay	1	6.9	100	26	86.34	12.51	(Rashid, 2011)
TS-16	400×150×430 - Clay	1	6.9	137.5	17.3	83.17	12.05	(Rashid, 2011)
TS-17	400×150×430 - Clay	1	6.9	125	17.3	78.79	11.41	(Rashid, 2011)
TS-18	400×150×430 - Clay	1	6.9	112.5	17.3	74.38	10.77	(Rashid, 2011)
TS-19	400×150×430 - Clay	1	6.9	100	17.3	69.95	10.13	(Rashid, 2011)
TS-20	500×200×345- Clay	1	14.1	322	18.8	182	12.91	(Bouassida and Porbaha, 2004)
TS-21	500×200×345- Clay	1	15.7	292	18.8	186.7	11.89	(Bouassida and Porbaha, 2004)
TS-22	500×200×345- Clay	1	9.4	259	18.8	132.7	14.12	(Bouassida and Porbaha, 2004)
TS-23	500×200×345- Clay	1	11	266	18.8	152	13.82	(Bouassida and Porbaha, 2004)
TS-24	500×200×345- Clay	1	12.6	357	18.8	181.3	14.39	(Bouassida and Porbaha, 2004)
TS-25	500×200×345- Clay	1	9.5	347	18.8	162.2	17.07	(Bouassida and Porbaha, 2004)
TS-26	900×300×900- Clay	1	3	425	12.6	81	27	(Yin and Fang, 2010)
TS-27	900×170×200- Clay	1	2.66	29.96	22	25	9.4	(Omine et al., 1999)
TS-28	900×170×200- Clay	1	2.66	29.96	42	39.2	14.74	(Omine et al., 1999)
TS-29	900×170×200- Clay	1	2.66	113.29	22	57.9	21.77	(Omine et al., 1999)

Table 1 Data for soft soil improved with end-bearing DCM columns (model A)

Note: TS = test; BCF = UBC of treated soil to undrained shear strength of soft soil

Table 2 Data for soft soil improved with floating	g DCM columns (model B)	
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Test number	Box dimensions (mm) & soil type	$L_r$	Cus (kPa)	Cuc (kPa)	α(%)	UBC (kPa)	BCF	Reference
TS-30	300×200×200 - Peat	0.25	9.5	85.8	13.1	53.5	5.63	(Dehghanbanadaki et al. 2016)
TS-31	300×200×200 - Peat	0.25	9.8	82.3	19.6	54.6	5.57	(Dehghanbanadaki et al. 2016)
TS-32	300×200×200 - Peat	0.25	9.1	80.7	26.2	59.3	6.52	(Dehghanbanadaki et al. 2016)
TS-33	300×200×200 - Peat	0.5	10.1	88.4	13.1	61.7	6.11	(Dehghanbanadaki et al. 2016)
TS-34	300×200×200 - Peat	0.5	10.3	88.3	19.6	63.5	6.17	(Dehghanbanadaki et al. 2016)
TS-35	300×200×200 - Peat	0.5	9.7	82.8	26.2	63.7	6.57	(Dehghanbanadaki et al. 2016
TS-36	300×200×200 - Peat	0.25	9.5	78.6	13.1	62.7	6.6	(Dehghanbanadaki et al. 2016
TS-37	300×200×200 - Peat	0.75	9.7	79.4	19.6	71.3	7.35	(Dehghanbanadaki et al. 2016
TS-38	300×200×200 - Peat	0.75	9.4	78.3	26.2	74.6	7.94	(Dehghanbanadaki et al. 2016
TS-39	400×150×430 - Clay	0.5	6.1	118.7	34.7	61.12	10.02	(Rashid, 2011)
TS-40	400×150×430 - Clay	0.5	6.2	68.98	34.7	61.85	9.98	(Rashid, 2011)
TS-41	400×150×430 - Clay	0.5	6.4	121.8	26.2	63.56	9.93	(Rashid, 2011)
TS-42	400×150×430 - Clay	0.5	6.4	87.21	26.2	67.14	10.49	(Rashid, 2011)
TS-43	400×150×430 - Clay	0.5	6.2	89.62	26.2	59.98	9.67	(Rashid, 2011)
TS-44	400×150×430 - Clay	0.5	6.4	61.88	26.2	62.09	9.7	(Rashid, 2011)
TS-45	400×150×430 - Clay	0.5	6.4	87.69	26.2	62.09	9.7	(Rashid, 2011)
TS-46	400×150×430 - Clay	0.5	6.3	87.66	17.3	62.01	9.84	(Rashid, 2011)

Note: TS = test; BCF = UBC of treated soil to undrained shear strength of soft soil

#### **3** Development of computational models

William and Rasmussen (2006) defined a Gaussian process model as a set of random variables, where any finite number of variables has a multivariate Gaussian distribution. The process f(x) is determined by the mean function and covariance (kernel) function k(x, x') evaluated for instances x and x'. While the mean and kernel functions are represented by vectors and matrices, respectively, Gaussian processes operate on functions. The details of the exact equations, training process and determination of the hyper-parameters of the GPR models have been explained by William and Rasmussen (2006).

It should be mentioned that one of the key advantages of GPR is its ability to handle small datasets more effectively than many other machine learning techniques. This is particularly useful in applications where data is expensive to collect or where the data is inherently limited in size. One reason GPR is well-suited to small datasets is that it allows for flexible modeling of the data without relying on a large number of parameters. In contrast, many other machine learning techniques, such as deep neural networks, may require a large number of parameters to effectively model complex relationships, which can lead to overfitting and poor generalization performance when applied to small datasets. Another reason GPR is well-suited to small datasets is that it can effectively incorporate prior knowledge or expert insights into the model. This is particularly useful in domains where expert knowledge is available, but the data is limited, as it allows the model to effectively leverage the available information to make accurate predictions. Therefore, due to limited experimental tests in the present research, it was decided to use this computational model to estimate the UBC.

In the current study, the developments of the GPR models were performed in MATLAB using the *fitrgp* function as in Eq. (6). The output of the model using new data was calculated using the *predict* function as in Eq. (7):

$$gprMdl = fitrgp(X,y)$$
 (6)

ypred = predict(gprMdl,Xnew)

where gprMdl is the GPR mode; X is the predictor, y is the response vector, Xnew is the new input data, and ypred is the output of the GPR model.

(7)

For the mean function, a simple constant (*c*) was chosen. Four kernel functions were used to find the best GPR model, which are expressed in Eqs. (8) to (11) (Momeni et al. 2020). The *k*-fold cross-validation approach for which k = 5 was considered in development of all GPR models.

1. Rational quadratic kernel function:

$$k(x, x') = \sigma_f^2 \exp[1 + \frac{d^2}{2\alpha l^2}]^{-\alpha}$$
(8)

2. Squared exponential kernel function:

$$k(x, x') = \sigma_f^2 \exp[\frac{-d^2}{2l^2}]$$
(9)

3. Matérn 5/2 kernel function:

$$k(x, x') = \sigma_f^2 \left( 1 + \frac{\sqrt{5}d}{l} + \frac{5d^2}{3l^2} \right) \exp\left[ -\frac{\sqrt{5}d}{l} \right]$$
(10)

4. Rational exponential kernel function:

$$k(x, x') = \sigma_f^2 \exp\left[\frac{-d}{l}\right] \tag{11}$$

where  $\sigma_f^2$  is the variance function of *f*, *d* is the Euclidean distance between instances *x* and *x'*,  $\alpha$  is the parameter of the rational quadratic covariance, and *l* is the length scale.

The GPR model was further optimized using Bayesian optimization in the regression learner app of MATLAB. This app utilizes an optimization scheme to minimize the model's mean squared error by trying out different hyperparameter combinations for a given model type. To optimize the GPR models, all possible kernel functions were explored, including nonisotropic and isotropic rational quadratic, squared exponential, Matern 5/2 and 3/2, and exponential kernels. The optimization process also searched for real values within the range of [0.001, 1] multiplied by the maximum value of the x-range (Matlab, 2018). Where:

$$XMaxRange = max (max(X) - min(X))$$
(12)

and X is the predictor data.

The GPR models' outcomes underwent comparison with ANFIS, MLP, and RBF models through novel experimental tests. ANFIS models were generated using grid partitioning (ANFIS-GP) and trained through a hybrid method. Gaussian membership functions were exclusively chosen for the ANFIS input parameters. MLPs were trained through the Levenberg-Marquardt back-propagation algorithm, and sensitivity analyses were conducted to determine the optimal number of hidden layers and neurons. RBFs were constructed in MATLAB using the newrb functions.

#### **4 Results**

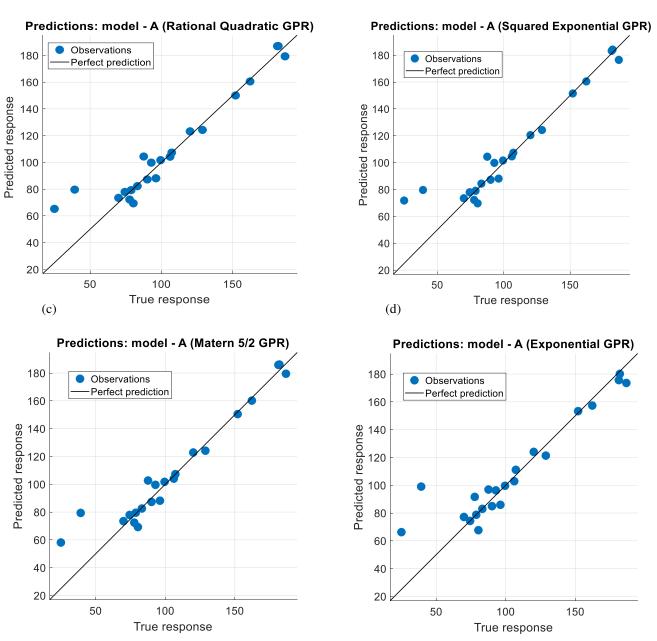
#### 4.1 Performance of GPR models with training data

Figs. 2(a) to 2(d) show the predicted UBCs versus the measured UBCs for all GPR models in the end-bearing DCM column condition (model A). It can be seen that there is a satisfactory agreement between the measured and predicted data for all GPR models. Figs. 3(a) to 3(d) show soil stabilized by floating DCM columns and the predicted UBCs versus the measured UBCs. In model B, the GPR models did not estimate the UBC as accurately as did model A in the end-bearing DCM column condition. The main reasons for this difference are the small data set used for physical modeling tests and the large variety of geotechnical properties of the basic soil and DCM columns.

Figs. 4(a) and 4 (b) show the optimization process of the GPR models. These results can be compared with the results of the actual UBCs shown in Figs. 5(a) and 5(b). Bayesian optimization improved the accuracy rate of the GPR in both models A and B. In the case of model A, the GPR model with basic constant kernel function of nonisotropic Matern 3/2 and a sigma value of 0.000010035 performance best. In model B, the optimization process revealed that the GPR model with basic linear kernel nonisotropic rational quadratic function and a sigma value of 0.00012356 showed the lowest MSE value.

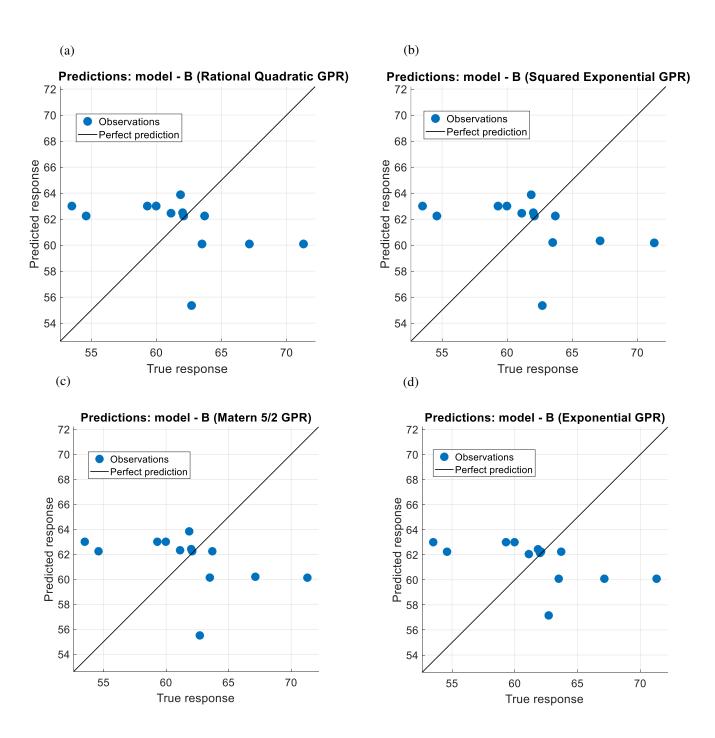
The training results of the GPR models for estimating UBC are listed in Table 3 (model A) and Table 4 (model B). As shown, the results in terms of R, RMSE, and MAE are similar. For both the end-bearing and floating DCM column conditions, the optimized GPR mode provided the best prediction performance compared to the other kernel types when estimating the UBC. This optimized GPR model in end-bearing DCM columns condition had an R value of 0.98, MSE of 7.13 kPa, and MAE of 5.15 kPa and in the floating DCM column condition had an R value of 0.4, MSE of 4 kPa, and MAE of 3.44 kPa. The Matern 5/2 kernel types performed second best.

(a)

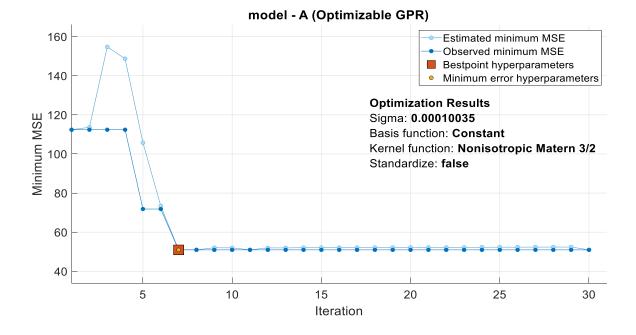


(b)

**Fig. 2** Measured and predicted UBC in end-bearing DCM column condition using kernel functions: (a) rational quadratic; (b) squared exponential; (c) Matern 5/2; (d) exponential



**Fig. 3** Measured and predicted UBC in floating DCM column condition using kernel functions: (a) rational quadratic; (b) squared exponential; (c) Matern 5/2; (d) Exponential



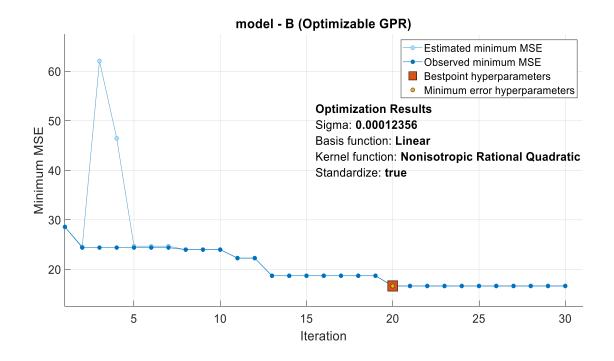


Fig.4 Optimization of GPR models: (a) model A; (b) model B

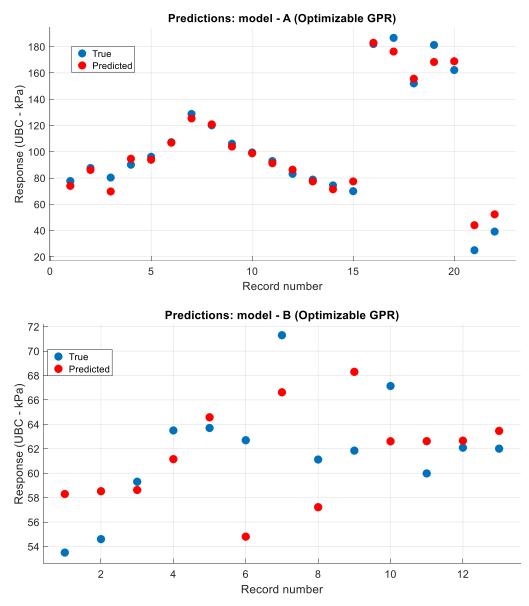


Fig. 5 Measured and predicted UBC for models A and B using optimized GPR models

Model	R	RMSE	MAE	Prediction speed	Training time (sec)	
Widder	К	(kPa)	(kPa)	(obs/sec)		
GPR-rational quadratic	0.92	13.46	7.88	180	12.6	
GPR-squared exponential	0.9	14.36	7.79	740	1.44	
GPR-Matern 5/2	0.93	12.36	7.35	560	1.65	
GPR-exponential	0.87	16.84	9.48	470	1.36	
GPR-optimized	0.98	7.13	5.15	600	212	

Table 3 Results of Gaussian regression models (model A) for training data

Model	R	RMSE (kPa)	MAE (kPa)	Prediction speed obs/sec	Training time (sec)
GPR-rational quadratic	-0.16	5.69	4.48	77	11.62
GPR-squared exponential	-0.14	5.64	4.45	360	1.21
GPR-Matern 5/2	-0.15	5.65	4.44	350	1.2
GPR-exponential	-0.08	5.49	4.18	310	1.23
GPR-optimized	0.4	4.07	3.44	460	220

Table 4 Results of Gaussian regression models (model B) for training data

#### 4.2 Performance of best GPR model and other computational models with test data

New experimental data (test data) was used to evaluate the performance of the best models. Nine tests were carried out in the end-bearing condition and five tests in the floating condition to determine the models' workability. Fig. 6(a) illustrates the workability in the end-bearing condition. Model A showed acceptable accuracy for estimating the UBC in tests TS-3, TS-10, TS-11, TS-15, TS-16, and TS-22, with an average difference of approximately 15% between the experimental and estimated UBC. On the other hand, Model B performed well in tests TS-33, TS-34, TS-38, TS-41, and TS-44, with an average difference of less than 10% between the experimental and estimated UBC. These results demonstrate the proposed model's reliability and effectiveness when dealing with new data.

The best models' overall data percentage of error is depicted in Fig. 7. However, the models' primary drawback was observed in tests TS-26, TS-27, and TS-29 in the end-bearing condition, with an average difference of up to 57% between the experimental and estimated UBC, which is unacceptable. One possible explanation for this considerable difference could be attributed to the different geotechnical properties of the stabilized tests for undrained shear strength of soil and DCM columns compared to the other tests. The ranges of these values were dissimilar to those of the other tests, and the model could not predict with acceptable accuracy. Fig. 7 provides more details regarding these geotechnical properties.

Table 5 presents a comparison of the performance indices of GPR models with other computational estimation models. The results indicate that while the ANFIS model can still be utilized for UBC prediction in both the end-bearing and floating DCM column conditions, the optimized GPR model outperformed the MLP, RBF, and ANFIS models to a certain extent using the test data. These findings suggest that implementing the optimized GPR model could be effective for UBC prediction in both the floating and end-bearing conditions of improved ground.

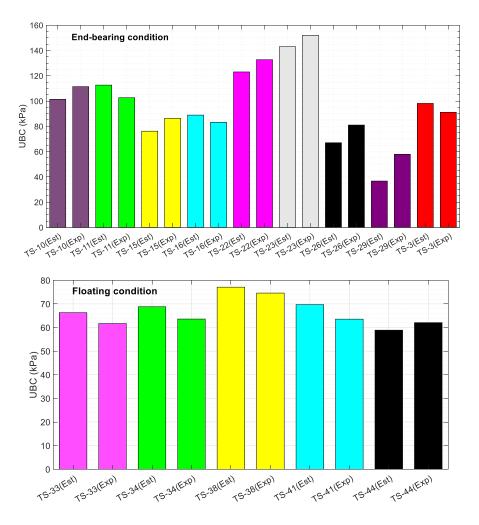


Fig. 6 Workability of best GPR models on test data: (a) end-bearing condition; (b) floating condition

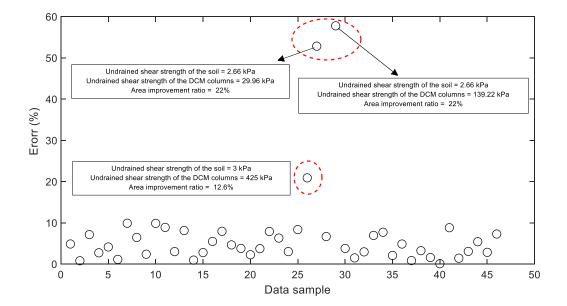


Fig. 7 Error for each test (%)

Model A	R	RMSE (kPa)	MAE	Model B	R	RMSE (kPa)	MAE
(end-bearing DCM)	K	KINDE (KFA)	(kPa)	(floating DCM)			(kPa)
GPR-rational quadratic	0.87	17.23	9.12	GPR-rational quadratic	-0.19	8.33	6.17
GPR-squared exponential	0.85	18.11	8.73	GPR-squared exponential	-0.17	8.3	6.12
GPR-Matern 5/2	0.88	16.2	8.23	GPR-Matern 5/2	-0.16	8.2	6.07
GPR-exponential	0.78	21.7	12.3	GPR-exponential	-0.11	8.12	5.7
GPR-optimized	0.93	9.12	6.7	GPR-optimized	0.22	6.5	4.12
MLP	0.73	22.3	13.5	MLP	-0.22	11.5	9.11
RBF	0.72	22.6	13.6	RBF	-0.2	11	8.93
ANFIS	0.81	17.12	8.8	ANFIS	-0.07	9.3	7.12

Table 5 Comparison of GPR models and other models with testing data

#### 6 A practical example

To evaluate the proposed models, a real case study was conducted on a low-weight wooden building located in Pontian, Johor, Malaysia, as shown in Fig. 9. These buildings are prevalent in the region, which is covered with various types of peat soil. The high organic content of such soft soil presents unique challenges in both geotechnical engineering and building construction. In Pontian, the peat soil depth can reach up to 10 m, with an average undrained shear of 10 kPa. Based on the Von Post classification system and in accordance with the degree of humification of the test samples, this peat can be categorized as H<sub>3</sub>. Fig. 8 shows that the peat soil in its unstabilized form could not tolerate the weight of a wooden building. Buildings constructed in this area have experienced differential settlement; thus, because of the geotechnical condition of the site, area of the building, applied vertical stress and undrained shear strength of cemented peat as a DCM column material, a stabilization program using model B (floating condition) was calculated based on Table 6. These calculations were based on the decision surfaces which were performed using the best GPR model, as shown in Fig .9. Table 6 presents the design results, which may be subject to changes depending on the availability of equipment and the discretion of the designer. These findings offer a new approach to designing DCM columns in peaty ground using an enhanced soft computing method.



Fig. 8 Typical low-weight wooden building located in Johor, Malaysia

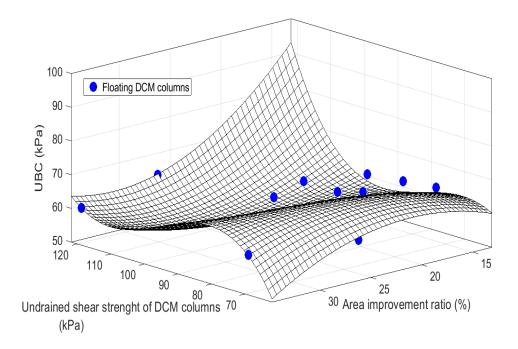


Fig. 9 Decision surface in floating condition

	Parameters and site condition		Design
٠	Building dimensions = $6 \text{ m} \times 12 \text{ m}$	•	Arrangement of DCM columns = rectangular
٠	Applied vertical stress to the soil = $20 \text{ kPa}$	•	Selected area improvement ratio = 19.6%
٠	Safe factor = $2$	•	Diameter of DCM columns $(D) = 1.5 \text{ m}$
٠	Soil classification = fibrous	•	Number of DCM columns = $8$
٠	Soil classification (Von Post) = $H_3$	•	Centre-to-centre in $x$ -dir = 1.5D
٠	Undrained shear strength of fibrous peat = 10 kPa	•	Undrained shear strength of each DCM columns = 100 kPa
٠	Average water content of the site = $495\%$	•	*Required cement content for DCM columns = $300 \text{ kg/m}^3$
٠	Void ratio = 11		(kg/m <sup>3</sup> : by mass of wet peat)
•	Organic content = 91%		
٠	Fibre content = $80\%$		(*Based on Dehghanbanadaki et al. 2019)
•	Bulk density (in situ) = $1 \text{ Mg/m}^3$		
٠	Specific gravity = 1.38		

### **6** Conclusions

This study aimed to develop an efficient model to estimate the UBC of soft ground improved with DCM columns. The research involved collecting data from 46 physical modeling tests in which soft clay and peat were improved with end-bearing (model A) and floating DCM columns (model B). The UBC of each model was determined through failure of the stabilized area under stress and strain conditions. Gaussian process regression (GPR) models with various kernel functions were created and optimized using Bayesian optimization. The optimized GPR model was then compared with three additional computational models (MLPs, RBFs, and ANFIS models) using test data. The inputs of the GPR models were the length of the DCM columns in the soil (floating condition), undrained shear strengths of the soil and DCM columns, and the area improvement ratio, with UBC chosen as the target. The optimized GPR model showed the lowest respective RMSE and MAE values of 7.13 kPa and 5.15 kPa in model A and 0.4 kPa and 4 kPa in model B, respectively. The results indicate that the proposed optimized GPR models have significant potential for estimating UBC in soft soil improved by end-bearing and floating DCM columns. To further discuss the limitations of the proposed model, it is important to note that the model was developed using a limited dataset of only 46 physical modeling tests. Therefore, the applicability of the model to a wider range of soil types and DCM column configurations is unclear. The model may not perform well when applied to different soil types or when different column configurations are used. Another limitation of the proposed model is the assumption that the soil and DCM columns have homogeneous properties. In practice, the soil and DCM columns can have heterogeneities that are difficult to capture in physical modeling tests. Therefore, the model's accuracy in predicting the ultimate bearing capacity of the soil and DCM columns with heterogeneous properties is unknown.

The current study provides a promising method for early prediction of stabilized ground UBC using GPR models. However, there are several future scopes and recommendations for other researchers to explore further.

- This study has only used a limited dataset of physical modeling tests. Future researchers could expand the dataset and include a wider range of soil and DCM column properties to further validate the proposed GPR model.
- The current study focused only on the estimation of UBC for end-bearing and floating DCM columns. Other geotechnical properties, such as settlement and lateral deformation, could also be estimated using GPR models.
- The decision-making model for the design of the geotechnical properties of the DCM columns could be expanded to include other factors, such as environmental and economic considerations.

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